DYNAMIC PROPERTIES OF SATURATED SAND UNDER CYCLIC LOADING CONDITIONS
(MODELING OF SHEAR STRESS-STRAIN RELATIONS)

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SUMMARY

On the basis of cyclic loading test results on saturated sand, the authors devised the shear stress-strain relations model of saturated sand in which the reduction of shear resistance by cyclic loading were considered. To verify this model was good or not, comparison was made between test results and calculated ones using this model when inputting real seismic waves to specimens. As a result, it became clear that this model could express dynamic behavior of saturated sand under cyclic loadings fairly well.

INTRODUCTION

On the assumption that a large scale earthquake occurs at sandy grounds, the authors have made an attempt at modeling the shear stress-strain relations of saturated sand in the process of liquefaction which is to be used for earthquake response analysis of the structures on sandy grounds. This time, in order to investigate the effects of the confining pressure, relative density, principal stress ratio, shear strain-history, etc. on dynamic properties of saturated sand, dynamic torsional shear test was performed using sinusoidal and random waves.

TEST SUMMARY

The test was performed using torsional cyclic loading triaxial apparatus. Specimens were 13.5cm height of hollow cylinder having outside diameter of 7.0cm and inside diameter of 3.0cm. Materials used in test were Toyoura Sand and Niigata Sand, and relative density(Dr) were 50% and 70%. Specific gravity (Gs), uniformity coefficient(Uc), the mean of grain size(D50), void ratio(e) and grain size distribution curves of these sand are shown in Table 1 and Fig. 1.

Test Procedures

After consolidating specimen at two kinds of principal stress ratio(\(\sigma_3/\sigma_1=0.5, 1.0\)), cyclic loading test was performed under undrained and strain controlled conditions using three types of waves as described later. The initial effective confining pressures(\(\sigma_0^'\))

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are 0.5, 1.0 and 2.0 kgf/cm² in case of Dr=50%, 0.5, 1.0, 2.0 and 4.0 kgf/cm² in case of Dr=70%.
The back pressures are 0.5 kgf/cm² in all cases.
Input waves are as follows:
Type I ---- The sinusoidal waves, which were kept
shear strain amplitude(Yd) constant,
were input 30-50 cycles continuously at
the frequency of 1Hz.
Type II ---- Waves which were made by connecting
the sinusoidal waves of different Yd
were input. As shown in Table 2, these
were 9 kinds in all. The typical exam-
ple is shown in Fig. 2.
Type III ---- Waves which were transformed from the
accelerogram of EL CENTRO and TOKACH-
OKI earthquake into displacement were
input. Time history of these waves are
shown in Fig. 3. Input level were
controlled by Ymax in these figures.

TEST RESULTS

τ-γ Relations

Fig. 4 shows an example of τ-γ hysteresis loops which were combined them at each Yd using
Type I. Fig. 5 shows skeleton curves which were
jointed the vertex of the first cycle at each Yd
Fig. 6 shows an example of τ-γ hysteresis loops
using Type II. Fig. 7 shows the mean of positive
and negative vertex of the first and last cycle
at each Yd, which correspond to Fig. 6. The arrow
head in Fig. 7 show loading sequence.

τ-γ Skeleton Curves

The relations between τ/(cm')0.9 (τ in Fig. 4
divided by (cm')0.9) and γ are shown in Fig. 8.
According to this figure, in case of the same Dr,
the relations between τ/(cm')0.9 and γ can be ex-
pressed by Eq. 1.

τ = v(γ)w(cm')0.9 (1)

where v, w = the parameters which depend on
properties of materials and Dr.

The Effects of Number of Cycles on τ-γ Relations

In case of loading by Type I, the reduction
ratio of shear stress amplitude(τd) with increase
of number of cycles(N) increase as Yd increase.
Fig. 9 shows the relations between the ratio of
shear stress amplitude of the Nth cycle to the
first cycle(τN/τ1) and N. According to Fig. 9, in
case of $N=1.5$ and $\gamma_d/\tau_1>0.2$, $\tau/\tau_1$ are nearly proportional to $\log N$. The gradients of straight lines (m) are different with materials, $\sigma_m$, $\tau$ and $\gamma_d$, however, in case of not having been affected by shear strain-history, $m$ are approximately proportional to $\log \gamma_d$ as shown in Fig.10. Thus, $m-\gamma_d$ relations can be expressed by Eq. 2.

$$m = A \cdot \log \gamma_d + B \quad (2)$$

On the other hand, Fig.11 shows the relations between the parameters $A$, $B$ and $\sigma_m$ on Toyoura Sand, $\log A$ and $\log B$ are approximately proportional to $\log \sigma_m$. The same result was obtained on Niigata Sand. It follows from above that the relations between $A$, $B$ and $\sigma_m$ can be approximated by Eq. 3.

$$A = a_1 \cdot (\sigma_m')^a_2$$
$$B = b_1 \cdot (\sigma_m')^b_2 \quad (3)$$

where $a_1$, $a_2$, $b_1$, $b_2$ = the parameters which depend on the properties of materials and Dr.

The Effects of Shear Strain-History

The $\gamma-\tau$ relations are very different with dimensions of previous $\gamma_d$ and their number of cycles even if the same materials, $\tau$ and $\sigma_m$. In a similar manner, the reduction ratio of shear stress amplitude ($m$) which correspond to $\gamma_d$ are different with the type of shear strain-history. Still more, in case of having been subjected to larger strain $\gamma_i$ than $\gamma_d$, $\gamma_d$ and $m$ are lower in comparison with them not having been subjected to $\gamma_i$ even if the same $\gamma_d$. In case of cyclic loadings at the same $\gamma_d$, $m$ using Type I are the greatest, and they decrease as $\gamma_d/\gamma_i$ and $\tau_d/\tau_i$ become lower. This fact, as is evident from shown later in Fig.14-16, is due to pore water pressure just before cyclic loading at arbitrary $\gamma_d$ are different with $\gamma_d$ and their number of cycles which specimens were subjected to previously. In other words, $\gamma_d$ and $m$ at arbitrary $\gamma_d$ depend on the values of $u/\sigma_m'$ and they become lower as the values of $u/\sigma_m'$ increase even if the same $\gamma_d$.

The Shapes of Hysteresis Loops

The shapes of hysteresis loops change from spindle into slip form with the reduction of shear resistance and strain-hardening phenomenon occurs, which is caused by the positive dilatancy of soil particles and becomes more remarkable as $\sigma_m'$, $\tau$ and $\gamma_d$ increase.

The Rules of History

As shown in Fig. 9, in case of loading by Type I, the gradient of straight
lines(m') which connect $\tau_1(N=1)$ and $\tau_{1.5}(N=1.5)$; number of cycles when the hysteresis curve get to the vertex of negative region at the first cycle) are nearly equal (0.4 ~ 0.5)m. Therefore, if not having been affected by shear strain-history, the relations between $\tau_1$ and $\tau_N$ at the same $\gamma_d$ can be expressed by Eq. 4.

$$N = 1.5 \quad \tau_{1.5} = (1-m' \log 1.5) \tau_1$$
$$N \geq 2 \quad \tau_N = (1-m \log N + (m-m') \log 1.5) \tau_1$$

On the other hand, according to the test results using Type II, when shear strain amplitude varied to $\gamma_{d2}$ after loading $N$ cycles at $\gamma_{d1}$, the first hysteresis curve at $\gamma_{d2}$ proceeded to the vertex ($\gamma_{d1}, \gamma(N+1)$) of the $(N+1)$th cycle at $\tau_d$ (Fig.12)

Relations between Damping Ratio(h) and $\gamma_d$

Fig.13 shows the h-$\gamma_d$ relations using Type I at the first cycles. According to this figure, the maximum values of h are hardly different with Dr and $\sigma_m'$. They range from 0.38~0.42, and even if $\gamma_d$ increase, h don't exceed their values. The range of shear strain where h are affected by $\sigma_m'$ and Dr are the range where h are in less than the maximum value. On the other hand, as shown in Fig.13(b), h decrease with the reduction of shear resistance by cyclic loadings, and they approach 20% ultimately. That is to say, in case of loading $N$ cycles at arbitrary $\gamma_d$, h can be approximated by Eq. 5.

$$h = (h_2-0.2) \cdot \tau_d/\tau_s + 0.2 \quad \text{(5)}$$

where $h_2$=damping ratio at $\gamma_d$ which have not affected by cyclic loadings.

$\tau_s$=shear stress on the skeleton curve at $\gamma_d$

The Tendencies of Pore Water Pressure Build-up

The typical tendencies of pore water pressure (u) build-up are shown in Fig.14~Fig.16. In case of loading by Type I, u build-up become more remarkable as $\sigma_m'$ and Dr become lower and $\gamma_d$ become larger. And the fluctuation amplitude of u increase as $\sigma_m'$, Dr and $\gamma_d$ increase. But, the gradients of $u$ build-up and the fluctuation of $u$ decrease gradually as $u$ build-up. In a similar manner, in case of loading by Type II and Type III, the gradients of $u$ build-up and their fluctuation amplitude increase as $\gamma_d$ increase, and they decrease as $u$ build-up. That is to say, in case of cyclic loading at the same $\gamma_d$, $u$ build-up and the fluctuation amplitude of $u$ depend on $u$ which accumulated previously. A noteworthy point is that $u$ have twice as much frequency as $\tau$ and $\gamma$. At each cycle except
the first cycle, when \( \gamma \) and \( \tau \) get to positive and negative vertexes \( u \) become the lowest, and when \( \tau \) become nearly zero \( u \) become the greatest.

**The Effects of The Confining Conditions**

According to Fig. 5, Fig. 10, Fig. 13 and Fig. 16, \( c_m' \) influence on the \( \tau - \gamma \) skeleton curves, the \( m - \gamma_d \) relations, the \( h - \gamma_d \) relations and the tendency of \( u \) build-up remarkably. On the other hand, in case of the same \( c_m' \) even if the different principal stress ratio, it can be considered that there is no effect of them on dynamic properties. Excepting special ground as the ground in the vicinity of structure having spread foundation and slope, it is expected that \( \sigma_3 / \sigma_1 \) range from 0.5 to 1.0, so that it can be considered that the effects of confining conditions on dynamic properties can be evaluated by only \( c_m' \). Therefore, in the \( \tau - \gamma \) relations model as described later, the effects of principal stress ratio isn't considered.

**FUNDAMENTAL THEORY OF ANALYZED MODEL**

The model consists of the \( \tau - \gamma \) skeleton curves, \( \tau - \gamma \) hysteresis loops, \( h - \gamma_d \) relations, the reduction ratio of shear stress and the rules of history. In the sections that follows, the fundamental theory of this model is described.

**\( \tau - \gamma \) Skeleton Curves**

Shear stress\((\tau_0)\) on skeleton curves at arbitrary \( \gamma \) are expressed by Eq. 6.

\[
|\tau_0| = \nu \cdot |\gamma| \cdot (c_m')^{0.9} \quad (6)
\]

where \( \nu, w \) = the parameters which depend on the properties of materials and \( \text{Dr} \).

In case of \( \text{Dr}=70\% \) on Toyoura Sand, \( \nu=2.08 \) and \( w=0.28 \). Substitution of these values into Eq. 6, the skeleton curves correspond to \( c_m' = 0.5, 1.0, 2.0, 4.0 \) kgf/cm\(^2\) are shown in Fig. 17.

**\( \tau - \gamma \) Hysteresis Loops**

As shown in Fig. 18, in case of assuming arbitrary two vertexes\((\gamma_1, \tau_1), (\gamma_2, \tau_2)\) of hysteresis loop, the \( \tau - \gamma \) relations of any point on this loop are expressed by Eq. 7.

\[
\frac{\tau - \tau_0}{\tau_1 - \tau_0} = a \left( \frac{y - \gamma_0}{\gamma_1 - \gamma_0} \right)^4 + b \left( \frac{y - \gamma_0}{\gamma_1 - \gamma_0} \right)^3 + \frac{\gamma - \gamma_0}{\gamma_1 - \gamma_0} \quad (7)
\]

\[\gamma_0 = (\gamma_1 + \gamma_2) / 2, \quad \tau_0 = (\tau_1 + \tau_2) / 2\]
The parameters $\alpha$ stands for how a swell of curves and can be expressed by Eq. 8.

$$\alpha = 15 \cdot \pi \cdot h / (24 \pi)$$  ----- (8)

$(1-\beta)$ stands for the gradient ratio between tangent which touches the hysteresis loop at the point corresponds to $Y_d$ and straight line which connects two vertexes in hysteresis loop. On the basis of test results, the parameter $\beta$ can be expressed by Eq. 9 as a function of $Y_d$.

$$\beta = 0.8 - (2.4 \times 10^{-3}) / (Y_d + 3.0 \times 10^{-3})$$  ----- (9)

$\gamma$-y Relations

Damping ratio which have not been influence by cyclic loadings are named $h_p$. $h_p-Y_d$ relations are expressed by Eq.10.

$$Y_d \times 10^{-4} = h_p = 0.05$$  
$$1 \times 10^{-4} \leq Y_d \leq Y_k$$  
$$h_p = 0.4$$  ----- (10)

$Y_k$ in Eq. 10 stands for shear strain amplitude when $h_p$ getting to the maximum value $= 0.4$, and is given by Eq.11.

$$Y_k = Y_{h1} \cdot (\sigma_m)^{0.5}$$  ----- (11)

where $Y_{h1}$ correspond to $\sigma_m = 1.0$ kgf/cm$^2$

The $h_p-Y_d$ relations on Toyoura Sand (Dr=70%) are shown in Fig.19. On the other hand, $h$ which have been influence by cyclic loading are expressed by Eq. 5 as a function of $h_p$ and $t_2/t_1$. However, in case of $h_p > 0.2$, it is assumed that $h = h_p$.

The Rules of History

In case of assuming that the hysteresis loops proceed to next vertex by $Y_2, t_2$ after turning over arbitrary vertex by $Y_1, t_1$, $Y_1-Y_2$ relations and $t_1-t_2$ relations are expressed by Eq.12. (Strictly, their relations are modeled in twenty cases.)

$$Y_2 = -Y_1$$  
$$t_2 = -t_1 (1 - m \cdot \log 1.5)$$  ----- (12)

In the case that the shear resistance have not been reduced, in other words, if $t_1$ are situated on the skeleton curve, $m$ is calculated by Eq. 1 and Eq. 2. On the other hand, in case of the shear resistance having been reduced, $m$ in Eq.12 is revised by Eq.13 according to the value of $t_1/t_2$.
\[ m_1 = (1 - \frac{\log \tau_1/\tau_s}{1.3}) \cdot m \quad (13) \]

More over, in case of having been subjected to larger strain than \( \gamma_1 \), \( m \) is reduced 20-50% according to the value of \( \tau_1/\tau_s \).

COMPARISON BETWEEN TEST AND CALCULATED RESULTS

To verify whether this model is good or not, comparisons were made between test and calculated results on the value of shear stress response which were obtained from inputting the waves of Type III. Fig.20 and Fig.21 show the comparison on a part of \( \tau-\gamma \) hysteresis loops and on the shear stress response. According to these figures, there are little different between test and calculated results in the tendencies of shear stress reduction and in shapes of the hysteresis loops, however, it seems that this model can simulated the reduction properties of \( \tau \) by cyclic loadings as appears in test results. Please note that each variable number used for analysis is shown in Table 3.

EARTHQUAKE RESPONSE ANALYSIS

Earthquake response analysis were performed using this model on imaginary sandy ground. It was assumed that the sand deposit was 24m in depth and was 1.8 t/m³ in density, the layer from G.L. = 0m to = 16m was 50% in relative density, the layer from G.L. = 16m to = 24m was 70% in relative density, the ground water level was G.L. = 0m. The multi-shear-model was used for analysis and the deposit was divided into six layers (each layer was 4m in thickness). On the other hand, it was assumed that the viscous damping factor was proportional to instantaneous stiffness and its value was 0.02. Wilson's 8 Methd was used for calculation and integral interval was 0.01 of a second. The fundamental natural period of this deposit was 0.41 of a second. Taft earthquake was chosen as input accelerogram because this earthquake contained a component of a time period about 0.41 of a second. The maximum value of input acceleration at base-rock was established at 138gal so that the maximum response value might
be calculated about at 200gal at surface layer.

**The Calculated Results**

Fig.22 shows the response time history of shear stress, strain and acceleration at surface layer and input acceleration at base-rock. Fig.23 shows the response hysteresis loops at surface layer. According to these figures, the excellence of long time period component caused by the reduction of stiffness appears about 4 seconds later. The rapid increase of shear strain and the reduction of shear resistance occurred about 6-7 seconds later, it seems that liquefaction began. Fig.24 shows distribution of the maximum response value of $\gamma$, $\tau$ and acceleration. At surface layer, $\gamma$ became about $10^2$, liquefaction concentrated on this layer. The maximum response acceleration became 223gal, its response magnification became about 1.6.

**CONCLUSIONS**

As existing dynamics models on saturated sand, some effective stress methods for predicting pore water pressure build-up were proposed previously. The authors devised a new method for predicting the $\tau$-$\gamma$ relations using the reduction ratio of shear stress which were obtained from cyclic loadings test. It became clear that this model could express dynamic behavior of saturated sand under cyclic loadings fairly well and there is good prospect in using it for earthquake response analysis of structures on sandy ground. Hereafter, the authors will investigate the adaptability of this model using earthquake accelerogram observed.