THE USE OF STRONG-MOTION INSTRUMENTS TO DETERMINE LOCAL MAGNITUDE

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SUMMARY

The use of strong-motion accelerometers and seismoscopes to determine local magnitude, $M_L$, is a recent development in strong-motion seismology. In the case of the accelerometers, the record of ground motion is used as an excitation in the equations of motion of the Wood-Anderson seismograph, thereby creating a synthetic seismogram. Seismoscope responses are used to determine the response of the Wood-Anderson instrument by applying an approximate formula developed from the theory of random vibrations. The methods have been applied to a number of recent earthquakes in which strong-motion records are available in the near field (Ref. 1, 2 and 3). This paper summarizes these results, which include a preferred measure of site-to-source distance for seismic risk studies and a recommended revision to the standard attenuation curve used to determine $M_L$.

INTRODUCTION

The concept of the magnitude of earthquakes was developed by C. F. Richter (Ref. 4) in 1935 to measure the size of earthquakes in Southern California. Since then the original idea has been modified and expanded so there are several magnitude scales in use, with the term local magnitude, $M_L$, reserved for the original concept. Of the various magnitude scales, the most relevant to engineering is the local magnitude, because it is defined in terms of the response of an instrument, the Wood-Anderson seismograph, whose period and damping are such as to make it broadly sensitive to motions in the frequency range of interest to engineering. In addition, it is determined closer to the source of the earthquake than the other scales, where the ground motion more nearly resembles that near the causative fault.

The standard Wood-Anderson seismograph has a natural period of 0.8 sec, 80 per cent of critical damping and a gain of 2800. By noting the observed decay of the peak response, $A$, of this instrument with distance, Richter developed his result, which can be written

$$M_L = \log_{10} A - \log_{10} A_0(\Delta)$$

in which $\Delta$ is the epicentral distance in kilometers and $A_0(\Delta)$ is the Wood-Anderson amplitude of an earthquake of magnitude zero. The shape of the $A_0$ vs. $\Delta$ curve was determined from data and its level was fixed by setting $A$ equal 1 mm at 100 km for $M_L = 3.0$. The sensitivity of the

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Wood-Anderson seismograph is such that it goes off-scale at about the level where the ground motion is humanly perceptible and the original attenuation curve given by Richter (Ref. 4) is defined only for $\Delta \geq 25$ km. For $\Delta \leq 25$ km, the instrument goes off scale for $M_\text{L}$ larger than about 4.5. To extend the standard attenuation curve to shorter distances, Gutenberg and Richter (Ref. 5) used data from the torsion seismometer, an instrument with a period of 10 sec, damping near critical, and a gain of 4. The extended attenuation curve is the one in common usage (Ref. 6).

The difficulty in using the local magnitude in engineering studies is that, except for the relatively uncommon 4X instrument, the seismographs all go off-scale in areas of strong shaking. Thus, the reported magnitudes for the bigger earthquakes are found from other methods, and the magnitude is frequently reported, even in the technical literature, without denoting the scale employed. It was the availability of near-field strong-motion data and the important use of the local magnitude in earthquake engineering that led the authors to perform the studies reported in ref. 1, 2 and 3.

THEORETICAL BACKGROUND

To use strong-motion accelerograms to compute $M_\text{L}$, it is only necessary to solve the equations of motion of the Wood-Anderson seismograph using the recorded acceleration as the excitation. Since the instrument is a single-degree-of-freedom oscillator, the calculation can be done by a minor change in programs used to calculate response spectra (Ref. 7). The procedure is valid because the strong-motion accelerograph records, without significant loss, the ground acceleration in the range of frequencies important to the displacement response of the Wood-Anderson instrument. An example of the calculated response is shown in Fig. 1. Note that the range of response of the synthetic seismogram is several meters. In the case of the seismoscope, a result from the theory of random vibrations is used to develop an approximate formula for estimating the Wood-Anderson response from that of the seismoscope. The seismoscope is also a single-degree-of-freedom oscillator with a period of 0.75 sec, about 10 per cent of critical damping, and a known gain. To correct for the difference in gains requires only a simple multiplicative factor. The required correction for frequency and damping is based on the mean square response of a oscillator to white noise excitation of spectral density $D_\text{n}$.

$$<x^2> = \frac{D_{\text{n}}}{16\zeta^2\pi^2}$$  \hspace{1cm} (2)

In Eqn. 2, $x$ is the response of the oscillator and $T_n$ and $\zeta$ are its natural period and damping, respectively. The application of this result to the response of the instruments involved is approximately valid if the ground motion is broad-band in character and long with respect to the two instrumental periods. As derived in Ref. 2, the Wood-Anderson response is estimated by

$$\frac{A_{\text{wa}}}{A_{\text{sc}}} = \frac{V_{\text{wa}}}{V_{\text{sc}}} \sqrt{\frac{\pi T_{\text{wa}}}{\pi T_{\text{sc}}}} \frac{r_{\text{sc}}}{r_{\text{wa}}} A_{\text{sc}}$$  \hspace{1cm} (3)

In which the subscript $\text{sc}$ refers to the seismoscope and $\text{wa}$ to the Wood-Anderson seismograph. $V$ refers to instrumental gain and $A$ is the maximum amplitude of response (one-half peak-to-peak). The accuracy of Eqn. 3 has
been verified by analysis of records at 23 sites where both accelerograms
and seismoscope records are available (Ref. 2).

APPLICATIONS

The procedures outlined above have been applied to records from several
important earthquakes, primarily in Southern California. The extensive set
of records from the May 2, 1983 Coalinga, California earthquake is now being
studied.

San Fernando, 1971: A total of 26 accelerograms, two components each, were
used to calculate \( M_L \) for this earthquake. The distance from the center of
faulting, inferred to be under Pacoima dam, varied from 0 to 146 km. The
areal coverage of the records used is shown in Fig. 2 and the values of \( M_L \)
vs distance are shown in Fig. 3. The local magnitude is 6.4, with a
standard deviation of approximately \( \pm \) unit.

Imperial Valley, 1979: This earthquake produced an excellent set of ground
motion data. For determining the local magnitude, 25 accelerograms from
California and 9 from Mexico were used. All but two of these sites, shown
in Fig. 4, produced useable records of both horizontal components of motion.
The values of \( M_L \) are plotted against distance in Fig. 5. The local magni-
tude is 6.4. The measure of distance used in Fig. 5 is discussed below.

Santa Barbara, 1978: This damaging earthquake was recorded in the near
field by 5 accelerographs and 2 seismoscopes. The distances ranged from 5
to 35 km, measured from the center of the fault plane. On the basis of
seismographic instruments, this earthquake has been assigned local
magnitudes of 5.1 (Caltech) and 5.7 (Berkeley). The earthquake produced
significant damage to several modern buildings and to freeway structures.
On the basis of the strong-motion records, the local magnitude is 6.0, with
the distance measured from the center of faulting. If the distances are
taken from the center of the zone of aftershocks, \( M_L \) is 5.9. In either case,
the magnitude from the strong-motion instruments is consistent with the
observed damage, which is more severe than is usually associated with
magnitude 5+ events.

Other California Earthquakes: The strong-motion records were also analyzed
for a number of California earthquakes. These are reported in Ref. 1, 3 and
8. Of particular interest are the larger earthquakes: Kern County, 1952,
\( M_L = 7.2 \); Borrego Mountains, 1968, \( M_L = 6.9 \); and Imperial Valley 1940, \( M_L = 
6.7 \) (using the revised attenuation curve of Ref. 3). The value of \( M_L = 7.2 
for the Kern County earthquake is the largest so far determined.

San Francisco, 1906: The records of two seismoscope-like instruments, a
Ewing duplex pendulum seismograph and a simple pendulum, were analyzed to
estimate \( M_L \) for this historic earthquake, whose surface-wave magnitude, \( M_S \),
is \( 8 \) (Ref. 2). Two of the Ewing seismographs were found and restored to
determine the period and damping of the instrument. On the basis of the
records obtained at distances of 65 km (Yountville pendulum) and 286 km
(Carson City Ewing seismograph), the local magnitude of the 1906 earthquake
was estimated to lie in the range 6-3/4 to 7.

Managua, Nicaragua, 1972: This damaging earthquake \( (M_S = 6.2) \) produced one
accelerogram and six seismoscope records, one of which was off-scale. The
distances from the center of faulting varied from 1.5 to 12 km. The strong-
motion instruments indicated a local magnitude of 6.2.

Guatemala, 1976: This large and destructive earthquake ($M_L = 7.5$) was
associated with extensive rupture along the Metagua fault and was particular-
ly destructive near Guatemala City. A seismoscope in Guatemala City produced
the only seismic recording near the fault. The appropriate distance to use
in determining $M_L$ is uncertain, but based on the discussion in Ref. 3,
$M_L = 6.9$ was estimated.

REVISION OF THE ATTENUATION CURVE

The plots of local magnitude vs distance shown in Figs. 4 and 5 indicate
there is a considerable scatter in the values of $M_L$ and that there may be a
systematic variation of the mean value as well. After studying this data
and similar, but less extensive, data from other earthquakes, the authors
have proposed a revision of the standard attenuation curve, $-\log_{10} A_0(L)$.
This revision is shown in Fig. 6; numerical values are given in Ref. 3.
Such a revision is supported by the numerical modelling reported by Hadley
et. al. (Ref. 9) and by independent studies presented by Luco (Ref. 10),
who has proposed a qualitatively similar correction. The amount of the
correction varies between $\pm 0.25$ units, which corresponds to a factor of
nearly 1.8 in the response of the Wood-Anderson instrument. Using the
revision increases the local magnitude calculated from records obtained
between 0-20 km from the fault by as much as 0.25. This implies, for
example, the shaking previously associated with a magnitude 6$\frac{1}{2}$ earthquake
at 15 km is more appropriately associated with $M_L = 6\frac{1}{2}$.

DISCUSSION

The local magnitude of an earthquake provides a single-parameter mea-
sure of the strength of shaking in the frequency range of importance to
engineering. By determining $M_L$ from strong-motion records obtained in the
near field, the use of magnitude in determining earthquake design criteria
will be made more consistent. The accuracy of correlations of other para-
eters of strong ground motion with magnitude should be improved, and the
scatter inherent in approximate formula for such correlations should be
reduced. However, a single parameter measure is fundamentally limited. $M_L$
accounts for the most important parameter, distance from the source, but does
not take into consideration the effects of radiation pattern, source mech-
anism, directivity, length of rupture nor geologic and site conditions. This
situation leads inevitably to scatter in the values calculated for the local
magnitude; the data we have examined indicate a range of $\frac{1}{2}$ unit or more is
to be expected.

As can be seen in Fig. 1, the Wood-Anderson response has a qualitative
resemblance to velocity and we found in Ref. 1 that the maximum velocity
occurred at the same time as the maximum Wood-Anderson response in 70 per-
cent of the cases. This helps understand why there is an excellent cor-
relation between Wood-Anderson response and velocity, which Boore (Ref. 11)
has used to develop a simple formula for determining $M_L$ from ground velocity.
Also, Espinoza (Ref. 12) has used peak acceleration to determine $M_L$.  

862
To determine \( M_L \) requires the epicentral distance, as originally proposed, (Ref. 4) or some other measure of distance. The measure chosen does not matter much for either large distances or very small earthquakes, but in the near-field, where the distance is comparable to the dimensions of the fault rupture or the depth of focus, the different measures of distance can give significantly different values of \( M_L \). This problem is of considerable importance in determining seismic design criteria for structures and facilities close to major faults. The strong-motion data from recent earthquakes, particularly the 1979 event in Imperial County, permit this problem to be studied. After examining the results for four different measures of distances, we concluded that the most consistent results were obtained by measuring the distance to the surface trace of the fault for sites located within a circle whose diameter is equal to the length of faulting. For sites outside of this circle, the choice of distance is not as important; the distance to the center of faulting (the center of the circle) is recommended.

Although of central importance to engineering use, the local magnitude is limited for seismological studies because the Wood-Anderson instrument is rather narrow-banded in seismological terms. Because it does not respond to energy radiating with long periods, other measures are preferred by seismologists when studying major and great earthquakes. Even the surface-wave magnitude, \( M_S \), which is typically determined by 20-sec period waves for distant, large earthquakes, fails to account for the significant amounts of energy released at 10's and 100's of second periods. For this reason, the moment magnitude, \( M_m \), has been proposed (Ref. 13). This measure includes all the energy of motion and can be found either from the area and amount of faulting or from far-field seismograms.

Correlations between the various magnitude scales have been studied and one important result is indicated in Fig. 8 which shows \( M_L \) plotted against \( M_w \). The figure shows that as the earthquakes get large, as measured by \( M_L \), \( M_w \) levels off, or saturates. The local magnitude appears to have a limit between 7 and 7.4. As noted above, the maximum recorded value of \( M_L \) is 7.2. Figure 7 and its supporting data provide seismological evidence for the idea that there is an upper-bound to the strength of shaking near the fault in large earthquakes. It has often been argued that the major difference between shaking close to the fault in an \( M_S = 6 \) to 7 earthquake and in an \( M_S = 8+ \) earthquake is in the duration of shaking and not in its intensity. Unfortunately, the data are not yet sufficient to determine a quantitative upper bound; near field records from major and great earthquakes are needed to resolve this important point.

Because the Wood-Anderson seismograph is a simple oscillator, and because the response spectrum is defined in terms of the response of a simple oscillator, the maximum Wood-Anderson response is essentially 2800 times the response spectrum ordinate for \( T = 0.8 \) sec and 80 per cent damping. There is a small difference because the Wood-Anderson maximum is read one-half peak-to-peak, whereas the response spectrum is determined by the maximum absolute value of response. One result of this correspondence between \( M_L \) and response spectra is that the local magnitude can be used to scale accelerograms for design studies. For example, if the design criteria call for a magnitude 6.5 earthquake at 15 km, then accelerograms recorded at approximately this distance and magnitude can be scaled so that if recorded at 15 km they would excite a Wood-Anderson seismograph to exactly the
amplitude required for $M_L = 6.5$. This approach for generating accelerograms and associated design spectra has been used in the design of major off-shore drilling platforms in Southern California.

The use of strong-motion instruments to determine local magnitudes and other near-field properties of strong-ground motion has been beneficial to both seismology and earthquake engineering. Future opportunities for study in this area are continually being presented as more strong-motion data accumulate. It may be possible, for example, to determine the nature of coherent sources of motion, or asperities, which are believed to be the sources of the large, destructive pulses seen in some near-field records.

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REFERENCES

Fig. 1. Acceleration, velocity, displacement and synthesized Wood-Anderson response from the 1968 Borrego Mountain earthquake.

Fig. 2. Locations of accelerograph sites used to determine $M_L$ for the San Fernando earthquake of 1971.

Fig. 3. Values of $M_L$ for the San Fernando earthquake. The error bar indicates the standard deviation of values from 4 seismographic stations at distances from 37 to 250 km.
Fig. 4. Location of accelerograph sites used to determine $M_L$ for the Imperial Valley earthquake of 1979.

Fig. 5. Values of $M_L$ for the Imperial County earthquake. The error bar indicates the standard deviation of values obtained at 7 seismographic stations at distances from 270 to 610 km.

Fig. 6. Revised attenuation curve, $-\log_{10} A_0$. Numerical values are in Ref. 3.

Fig. 7. Relation between local magnitude, $M_L$, and moment magnitude, $M_W$, showing saturation of $M_L$. ($M_S$ is used for $M_W$ for 2 cases where $M_W$ is not available).