AN ACCURATE ESTIMATION OF THE FUNDAMENTAL PERIOD OF REGULAR TALL BUILDINGS

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SUMMARY

The modeling of a building structure by a continuum gives a fast and sufficiently accurate method for the determination of the fundamental periods. The stiffness properties of every structural element are taken into account as well as the mass of the building and the foundation rotation. The procedure is explained and the results of its application to four chilean tall buildings are compared with those obtained by a matrix modal analysis, by using transfer matrices and by the formula of the Uniform Building Code. A different approach for buildings without shear walls is also shown.

INTRODUCTION

A method for the approximate dynamic analysis of a building structure based on its modeling by a continuum has been already reported (Ref.1). This model takes into account variations in the story heights, in the stiffness of the structural elements and in the mass per unit height. The solution is obtained by using the technique of transfer matrices and it can be processed by a microcomputer. In the case of regular buildings the solution is obtained from graphs and tables and the fundamental periods of the building in two directions can be found by hand calculations in about three hours with a good accuracy.

The main hypothesis is that the shear force in any vertical substructure j such as a wall or a rigid frame is of the form $Q_j(z) = C_{1j}y'(z) - C_{2j}y'''(z)$, where C_{1j} and C_{2j} are stiffness coefficients. Fig.1 shows the coordinate system. The equilibrium equation at any level requires that the total shear force of the building at that leves be $Q(z) = C_1y'(z) - C_2y'''(z)$, where $C_1 = \Sigma C_{1j}$ and $C_2 = \Sigma C_{2j}$.

If the structure vibrates in a normal mode, the deflection is given by $y(z)\ \sin(\omega t + \beta)$, where $y(z)$ is the mode shape, ω the natural frequency of the mode and β a phase angle. By derivation with respect to z and with the substitution of $-Q'(z)$ by the inertial forces per unit height $\mu\omega^2y(z)\sin(\omega t + \beta)$, μ beeing the mass per unit height, the equation for the normal mode is obtained. After eliminating the time factor and with the change of z by the non dimensional variable $s = z/H$, where H is the total height of the building, this equation becomes

$$y^{IV}(s) - \alpha^2y''(s) - \delta^2y(s) = 0$$

with the boundary conditions $y(0) = 0$, $y'(0)$, $y''(1)$ and $Q_1 = C_1y'(1) - C_2y'''(1) = 0$. α is a parameter of the structure and δ a frequency factor that is a function of α. Their values are given by the relations

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$$\alpha^2 = C_1 H^2/C_2 \qquad\qquad \delta^2 = \omega^2 \mu H^4/C_2$$

The inclusion of the foundation rotation leads to the same equation with a second member that is not zero; the boundary conditions are also different but the solution is similar. A second parameter ε appears, expressed by

$$\varepsilon = C_2/k_\phi I_F H$$

This model requires that the foundations are equivalent to one single foundation. I_F is the inertia of the contact area about an axis through its centroid that is transverse to the direction of motion. k_ϕ is the elastic modulus of the soil for rotation of the foundation about an horizontal axis under a dynamic overturning moment: the product $k_\phi I_F$ is equal to the ratio overturning moment: angle of rotation in radians. The graph of Fig.2(Ref.2) gives the frequency factor δ_1, that corresponds to the frequency ω_1 of the first mode of vibration, in function of the two parameters α and ε. The curve $\varepsilon = 0$ means that the soil is infinitely stiff and no rotation occurs. The four buildings that will be mentioned in this work, founded on a dense gravel, have values of ε in the range 0.01 to 0.03.

The curves of Fig.2 were obtained for $C_1(z)$, $C_2(z)$ varying with the law $C_i(z) = C_i(9/8 - z/4H)$, $i = 1$ or 2, and for μ constant. Our tall buildings in Chile usually have a width of walls and beams that is staggered in the height of the building. One extreme case is that of constant width. The opposit extreme case has widths at the top that are 50% of the width at the base. The formula above mentioned can be used for all buildings in this range with an error less than 5%.

DETERMINATION OF THE STIFFNESS CONSTANTS C_1j and C_2j

An uncoupled wall has a shear $Qj(z) = -EI_j y'''(z)$, so $C_{1j} = 0$ and $C_{1j} = EI_j$, the product of the modulus of elasticity and the inertia of the cross sectional area about a transverse axis through its centroid.

Fig.3 shows two coupled walls. The main hypothesis above mentioned requires that the walls deflect only by bending moments. No deformations under axial load or shear forces can be considered. By accepting this, the relative displacement of the end sections of a beam are Ly', as shown in the figure. L is the distance between the axis of the walls. The shear force T in the beam is kLy', k beeing the beam stiffness. The counter-flexure point of the beam is located at a distance L_a from the axis of wall "a"; the beam acts with a bending moment $TL_a = kLL_a y'$ on the wall, with a sign that is opposite to that of the bending moment originated by the lateral forces acting on the wall. The bending diagram of the wall due to the actions of beams is a staggered one, but in the modelation by a continuum it becomes a continuos line with a slope $-(kLL_a/h)y'$, where h is the story height.

By deriving the equation $-EI_a y''(z) = M(z)$, the equation $-EI_a y'''(z) = Q_a(z) - (kLL_a/h)y'$ is obtained and it is readily seen that $C_{1a} = kLL_a/h$ and $C_{2a} = EI_a$. When two beams are connected to the wall, one to each side, their effects are added in the case of C_{1a}, that is now expressed by a sum.

Beams that arrive at right angle to the plan of the wall may add new terms to C_{1a}. Figures 4 and 5 show two of such cases and are self-explaining. In Fig.4, wall "a" has a $C_{1a} = 2kLL_a/h$ and each of the walls "b" a $C_{1b} = kLL_b/h$. In the case of Fig.5, $L = L_a - L_b$; each of the walls "a" have a $C_{1a} = kLL_a/h$ and each of the walls "b" a $C_{1b} = -kLL_b/h$. The sign minus is because the action of the beam charges the walls "b" down, acting with a bending moment that has the same sign than that originated by horizontal loads.

A column c whose ends have a relative interstory displacement Δ takes a shear force $Q_c = K_c\Delta$. The stiffness K_c may be computed for instance by Dr. Muto's method in order to include the effect of end rotations, stiff end sections and shear deformation. If $Q_c = K_c h\Delta/h$ is replaced by $Q_c = K_c hy'(z)$, it is found that $C_{1c} = K_c h$ and $C_{2c} = 0$. The sum of the stiffness coefficients C_{1c} over all the frame columns gives the value of C_{1j} for the frame j.

If the building is regular, as those shown in Fig.6,7 and 8, C_1 and C_2 may be variable in the height. Their values can be averaged by $C_1 = (\Sigma C_{1k}h_k)/H$, $C_2 = (\Sigma C_{2k}h_k)/H$, where C_{1k}, C_{2k} are the values at a story k and h_k the story height. Then the value of α and ε are calculated and the graph of Fig.2 gives δ_1. With δ_1, the determination of ω_1 and $T_1 = 2\pi/\omega_1$ is inmediate.

APPLICATION OF THE METHOD TO TALL CHILEAN BUILDINGS

Four tall reinforced concrete chilean buildings of 21 and 22 stories were analized in the two main directions, (Ref.3), by three methods. No foundation rotation was considered in this study. The results appear in table N°1, where (M) means matrix modal analysis, (T) means approximate analysis by using transfer matrices, (G) means approximate determination of periods by the graph of Fig.2 and (UBC) means the period determined by UBC formula $T = 0.05\ H/\sqrt{D}$, D beeing the lenght of the building in the direction of the movement, H and Δ beeing expressed in feets.

Building N°4 is not shown and resembles building N°2. From the tables it can be seen a good agreement of the procedure (G) with the matrix analysis (M); the use of transfer matrices is even closer, with few exceptions. It seems not fair to compare this results with the UBC formula, apparently chilean tall buildings are stiffer than those in California. In any case a procedure that takes into account the stiffness of every structural element seems more reliable that empirical formulae that may have large dispersions in their results.

Building N°4 was tested by forced vibration, finding the periods $Tx = 0.69$ sec, $Ty = 0.95$ sec, after the construction was finished and the building was empty. The computed values (M), (T) and (G) suppose a liveload equivalent to a 7% of the dead load, so the experimental values should be increased in about 3,5% before comparing them with those in the table.

The assumption that the walls are not extensible under axial loads introduces an error by supposing a stiffer structure than the actual one. This error is small for $C_1'H^2/C_2 < 3$, where C_1' is the component of C_1 due to the action of beams on walls. Anyhow, this error is in the safe side in the computation of the base shear.

ESTIMATION OF THE FUNDAMENTAL PERIOD OF BUILDINGS COMPOSED BY RIGID FRAMES

If there are no walls, $C_2 = 0$ and the equation of normal modes and the boundary conditions change. The problem is equivalent to that of a shear beam. By using the average value of C_1 in the height of the structure, the fundamental period T is given by

$$T = 4\sqrt{\frac{\mu H^2}{C_1}} \qquad \text{(a)}$$

In a seven story high reinforced concrete building, this formula was checked against (b) matrix modal analysis (c) Rayleigh method, (d) UBC formula $T = 0.1\,N$, where N is the number of stories, (e) ATC-3 formula $T = 0.08\,N \pm 0.02\,(N-3)^2$ in a preliminary version and (f) ATC-3 formula $T = 0.025\,h_n^{3/4}$, where h_n is the height of the building in feets. The results are shown in the table N°2.

TABLE N°1: DIMENSIONS AND FUNDAMENTAL PERIODS OF FOUR BUILDINGS

Note: In the drawings, the horizontal axis of the plan is designed with x, the vertical one with y. Tx and Ty are the fundamental periods in these directions.

Building N°	1	2	3	4
H(m)	63.04	57.10	59.47	57.65
D_x(m)	47.50	20.80	23.46	23.30
D_y(m)	29.30	24.70	23.00	19.92
T_x(M)	0.45 sec.	0.81 sec.	1.16 sec.	0.74 sec.
T_x(T)	0.45 sec.	0.82 sec.	1.11 sec.	0.77 sec.
T_x(G)	0.48 sec.	0.82 sec.	1.10 sec.	0.81 sec.
T_x(UBC)	0.81 sec.	1.13 sec.	1.12 sec.	1.08 sec.
T_y(M)	0.55 sec.	0.83 sec.	0.70 sec.	1.13 sec.
T_y(T)	0.55 sec.	0.76 sec.	0.70 sec.	1.12 sec.
T_y(G)	0.66 sec.	0.78 sec.	0.70 sec.	1.14 sec.
T_y(UBC)	1.03 sec.	1.04 sec.	1.03 sec.	1.17 sec.

TABLE N°2: FUNDAMENTAL PERIODS OF VIBRATION T(sec.)

Method	(a)	(b)	(c)	(d)	(e)	(F)
T_x	0.715	0.75	0.725	0.70	0.88	0.56
T_y	0.89	0.98	0.93	0.70	0.88	0.56

REFERENCES

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2. Castro, René, Civil Engineering Thesis, Dept. of Civil Engineering, Univ. of Chile, 1980.

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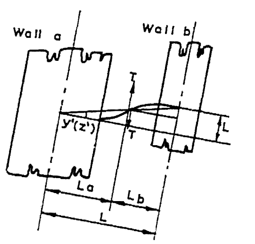

FIG.3 COUPLED SHEAR WALLS

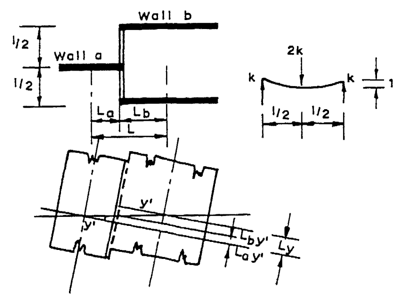

FIG.4 BEAMS NORMAL TO THE WALLS

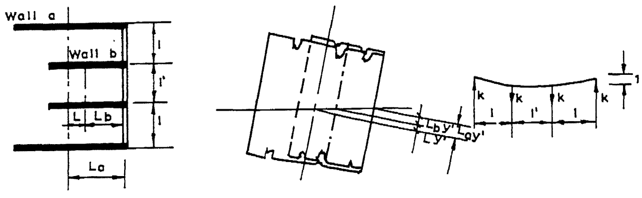

FIG.5 BEAMS NORMAL TO THE WALLS

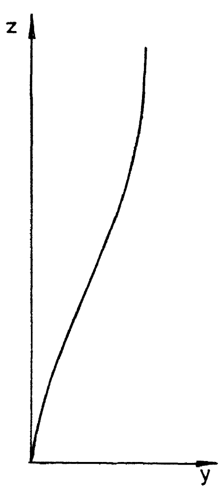

FIG. 1

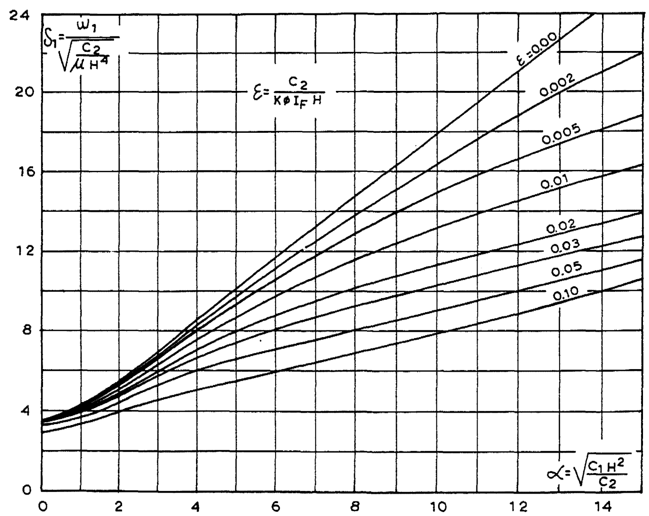

FIG. 2 FREQUENCY FACTOR δ_1

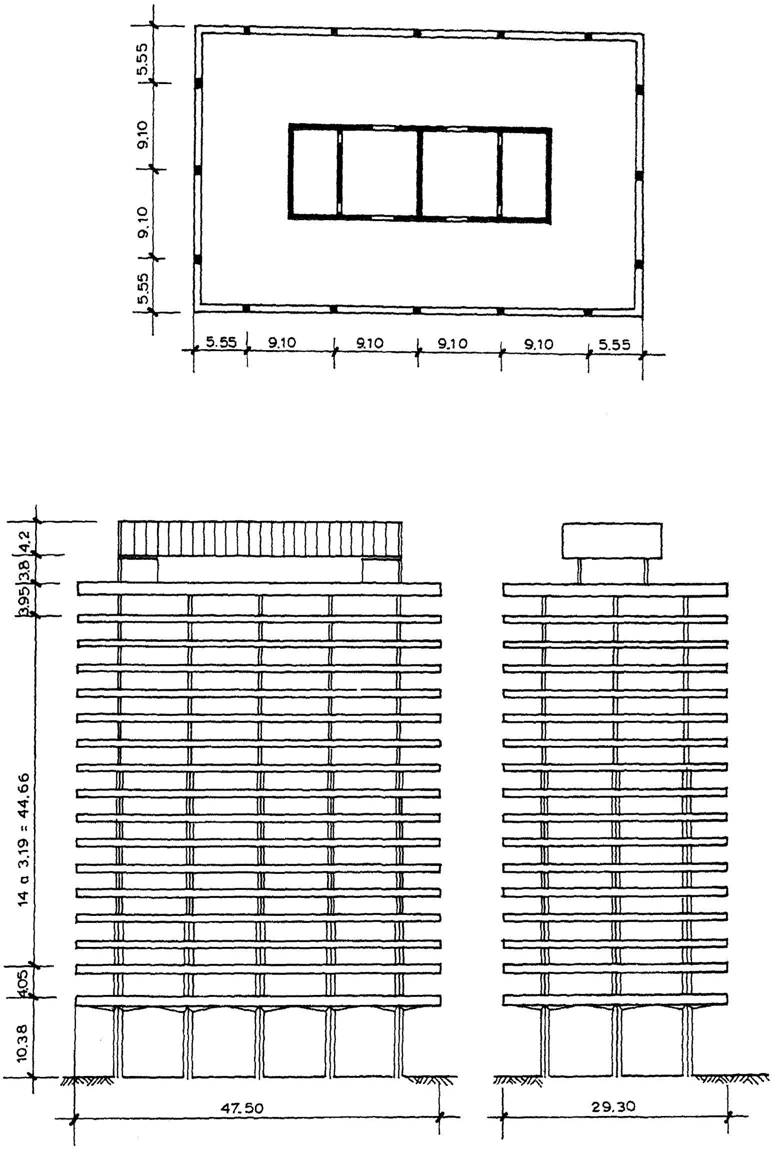

FIG. 6 BUILDING N°1 PLAN AND ELEVATIONS

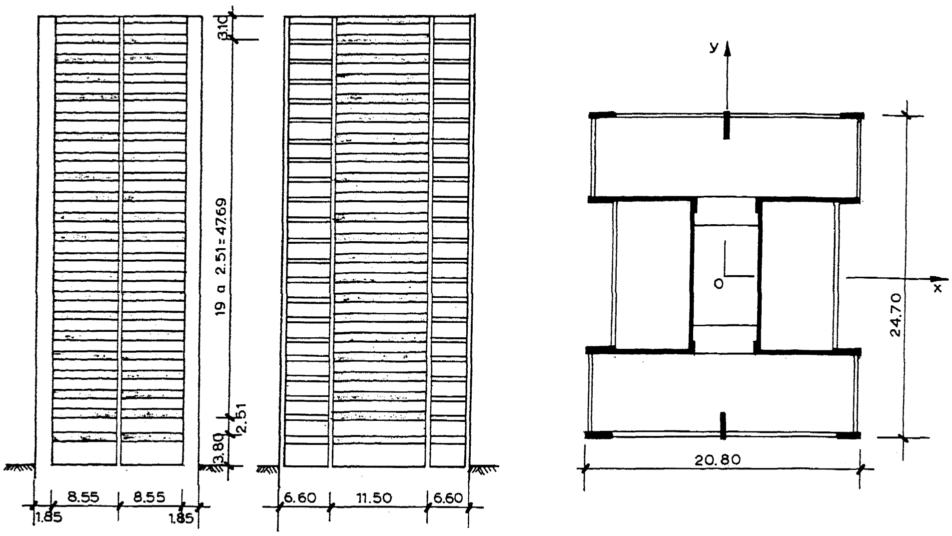

FIG. 7 BUILDING N°2 PLAN AND ELEVATIONS

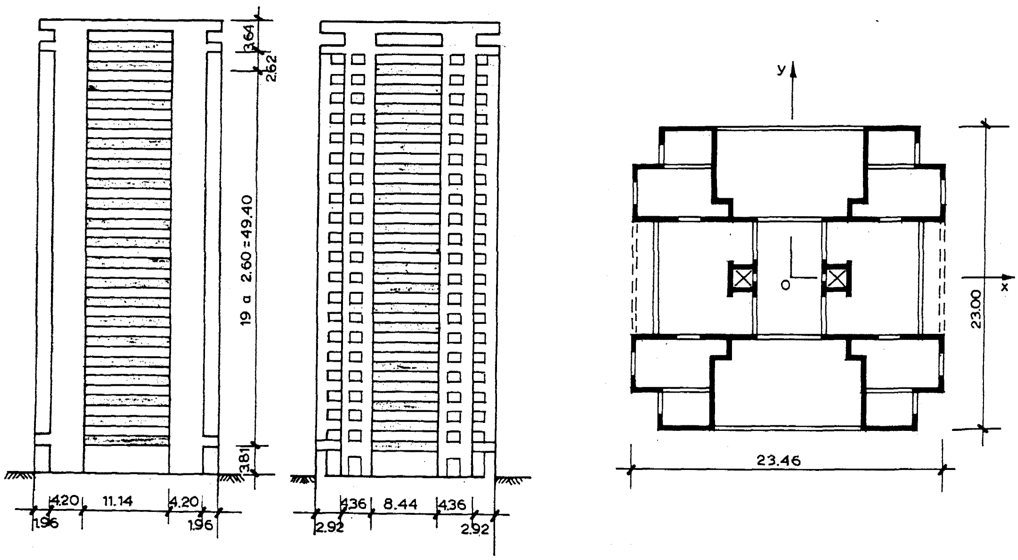

FIG. 8 BUILDING N°3 PLAN AND ELEVATIONS