SYNTHETIC ACCELEROMETERS FOR TESTING PROCESSING PROCEDURES

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SUMMARY

The processing applied to digitized accelerograms is often adapted from other fields. The accuracy of some of the processing steps, particularly integration and long-period filtering, is often brought into question. The derivation of synthetic accelerograms with which to measure the accuracy of these and other processing steps is described in this paper. The fundamental characteristics of the simulations are that they have the appearance of recorded accelerograms, their integrals are known and approach zero after some time, and they remain integrable after particular frequency components have been removed.

INTRODUCTION

During the course of continuous upgrading of processing schemes to produce corrected versions of digitized accelerograms and to produce response and Fourier spectra, there has always been access to standard, well-known, accelerograms (for example, the El Centro, 1940, record) with which to test new processing ideas. These tests are usually carried out to put the final polish on a particular processing step after sufficient theoretical work has been done and after comparisons with other methods have been thoroughly explored. But this final processing of a standard accelerogram actually proves nothing about the quality of the processing; it merely serves to show how the processing affects that specific record. This point is indeed verified by the very appearance of new processing ideas. The new ideas are necessary because, at some point, any standard processing scheme eventually meets an accelerogram record that has some characteristic implanted during its recording, digitizing, or preliminary preparation, with which the processing is incompatible.

Throughout the USGS experience since 1975, advances in instrument deployment, the quality of film recording, automatic digitization and computer processing are all met and challenged eventually by the records obtained during the next interesting earthquake. Indeed, we might well admit there has not yet been a significantly recorded earthquake that did not provide more questions than answers in the narrow field of the processing of those records.

Current Processing

The standard or routine processing included in the Caltech digitization and analysis project of the 1970's has been adjusted over the subsequent years by all users of the programs. These adjustments have been only in the detail of the way in which the individual processing steps are accomplished. The basic steps and the order in which they are done are

straightforward. Obvious geometrical, physical and optical corrections are applied. The digitization of reference traces and time marks are used to determine where the time-axis is to be placed, and to fix the time coordinates. High-frequency noise, predominantly digitizing noise, is removed. Instruments whose response is not flat over the frequency range of interest have their effects removed. Long-period noise from any of a number of sources are removed with the understanding that perhaps important signals with the same period range must be removed at the same time. The resulting acceleration is then ready for integration for velocity and displacement. The calculation of response and Fourier spectra is based on the corrected acceleration.

The details within each step are adjusted, when required, to process data of either improved, or unexpectedly lower, quality, to make use of increased knowledge in time series analysis, and to process particular records showing resistance to the usual processing.

In the USGS processing package (Ref. 1) many of the changes in detail have become options in our overall routine scheme. The questions that are raised by the availability of these options, and asked by an operator faced with the processing of a set of recordings, include questions concerning the accuracy of each of the options, or if the accuracy can be adjusted, then the efficiency (in time and memory) of the options at a chosen, optimum, accuracy. In this report we introduce a scheme for simulated accelerograms and provide some preliminary test results.

Previous Tests and Simulations

Almost all the processing techniques in use at the present time have been tested before being recommended for use, or actually used, on specific data sets (see, for example, Ref. 2) None of these have been tested with simulated records which are similar in appearance to recorded accelerograms, and whose frequency content and integrability for correct velocity and displacement are known beforehand.

Simulated earthquake accelerograms with a known integrability for velocity and displacement are well-reported in the literature. Those that might have offered some hope of successful checking of integrating techniques had they been developed for that purpose are described in Refs. 3, 4, and 5. Ipek (Ref. 5) describes a simulation formed by summing sections of sinusoids of various frequency, starting times, and stopping times. The simulated records described in this report are closely related to those in Ref. 5.

SPECIFICATIONS FOR A SIMULATED ACCELEROGRAM

A scheme for producing simulated accelerograms that can test the digitizing and computer processing in the current U.S. Geological Survey processing package is described in the following sections of this paper. Some of the characteristics of such a simulated accelerogram are listed below:

1. The simulation must be able to match an existing accelerogram, or at least those parts of it that are considered to be the source of incipient processing problems.
2. The simulation is to be made up of elements that are algebraic functions of time, are readily integrable twice, and contain only one predominant frequency.

3. The total simulation is to contain specific frequencies, appearing in the record at specified times. In order to meet item 1 above, the amplitudes of an element with a particular frequency component are to fit within a describable envelope.

4. After a sufficiently long time, but not so long as to cause run-time difficulties, the acceleration, velocity and displacement are to approach zero arbitrarily closely.

5. The simulation of a displacement or velocity element is to be possible.

With these guidelines we considered an individual displacement element with the form:

$$D(t) = A t^2 e^{-kt^2} \sin \omega t$$  \hspace{1cm} (1)$$

where \(A\), \(k\) and \(\omega\) are constants, and the corresponding velocity and acceleration, obtained by differentiation:

$$V(t) = A t e^{-kt^2} [2(1-kt^2) \sin \omega t + \omega t \cos \omega t]$$  \hspace{1cm} (2)$$

$$A(t) = A e^{-kt^2} [4\omega t (1-kt^2) \cos \omega t + (2-(10k+\omega^2) t^2 + 4k^2 t^4) \sin \omega t]$$  \hspace{1cm} (3)$$

The shape and scaled amplitude of an acceleration element are controlled by the parameters \(A\), \(k\), and \(\omega\). The role of \(A\) and \(\omega\) as scaling factor and angular frequency, respectively, is clear. The parameter \(k\) can be chosen to ensure that the particular peak value attained by this element occurs after a predetermined number of local peaks have already occurred. Alternatively, \(k\) can point to the time elapsed from the start of this element until its peak is reached. Any number of elements can be added together with different starting times.

**TYPICAL EXAMPLES OF SYNTHETICS USE**

A simulated accelerogram is devised to test three specific cases: (1) to confirm the accuracy of a standard integration package, handling frequencies between 0.33 and 50 Hz in a 20 sec-duration record; (2) to check the overall accuracy of the multistep process of (a) computer plotting the simulation, (b) photographically preparing it to resemble a recorded accelerogram, (c) digitization, and (d) routine processing including integrating for velocity and displacement; and (3) a preliminary frequency domain check on the quality of various low-frequency filters, and the quality of the digitization of the processed film record in 2 above.
By suitably choosing values for the parameters for 14 elements, an accelerogram has been prepared with a duration of 20 sec and containing no particularly demanding properties (from a digitizing standpoint) except a burst of 50 Hz signal. Table 1 lists properties of these elements leading to choices of $A$, $k$ and $\omega$. Figure 1 shows one component element in the simulation, with a frequency parameter of 1 Hz, and having its 4th local peak with a maximum of 0.22 g occurring at 8 sec. The complete simulation is represented by the first trace of figure 2. The lowest and highest frequencies present, 0.33 and 50 Hz, are targeted for removal by filtering in subsequent tests, and can also be removed from this list to produce a 12-element simulation. The last 4 elements in Table 1 are present solely to sharpen the peak provided by element No. 5 at 6 sec.

Integration Check

Figure 2 shows the simulated accelerogram and its processing, as though it were a recorded accelerogram, by the USGS processing option that includes integration for velocity, removal of a least-square-fitted straight line from the velocity, and subsequent integration again for displacement. No filtering of any kind has been applied. The corresponding simulated displacement, calculated from the sum of the 14 elements described by equation 1, has not been presented, but its plot is indistinguishable from the third (displacement) trace. The peak value is 26.82 cm compared with the integrated value of 26.90 cm, showing that simple integration schemes are satisfactory for this range of frequencies.

Multistep Check

This is a preliminary check of the multistep process consisting of plotting the computed simulation, photographically preparing this plot to resemble an actual recorded accelerogram with time marks and reference trace, and digitization through our normal channels, using a computer-controlled, trace-following laser scanner. The film version was digitized in two frames, and the trace reassembled using a vertical butting line digitized in each frame. Processing was restricted to the high-frequency range where digitizing noise might be a problem. In a one-step filtering algorithm applied in the time-domain, the following steps are accomplished:

1. An instrument correction from 0 to 50 Hz, using the instrumental constants for the 230° component of El Centro array no. 7. (This record of the 1979 Imperial Valley, California, earthquake, provided a guide on which to base the simulation);

2. A cosine taper to zero between 50 and 100 Hz;

3. High-cut filter from 100 to 300 Hz; and

4. Decimation to 200 sps.

Figure 3 shows the resulting acceleration, velocity and displacement, where the velocity and displacement are calculated by integration as they were for the integration check above.
The significant differences between these plots and those of figure 2 are as follows:

1. The 50 Hz content peaking at 6.3 sec has been magnified by the instrument correction (25.6 Hz natural frequency, 0.59 critical damping).

2. The peak value at 6 sec has a lower amplitude (573.69 cm/sec²) than the analytical simulated value (577.06 cm/sec²). This difference is not addressed here, for we are not concerned with high-frequency tests; the difference corresponds to the third significant figure accuracy of the instrument sensitivity.

3. The displacements compare favorably except for a visible significant long-period component. Careful comparison indicates a 16-sec component with a peak amplitude of 7.5 cm. This translates to a component in the acceleration with a peak of 1.15 cm/sec².

Checks on Low-Frequency Filters

The Fourier spectrum of the 14-element simulation is shown in figure 4, labelled with "no filter, 14 elements". Removal of the 0.33 and 50 Hz elements (Table 1) results in a 12-element simulation which should be a reasonable base to check the effects of a high-frequency 25 Hz filter and a low frequency 0.667 Hz filter applied to the 14-element simulation. The comparison is shown in figure 4 where we concentrate our attention only on the effects at low frequencies.

Three filters from among the options available in the USGS processing package were applied to the 14-component synthetic accelerogram. The corner frequency for two Butterworth filters, with order 4 and 8, was 0.667 Hz. The cutoff for a frequency domain FFT filter was also 0.667 Hz, with a cosine taper transition width of 0.1 Hz. The filter's expected performance at this frequency is shown in Figure 4 in the steep-sloped section at log f = 0.2. The noise levels in the stop band for the three filters are different, the 4th order Butterworth having the lowest levels.

The spectrum of the 12-element simulation, with the 0.33 and 50 Hz elements removed, is also shown in Figure 4. Its character is different from that of the filtered 14-element spectra, and shows the wide frequency range actually present in the nominally 1 Hz element.

Figure 4 also contains the spectrum of the digitized 14-element simulation with no low-frequency filtering. As discussed above, the 16-second noise content is prominent, at log f = 1.2. Up to frequencies as high as the 20 Hz element (log f = 1.3), the spectrum of the digitized simulation is otherwise very close to the plotted spectra of the actual simulation. Between 20 and 50 Hz, however, the instrument correction commences to have an effect, and the spectral level of the corrected data is indicated. The high frequency cutoff of 50 Hz is indicated by the Fc arrow.

Further investigation of this or similar simulations will allow a test of the accuracy of the FFT calculations. At the present time, the
simulation provides a basic record on which various filters can be compared.

CONCLUSIONS

The simulation process described here can, at the present time, provide a testing mechanism for most of the processing steps in the various routine and special processing schemes in operation at the USGS and elsewhere. Further investigation with this simulation will enable us to specify in a useful way the accuracy of integration, digitization, filtering, and spectral procedures, when used specifically on typical or unusual strong-motion recordings.

REFERENCES


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Figure 1. Acceleration element with frequency parameter of 1 Hz, reaching a maximum of 0.22 g on its 4th peak at 8 sec.

Figure 2. Simulated accelerogram, with integrated velocity and displacement. The 14 elements are listed in table 1. Peak values, as indicated, are -577.06 cm/sec/sec, 98.71 cm/sec, 26.90 cm. Accurate representation of 50 Hz content attained with 1000 sps.
Figure 3. Digitization of the prepared simulation, represented by the first trace of figure 2. High frequencies are treated routinely (see text), while further processing for velocity and displacement is identical to that of figure 2. Peak values, as indicated, are -571.69 cm/sec/sec, 97.55 cm/sec, 29.56 cm.

Figure 4. Fourier spectra for: 1) the 14- and 12-element simulations with no low-frequency filter; 2) three filters of the 14-element simulation; and 3) the digitized version with no low-frequency filter but instrument correction at high frequencies.