PREDICTION OF VIBRATORY SHEAR MODULUS AND DAMPING RATIO
FOR COHESIVE SOILS

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SUMMARY

The purpose of this paper is to find out the relationships between the plasticity index and the shear modulus or the damping ratio for cohesive soils. A resonant-cyclic triaxial test apparatus has been developed to measure the shear modulus and the damping ratio for the wide range of strain amplitudes. Some results from laboratory experiments are compared with those from in-situ seismic survey. It is shown that the plasticity index is a very useful parameter in predicting the shear modulus and the damping ratio for cohesive soils.

INTRODUCTION

There are many works regarding the vibratory shear modulus and the damping ratio of soils. They are, however, mainly researches on the factors which affect the shear modulus and the damping ratio, such as the void ratio, the confining pressure and the consolidation time. In this paper, the shear modulus and the damping ratio for various types of cohesive soils are investigated in relation to the plasticity index; the effects of the plasticity index on (1) the shear modulus at low-strain amplitudes Go, (2) the stress-strain relationships in terms of the G/Go-Y curves, (3) the damping ratio h and (4) the time effects. The range of the plasticity index used in the study is between 0 and 87.

EQUIPMENTS AND TEST PROCEDURE

The laboratory experiments are performed with both the resonant column technique and the cyclic triaxial test technique. The former is used to apply low-strain amplitudes. On the other hand, the later is for high-strain amplitudes. It is, however, not so efficient preparing two specimens for tests by both techniques that a resonant-cyclic triaxial test apparatus has been developed to measure the shear modulus and the damping ratio for the wide range of strain amplitudes with one specimen in the triaxial cell. Fig. 1 shows the schematic diagram of this apparatus. The torsional vibration in the resonant column technique and the axial cyclic strain in the cyclic triaxial test are applied to specimens. The shear strain amplitudes in the resonant column tests are determined from the mean shear strain of the specimen, whereas in the cyclic triaxial tests those are determined from Eq. (1). The shear modulus is calculated by Eq. (2) assuming that the Poisson's ratio is equal to 0.45.

\[ Y = e \cdot (1 + \nu) \quad \ldots \ldots (1) \]
\[ G = E / 2 \cdot (1 + \nu) \quad \ldots \ldots (2) \]

where,
- \( Y \): shear strain amplitude
- \( G \): shear modulus
- \( E \): Young's modulus
- \( \nu \): Poisson's ratio

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The shear moduli from both techniques are found consistent with each other when the shear strain amplitudes are identical (Ref. 1). Solid cylindrical specimens with diameters of approximately 50mm and length of about 125mm are used in the laboratory tests. The applied confining pressures are 40 kN/m² to 700 kN/m².

The damping ratio is calculated from the stress-strain relationships for the 5th loading cycle according to the following equation:

\[ h = \Delta W/4\pi W \ldots (3) \]

where, \( \Delta W \); energy loss per cycle \( W \); energy stored in specimen per cycle

In the fields, the cross-hole and/or the down-hole methods are used to measure the velocity of the P-waves and S-waves.

**TEST MATERIALS**

Five remolded soils are artificially prepared by mixing Honmoku clay with crushed Toyoura sand to obtain various types of cohesive soils. Undisturbed materials are Kurima clay, Ishinomaki clay, Kinkai clay, Port island clay, Rokko island clay and Osaka clay, which are sampled from various port and harbour areas in Japan. The physical properties of each sample are listed in Table-1.

**RESULTS AND DISCUSSIONS**

Shear modulus at low-strain amplitudes;(During primary consolidation)

The shear moduli at low-strain amplitudes are calculated from the following empirical equation.

\[ G_0 = A \cdot F(e) \cdot (\sigma_c')^{1/n} \ldots (4) \]

where, \( e \); void ratio \( \sigma_c' \); confining pressure

The parameters \( A \), \( F(e) \) and \( n \) in Eq. (4), which are proposed by several researchers, are tabulated in Table-2. As it is clear from Table-2, the parameter \( n \) can be considered having the value of around 0.5 to 0.6, but the parameters \( A \) and \( F(e) \) proposed by each study are not same. Fig.-2 shows the relationships between the \( A \), which is obtained by dividing the measured \( G_0 \) with \((2.73-e)^2 \cdot \sigma_c'/(1+e)\), and the plasticity index. The data encircled by a dotted line in Fig.-2 mean that the \( G_0 \) is measured for the specimens prepared from the disturbed samples. It is found that the \( A \) is not constant even when the \( F(e) \) is assumed to be equal to \((2.97-e)^2/(1+e)\), but it depends on the plasticity index; as the plasticity index increases, the \( A \) becomes larger.

In Eq. (4), the \( G_0 \) being represented by the void ratio and the confining pressure, it is reasonable to think that the \( G_0 \) for saturated natural soils is governed by either of them, if relationships between the void ratio and the confining pressure are determined linearly. Based on this idea, the \( G_0 \) is divided with \( \sigma_c' \) and plotted against the plasticity index. Figs.-3 (a), (b) and (c) show that the \( G_0/\sigma_c' \) is governed by the plasticity index and the confining pressure. The effects of the confining pressure in Fig.-3 can be attributed to the fact.
that the parameter \( n \) is not always equal to 0.5 and the \( G_0/\sqrt{\nu} \) still includes the dimension of the confining pressure. Figs.3 (a), (b) and (c) show that the \( G_0 \) for cohesive soils can be represented by the following equation taking account of the types of soils in terms of the plasticity index.

\[
G_0 = F(I_p) \cdot \sqrt[\nu]{c'} \quad (5)
\]

where,

\( F(I_p) \); values given by Figs.3 (a), (b) and (c)

The comparison between the \( G_0 \) measured and the \( G_0 \) predicted from Eq.(5) is shown in Fig.-4 indicating that the predicted \( G_0 \) gives a good agreement with the measured one.

Comparison of Eq.(5) with in-situ data

In order to make sure the accuracy of results of in-situ seismic surveys, two kinds of techniques are employed in this study; cross-hole technique and the down-hole technique. The distinguishing characteristics of the cross-hole technique exist in the measurement of waves travelling horizontally in the stratum. On the other hand, the characteristics of the down-hole technique is that it reproduces the same conditions with earthquakes by making waves travel vertically to the strata. Fig.-5 shows that both techniques give the almost same values, irrespective of the different ways of measurements.

The \( G_0 \) measured with the resonant column technique shown with white circles in Fig.-5 are fairly high when compared with those from the in-situ survey. The reason is that effective overburden pressures in the layer are smaller than those calculated from the embankment on the test-field since the layer is still under the primary consolidation. So the \( G_0 \) measured with resonant column technique are corrected to the appropriate in-situ mean effective principal stresses taking account of the degree of consolidation and assuming the coefficient of lateral earth pressure \( K \) is equal to 0.5. In this case, however, the void ratio is not corrected because the \( G_0 \) may be governed by the confining pressure as shown in Eq.(5) if the void ratio and the consolidation pressure have the linear relationship. Solid circles in Fig.-5 are values modified with the mean effective principal stresses. Modified results show good agreements with the in-situ data without the correction of the void ratio.

Effect of the plasticity on the \( G/G_0-\gamma \) curves

It is well known that the shear modulus decreases as the shear strain amplitude increases. The ratio of decrease is usually expressed by the \( G/G_0-\gamma \) curves in which the \( G \) represents the shear modulus at arbitrary strain. As the effects of the confining pressure on the \( G/G_0-\gamma \) curves for cohesive soils are reported negligible (Ref.1,8), the effects of the plasticity index on the \( G/G_0-\gamma \) curves are shown in Fig.-6. It shows the \( G/G_0-\gamma \) curves shift to the righthand side in Fig.-6 as the plasticity index becomes larger. The solid lines are the normalized relationships between the plasticity index and the \( G/G_0-\gamma \) curves.

Plasticity index and reference strain for Hardin-Drnevich model

Hardin-Drnevich have proposed the vibratory stress-strain relationship
based on the hyperbolic model (Ref.9).

\[
\frac{G}{Go} = \frac{1}{(1 + \gamma / \gamma_r)} \quad \ldots (6) \quad \frac{1}{\gamma_r} = Go / \tau \quad \ldots (7)
\]

where,

\( \gamma_r \); reference strain  
\( \tau \); shear stress at strain 1.0%.

As it is necessary to estimate the reference strain in order to calculate the G/Go-\( \gamma \) curves from Eq. (6), the relationship between the plasticity index and the reference strain are investigated and shown in Fig.-7. The Go/\( \tau \), however, is used in Fig.-7 instead of the reference strain, in which \( \gamma \) is the values determined conveniently from the shear stress at the shear strain amplitude 1.0%. Fig.-7 shows that the Go/\( \tau \) can be estimated from the plasticity index, and when the plasticity index is more than 35, the Go/\( \tau \) is given irrespective of the confining pressure. The computed from Eq.(6) using the values shown in Fig.-7 are compared with measured values in Fig.-8. Hardin-Brunevich model can be used to fit most of the G/Go-\( \gamma \) curves by adopting the Go/\( \tau \) in Fig.-7 except soils with low plasticity indexes which are affected by the confining pressure.

**Plasticity index and damping ratio**

It is found in Fig.-9 that the damping ratio for cohesive soils is not so affected by the confining pressure but depends on the plasticity index. Fig.-10 shows the relationship between the plasticity index and the damping ratio. The damping ratio has the minimum values at the plasticity index 40 to 60. This inclination is the same with that reported by Kokusho et al.(Ref.7).

**Time effect**

The time effect on the vibratory deformation characteristics is one of the very important factors, especially when the data from laboratory experiments are applied to the fields where the geographical conditions are quite different from test conditions in the laboratory (Ref.6). The time effects on the Go measured with the resonant column technique are shown in Fig.-11 as the rate of increase to the logarithmic scale of time after primary consolidation. The Gt is the shear modulus at the end of the primary consolidation. Fig.-11 shows the relationships between the increase rate and the plasticity index. The increase rate lies between 5% and 30% and seems to become larger as the plasticity index increases.

**PREDICTION OF SHEAR MODULUS AND DAMPING RATIO**

When vibratory experiments are not available due to some reason, the plasticity index can be used, as mentioned above, to predict the Go, the G/Go-\( \gamma \) curves and the h-\( \gamma \) curves for cohesive soils. Prior to the prediction, however, we should note that objective grounds are wheather under or after the condition of the primary consolidation. Time effects on them are so important that it would be necessary to accumulate more data in the fields as well as those in laboratory.

**REFERENCES**

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shear modulus of clayey soils with various plasticities", Proc. of 5th JEES.
6) Trudeau, P.J., et al. (1974); "Shear wave velocity and modulus of a marine clay", Jour. of BSCE, Vol. 61, No. 1.
8) Umehara, Y., et al. (1982); "Laboratory tests and in-situ seismic survey on vibratory shear moduli of cohesive soils", Proc. of 6th JEES.

Table-1 Physical properties of samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>No.</th>
<th>G</th>
<th>Ip</th>
<th>D50</th>
<th>Clay content (%)</th>
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<tbody>
<tr>
<td>Mixture</td>
<td>1</td>
<td>2.712</td>
<td>63.8</td>
<td>0.0064</td>
<td>47.7</td>
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<tr>
<td></td>
<td>2</td>
<td>2.706</td>
<td>77.9</td>
<td>0.0122</td>
<td>39.0</td>
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<tr>
<td></td>
<td>3</td>
<td>2.909</td>
<td>25.1</td>
<td>0.0055</td>
<td>28.7</td>
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<tr>
<td></td>
<td>4</td>
<td>2.955</td>
<td>16.2</td>
<td>0.0035</td>
<td>24.0</td>
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<td></td>
<td>5</td>
<td>2.644</td>
<td>9.4</td>
<td>0.0044</td>
<td>21.2</td>
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<td></td>
<td>6</td>
<td>2.595</td>
<td>9.9</td>
<td>0.0044</td>
<td>11.0</td>
</tr>
</tbody>
</table>

Tahimikai 81-11-1  2.349  62.2  0.0074  34.5
81-11-5  2.666  63.1  0.0081  30.0
81-11-7  2.747  67.7  0.0099  28.0
81-11-10  2.747  71.0  0.0155  33.5
81-11-12  2.745  76.2  0.0155  37.5
81-11-13  2.747  56.5  0.0099  43.5
81-11-15  2.747  52.5  0.0099  43.5
81-11-17  2.687  44.7  0.0083  60.5

Rokku 13-1  2.69  63.9  0.0057  48.0
Indonesia 53-11-1  2.72  58.2  0.0076  55.0
53-11-18  2.66  76.5  0.0139  51.0
53-11-3  2.66  63.1  0.0064  53.0
53-11-5  2.67  76.1  0.0076  58.0
53-11-7  2.69  58.2  0.0076  55.0

Rokku II 56-11-1  2.741  72.6  64
56-11-2  2.66  69.8  55
56-11-6  2.720  63.1  44
56-11-7  2.658  77.6  50
56-12-1  2.686  82.6  43
56-12-2  2.765  58.7  0.0058  47
56-12-3  2.321  48.7  0.0075  41

Table-2 Parameters n, A and P(e)

<table>
<thead>
<tr>
<th>Sample</th>
<th>n</th>
<th>A</th>
<th>P(e)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardin-Black (1968)</td>
<td>2.397</td>
<td>2.397</td>
<td>2.397</td>
<td>7</td>
</tr>
<tr>
<td>Humphries-Wals (1968)</td>
<td>0.90</td>
<td>10</td>
<td>350-130</td>
<td>3</td>
</tr>
<tr>
<td>Marquis-White (1972)</td>
<td>0.75</td>
<td>450</td>
<td>4.6x10^2</td>
<td>4</td>
</tr>
<tr>
<td>Afifi-Richard (1973)</td>
<td>0.55</td>
<td>8500</td>
<td>2.9x10^2</td>
<td>5</td>
</tr>
<tr>
<td>Trudeau-Whitman (1974)</td>
<td>0.55</td>
<td>4.50</td>
<td>2.9x10^2</td>
<td>6</td>
</tr>
<tr>
<td>Kokusho et al. (1982)</td>
<td>0.66</td>
<td>90</td>
<td>7.3x10^2</td>
<td>7</td>
</tr>
</tbody>
</table>

Fig. 1 Equipment

1. Displacement transducer
2. Velocity meter
3. Driving cell
4. Pressure transducer
5. Load cell
6. Specimen
7. Hydraulic control and recording
8. Load cell
9. Amplifier
Fig. -2 Parameter A in Eq.(4)

Fig. -3 (a) Parameter F(Ip) in Eq.(5)

Fig. -3 (b) Parameter F(Ip) in Eq.(5)

Fig. -3 (c) Parameter F(Ip) in Eq.(5)
Fig.-4 Comparison of evaluated shear modulus with measured one

Fig.-5 Comparison of data from resonant column test with in-situ seismic survey (Kurihama test-field)

Fig.-6 Effect of plasticity index on G/Go-γ curves
Fig. 8 Expression of $G/\gamma$ curves by Hardin-Drnevich model with plasticity index

Fig. 7 Relation between $G/\gamma$ and $I_p$

Fig. 9 Effect of confining pressure on damping ratio

Fig. 10 Effect of plasticity index on damping ratio

Fig. 11 Time effect