ENERGY DISSIPATION CAPACITY OF STIFFENED STEEL
SYSTEMS UNDER HIGH-INTENSITY EARTHQUAKE

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SUMMARY

The behaviour of steel cross brace, subjected to a high-intensity quake is analysed here. A step-by-step computer program has been developed for this purpose, using a simulation model for the brace under traction or compression, based on experimental data.
An application to a realistic case is given, for several degrees of stiffness, in order to study the dissipation capacity of the system and to obtain indications on its structural ductility.
In conclusion, through energy-based considerations, an equivalent model of static energy sink is proposed, to be used for design purposes.

INTRODUCTION

Among steel frame structures the "pendular" type is much used because of the constructive simplifications that can be obtained through them. These structural systems have however been often criticized because they rely exclusively on the plastification of the diagonal braces, subjected to alternative axial stresses, for the dissipation of energy during high-intensity quakes.
As a consequence high resistance joints must be envisaged, that can transmit entirely the forces to the braces, avoiding local brittleness, as well as slender braces, that have been found (Ref. 1) to behave in a ductile manner.
Theoretical studies and investigations on models (Ref. 2), (Ref. 3) have been carried out and aimed mainly at analysing the behaviour of the joints, both of bolted and of welded construction - for the latter special constructive techniques must be used - with attention to their stability and ductility.
It appears that only a small number of authors (Ref. 4), (Ref. 5), have examined the dissipation capacity of the braced system, and they have done this on the basis of the incomplete information hereto available.
As a consequence this type of system is considered suitable only for structures having a limited number of storeys.
The energetic aspects of the behaviour of these structures have therefore been investigated here with the aim of developing criteria for dimensioning the brace during the design stage.

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The model of behaviour proposed in Ref. 6, has been found to be in accordance with the actual behaviour and has been adopted here.

MODEL OF BEHAVIOUR AND COMPUTER PROGRAM

Fig. 1 shows the stress-strain diagram for the two diagonal braces based on the adopted model. The characteristic points and the angles $\alpha_1$, $\alpha_2$ and $\alpha_3$ are a function of the slenderness.

![Diagram](image)

**FIG. 1**

![Diagram](image)

**FIG. 2**
According to Ref. 6 the following values have been adopted, resulting from experimental analyses on full-size models, according to diagram shown in Fig. 2, and to Table I.

<table>
<thead>
<tr>
<th></th>
<th>$\lambda = 100$</th>
<th>$\lambda = 150$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tan \alpha_1$</td>
<td>0.0175</td>
<td>0.015</td>
</tr>
<tr>
<td>$\tan \alpha_2$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$X_3$</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_C/F_y$</td>
<td>0.4±0.5</td>
<td>0.25±0.3</td>
</tr>
</tbody>
</table>

Table I

During one half-cycle the resultant stiffness is the sum of the actions of the brace under traction and the one under compression, according to a schematization of the type illustrated in Fig. 3. Which shows one of the possible combinations obtainable from the relative positions of $X_1$ and $Y_1$. Since the abscissae $X_2, X_3, Y_2, Y_3$ and $Y_4$ can be calculated once $X_1$ and $Y_1$ and the $X_r$ relative to the previous half-cycle are known, the shape of the two diagrams and their relative positions are completely defined.

This allows the calculation of the overall stiffness of the system in the different sections. Obviously the $X_r$ for each half-cycle depends on the seismic input and is determined by the change of sign of the velocity.

Once the stiffnesses are known and the equation of motion at the time $i$ is defined one obtains the system of equations

\[
\begin{align*}
F_1(x_i, \dot{x}_i, \ddot{x}_{i-1}, \Delta t) &= 0 \quad (1) \\
F_2(\dot{x}_i, \ddot{x}_i, \dddot{x}_{i-1}, \Delta t) &= 0 \quad (2) \\
F_3(x_i, \dot{x}_i, \dddot{u}_i) &= 0 \quad (3)
\end{align*}
\]

that can be solved once the values of $\Delta t$, $\dddot{u}_i$ (soil acceleration) and the vector

\[
(i - 1) = \begin{bmatrix} x_{i-1} \\ \dot{x}_{i-1} \\ \dddot{x}_{i-1} \end{bmatrix}
\]

relative to the previous step have been assigned.

It can be seen that in $F_3$, which represents the equation of motion, $\dddot{x}_i$ is absent, since the damping action is considered negligible.

The above considerations have been introduced in a computer program which allows to follow the displacements, the velocities and the accelerations of the structure as a function of time, for single degree of freedom systems, as well as the stresses in each of the two diagonal braces, once the seismic input has been assigned.
As far as the values of $\Delta t$ are concerned, suitably small increments are chosen both for the elastic (Ref. 7), (Ref. 8), and for the plastic phase; this has required a preliminary phase of interpolation between successive values of the acceleration, dependent on the adopted seismic input.

**APPLICATION**

The procedure is applied to the configuration shown in Fig. 4; the system has been loaded with the acceleration curve recorded in Tolmezzo on May 5, 1976 for the component E-W, shows in Fig. 5.

![Fig. 4](image)

![Fig. 5](image)

This application is described in Ref. 1, where simplified simulation models were used for the behaviour of the diagonal braces. The analysis was carried out for the following values of the cross-section $A_d$ of the braces: 0.814 sq in. (5.25 cm$^2$), 0.387 sq in. (2.50 cm$^2$) and 1.24 sq in. (8.00 cm$^2$). Fig. 6 shows the displacements of the horizontal beam vs. time for the value of cross section 0.814 sq in.

![Fig. 6](image)

The maximum displacements are smaller for the larger sections, as already observed in Ref. 1 with simplified simulation models. This is shown in Fig. 8. Fig. 7 shown separately the cycle diagrams for each brace with $A_d = 0.814$ sq in.
FIG. 7
Fig. 9 shows the seismic energy for the different values of $A_d$.

### Equivalent Static Energy Sink

Assuming that the energy calculated for the three cases is absorbed during one displacement of the system (seismic braking), a quasi-static loading process can be envisaged, neglecting the modest contribution given by the brace working in compression. The diagonal brace under traction, after its elastic deformation, will undergo a plastic deformation that will dissipate the energy that has been calculated for each system, as shown in Fig. 10.

Indicating with $E_{el}$ the maximum elastic energy for the considered system and with $E_p$ the energy that must be dissipated by the plastic deformation, the structural ductility can be defined as

$$\Delta = \frac{E_{el} + E_p}{E_{el}} \quad (4)$$

For the three cases considered here one obtains the values given in Table II. One observes that the ductility required from the "equivalent static energy sink" decreases for increasing values of $A_d$, in spite of the fact that more rigid systems must dissipate larger quantities of energy (Fig. 9).

This is in accordance with the observation that for sufficiently large values of $A_d$ one has $\Delta = 1$. $E_m$ being the energy induced in the system for the medium strength quake considered in the standards, it can be assumed that an $n$ times stronger quake will produce the energy $E_n = n^2 E_m$; consequently, indicating with $\alpha$ the plastic threshold, i.e. the ratio of stress for a medium quake over $F_y$, one obtains

<table>
<thead>
<tr>
<th>$A_d$ (cm$^2$)</th>
<th>$E_{el}$ (J)</th>
<th>$\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>285</td>
<td>37</td>
</tr>
<tr>
<td>5.25</td>
<td>599</td>
<td>31</td>
</tr>
<tr>
<td>8</td>
<td>912</td>
<td>25</td>
</tr>
</tbody>
</table>

Table II
\[ \Delta = \frac{n^2}{\alpha^2} + 1 \]  

(5)

This relation, pertaining to the schematization of the "equivalent static energy sink", enables one to design \( A_d \) through a static analysis based on the value of \( \Delta \) considered admissible, and on the anticipated value of \( n \). For the above example, assuming a seismic coefficient \( c = 0.1 \) and a structural coefficient equal to 1.2, the load in the diagonal brace, for a medium-strength quake will be \( N_m = 19.06 \text{ kips (84.82 kN)} \) and consequently \( \alpha = 1.825 \) \( A_d \). Assuming \( \Delta = 31 \) for \( A_d = 0.818 \text{ sq in.} \), as obtained before, one finds from equation (5) \( n = 8.22 \); for \( \Delta = 25 \) and \( A_d = 1.24 \text{ sq in.} \), one obtains \( n = 11.21 \). The case of \( A_d = 0.387 \text{ sq in.} \) is not significant, since it brings about \( \alpha < 1 \) and therefore is not in accordance with sound design practice with regard to the elastic phase.

It should be pointed out, as a confirmation of the validity of the proposed criterion, that the values of \( n = 8 \times 11 \) found are very close to the maximum values of the acceleration response spectrum for the considered quake and for small natural periods of vibration, like those found in the application described here.

CONCLUSIONS

The present investigation yields useful informations on the dissipation behaviour of the considered system. Using slender and therefore sufficiently ductile braces, the plastification of the tensed diagonal can absorb a relevant quantity of energy during each cycle. However, when the braces are suitably dimensioned, as they can be by using the criterion proposed here, the maximum displacements can be kept within limits that are acceptable in consideration of the properties of the material.

The energy transmitted by the quake to the system does not increase proportionally with the stiffness, while the energy that can be dissipated plastically with the same \( \varepsilon_{\text{max}} \) does; consequently stiffer systems have a larger reserve of ductility, although they are more affected by quakes. Economic considerations suggest however to use an optimization procedure for the design of the braces.

The simulation model used here is in close accordance with the actual behaviour and gives more weight to ductility them to resistance under compression of the braces, thus excluding the possibility of using a single diagonal brace; this is clearly shown by the following results obtained from the application illustrated above for a single brace with \( A_d = 0.814 \text{ sq in.} \) (5.25 cm²): number of cycles = 14, \( \varepsilon_{\text{max}} = 2.60\% \), corresponding to an horizontal displacement of the beam equal to 8.70 in. (22.09 cm).

A complete experimental verification is considered necessary in order to evaluate the fatigue damage, although the number of cycles obtained is small. Any damage to the section due to excessive compressive strain will reduce the stability and the dissipation capacity of a brace during the following cycle.
(Ref.4), (Ref.5), (Ref.2); this must be considered when estimating the energy that can actually be dissipated. Another important aspect that should be examined is the P-Δ effect, that weakens further the hysteresis cycles and that has been hereto considered negligible for small displacements. On the basis of these considerations, the authors plans to extend this investigation to different seismic inputs and to different values of the slenderness of the diagonal braces in order to develop a design criterion based on the proposed model of the "equivalent static energy sink" and suitable for multistorey systems.

REFERENCES