

PROBABILISTIC MODAL COMBINATION FOR EARTHQUAKE LOADING

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SUMMARY

A probabilistic method for evaluating the earthquake response of multi-degree-of-freedom structures using the response spectrum approach is developed. Various statistical quantities of the response, including the root-mean-squares of the response and its time derivative and the cumulative distribution and the mean and variance of the peak response, are directly obtained in terms of the response spectral ordinates and the modal properties of structure. The procedure is applicable to structures with closely spaced frequencies for which the existing SRSS method for the mean response is in gross error. For an example structure with closely spaced frequencies, the proposed response spectrum method produces results that are in close agreement with simulation results based on time-history computations.

INTRODUCTION

Earthquake induced loads on structures are stochastic in nature. Therefore, a probabilistic approach for the analysis of structural response to earthquakes and the assessment of safety is essential. Random vibration techniques have successfully been used to determine the statistical quantities of response to stochastic inputs typical of earthquake motions [1,6]. In this approach, the input excitation is usually described through a power spectral density function. This description, however, is not the most convenient in earthquake engineering practice. Instead, a description of the ground motion in terms of an average response spectrum is found to be more convenient and is commonly used in design applications and code specifications. Based on concepts from the theory of random vibrations, Rosenblueth et al. [7] were the first to develop a method for determining the responses of multi-degree-of-freedom (MDF) structures to earthquakes using the response spectrum approach. Their method, as well as other methods that were subsequently developed [8,10], are limited to a determination of the mean value of the maximum response.

In this paper, a probabilistic method for evaluating the responses of linear MDF structures to earthquakes using the response spectrum approach is developed. Various statistical quantities of the response, including the root-mean-squares of the response and its time derivative and the cumulative distribution and the mean and variance of the peak response, are directly obtained in terms of the response spectrum ordinates and the modal properties of the structure. The method includes the correlation between modal responses; thus, it is applicable to structures with closely spaced frequencies. The required expressions are all in closed form and require little computational effort.

The development in this paper is based on the previous works of Vanmarcke [11,12] and the author [3]. In Ref. 11, it is shown that most statistical quantities of interest for a stationary process are obtained in terms of the first three moments of the power spectral density function. In Ref. 12, these moments are used to determine the cumulative distribution of the first-passage time for a Gaussian process, which is also the distribution of the peak over a specified duration. In Ref. 3, closed-form expressions for the first three spectral moments of the response of MDF structures to white-noise and filtered white-noise input excitations are derived. The significance of closely spaced modes, which result in correlated modal responses, is included in this derivation. Through comparisons of results for the two types of inputs, the range of applicability of the white-noise model as an approximation for wide-band inputs is also

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determined in this reference. These results will subsequently be used in developing the response spectrum method.

RESPONSE OF MDF SYSTEMS TO STATIONARY GAUSSIAN EXCITATION

Consider an n -degree-of-freedom, viscously damped, linear system having classical modes. Let $\omega_i, \zeta_i, i=1, 2, \dots, n$, represent its natural frequencies and damping coefficients, respectively. Any response of such a system can be expressed in terms of its modal responses as

$$R(t) = \sum_i R_i(t) = \sum_i \Psi_i S_i(t) \quad (1)$$

where $R_i(t) = \Psi_i S_i(t)$ is the response in mode i , in which Ψ_i is the effective participation factor, a constant in terms of the modal vectors and the mass matrix, and $S_i(t)$ is the i -th normal coordinate representing the response of an oscillator of frequency ω_i and damping coefficient ζ_i to the given input [1]. For a zero-mean stationary Gaussian input, $F(t)$, described through a one-sided power spectral density, $G_F(\omega)$, the corresponding power spectral density for the stationary response is

$$G_R(\omega) = \sum_i \sum_j \Psi_i \Psi_j G_F(\omega) H_i(\omega) H_j^*(\omega) \quad (2)$$

where $H_i(\omega) = 1/(\omega_i^2 - \omega^2 + 2i\zeta_i\omega)$ is the complex frequency response function of mode i and the asterisk denotes a complex conjugate. Using this relation, the first three moments of the response power spectral density, as defined by Vanmarcke [11], are

$$\lambda_m = \int_0^\infty \omega^m G_R(\omega) d\omega = \sum_i \sum_j \Psi_i \Psi_j \lambda_{m,ij}, \quad m=0, 1, 2 \quad (3)$$

where

$$\lambda_{m,ij} = \text{Re} \left[\int_0^\infty \omega^m G_F(\omega) H_i(\omega) H_j^*(\omega) d\omega \right], \quad m=0, 1, 2 \quad (4)$$

are the cross-spectral moments of the normal coordinates associated with modes i and j [3]. Introducing the coefficients $\rho_{m,ij} = \lambda_{m,ij} / \sqrt{\lambda_{m,ii} \lambda_{m,jj}}$, $m=0, 1, 2$, Eq. 3 can be written in terms of uni-modal spectral moments as

$$\lambda_m = \sum_i \sum_j \Psi_i \Psi_j \rho_{m,ij} \sqrt{\lambda_{m,ii} \lambda_{m,jj}}, \quad m=0, 1, 2 \quad (5)$$

It is noted that $\rho_{0,ij}$ and $\rho_{2,ij}$ are the correlation coefficients between $S_i(t)$ and $S_j(t)$ and between their time derivatives, $\dot{S}_i(t)$ and $\dot{S}_j(t)$, respectively. A similar interpretation of $\rho_{1,ij}$ is not possible; however, the behavior of this coefficient is also similar to a correlation coefficient. Whereas the spectral moments, $\lambda_{m,ii}$, are in general sensitive to the shape of the input power spectral density, the coefficients, $\rho_{m,ij}$, remain relatively indifferent for wide-band inputs. For the response to a white-noise input, approximate expressions for these coefficients from Ref. 3 are

$$\rho_{0,ij} = \frac{2\sqrt{\zeta_i \zeta_j} \left[(\omega_i + \omega_j)^2 (\zeta_i + \zeta_j) + (\omega_i^2 - \omega_j^2) (\zeta_i - \zeta_j) \right]}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2 (\zeta_i + \zeta_j)^2} \quad (6)$$

$$\rho_{1,ij} = \frac{2\sqrt{\zeta_i \zeta_j} \left[(\omega_i + \omega_j)^2 (\zeta_i + \zeta_j) - 4(\omega_i - \omega_j)^2 / \pi \right]}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2 (\zeta_i + \zeta_j)^2} \quad (7)$$

$$\rho_{2,ij} = \frac{2\sqrt{\zeta_i \zeta_j} \left[(\omega_i + \omega_j)^2 (\zeta_i + \zeta_j) - (\omega_i^2 - \omega_j^2) (\zeta_i - \zeta_j) \right]}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2 (\zeta_i + \zeta_j)^2} \quad (8)$$

A comparison of these results with the corresponding values for response to filtered white-noise inputs has demonstrated that these expressions can be used for responses to wide-band inputs

typical of earthquake ground motions [3]. These expressions for $\rho_{m,ij}$ are plotted in Fig. 1 against the frequency ratio for selected values of damping.

In terms of the moments of the response power spectral density, λ_{p_i} , the statistical quantities of response are: the root-mean-square (rms) response, $\sigma_R = \sqrt{\lambda_0}$; the rms of the time derivative of the response, $\sigma_{\dot{R}} = \sqrt{\lambda_2}$; the response mean zero-crossing rate, $\nu = \sqrt{\lambda_2/\lambda_0}/\pi$; and the cumulative distribution of the maximum absolute response over duration τ ,

$$R_\tau = \max_t |R(t)| \quad (9)$$

as

$$F_{R_\tau}(r) = \left[1 - \exp(-s^2/2) \right] \exp \left[-\nu\tau \frac{1 - \exp(-\sqrt{\pi/2}\delta_e s)}{\exp(s^2/2) - 1} \right], \quad r > 0 \quad (10)$$

in which $s = r/\sigma_R$, $\delta_e = \delta^{1.2}$, and $\delta = \sqrt{1 - \lambda_1^2/\lambda_0\lambda_2}$ [11,12]. The mean and standard deviation of the maximum response may, in general, be obtained as $R_\tau = p\sigma_R$ and $\sigma_{R_\tau} = q\sigma_R$, respectively, where p and q are peak factors. Empirical expressions for p and q that are consistent with the distribution in Eq. 10 were obtained in Ref. 3 as

$$p = \sqrt{2 \ln \nu_e \tau} + \frac{0.5772}{\sqrt{2 \ln \nu_e \tau}} \quad (11)$$

$$q = \begin{cases} \frac{1.2}{\sqrt{2 \ln \nu_e \tau}} - \frac{5.4}{13 + (2 \ln \nu_e \tau)^{3.2}}, & \nu_e \tau > 2.1 \\ 0.65, & \nu_e \tau \leq 2.1 \end{cases} \quad (12)$$

where

$$\nu_e = \begin{cases} (1.63\delta^{0.45} - 0.38)\nu, & \delta < 0.69 \\ \nu, & \delta \geq 0.69 \end{cases} \quad (13)$$

is an equivalent rate of statistically independent zero crossings. These expressions are valid in the ranges $0.1 \leq \delta \leq 1$ and $5 \leq \nu\tau \leq 1000$, which are of interest in earthquake engineering. Fig. 2 shows plots of p and q versus $\nu\tau$ for selected values of δ . (It is noted that the distribution in Eq. 10 and the peak factors in Eqs. 11 and 12 include the effect of the dependence between crossings of the response process and, in this respect, are superior to similar results previously given by Davenport [2]).

DEVELOPMENT OF THE RESPONSE SPECTRUM METHOD

Let $\bar{S}_\tau(\omega, \zeta)$ represent the mean of the maximum absolute response of an oscillator of frequency ω and damping ζ to a given input, $F(t)$, over duration τ . A plot of $\bar{S}_\tau(\omega, \zeta)$ for all ω and ζ is defined herein as the response spectrum associated with the input $F(t)$ and the duration τ . It is the intention here to develop a method for approximate evaluation of the response of an MDF structure when the input is described through its response spectrum. This development is based on the assumption that the input is a wide-band process, i.e., that it has a smoothly varying power spectral density over a wide range of frequencies.

From the definition of the response spectrum, it is clear that $\bar{S}_\tau(\omega_i, \zeta_i)$ is the mean of the absolute maximum of the i -th normal coordinate, $S_i(t)$. Thus, if p_i denotes the peak factor for this process, using the relation $\bar{S}_\tau(\omega_i, \zeta_i) = p_i \sqrt{\lambda_{0,ii}}$, one has

$$\lambda_{0,ii} = \frac{1}{p_i^2} \bar{S}_\tau^2(\omega_i, \zeta_i) \quad (14)$$

It is shown in Ref. 3 that for responses to a wide-band input

$$\nu_i = \frac{1}{\pi} \left(\frac{\lambda_{2,ii}}{\lambda_{0,ii}} \right)^{\frac{1}{2}} \approx \frac{\omega_i}{\pi} \quad (15)$$

$$\delta_i = \left[1 - \frac{\lambda_{1,ii}^2}{\lambda_{0,ii}\lambda_{2,ii}} \right]^{\frac{1}{2}} \approx \left[1 - \frac{1}{\sqrt{1-\zeta_i^2}} \left(1 - \frac{2}{\pi} \tan^{-1} \frac{\zeta_i}{\sqrt{1-\zeta_i^2}} \right) \right]^{\frac{1}{2}} \approx 2 \left(\frac{\zeta_i}{\pi} \right)^{\frac{1}{2}} \quad (16)$$

These relations are exact for responses to white-noise inputs and are close approximations for responses to earthquake-type, wide-band inputs. Using these expressions in Eqs. 11 and 13, the peak factor, p_i , for the i -th normal coordinate is computed in terms of the corresponding modal frequency and damping coefficient. Substituting this factor in Eq. 14, the first moment, $\lambda_{0,ii}$, is obtained directly in terms of the response spectrum ordinate associated with mode i . Furthermore, using Eqs. 14-16, the second and third spectral moments are also obtained in terms of the response spectrum ordinate as

$$\lambda_{1,ii} = \frac{\omega_i \sqrt{1-4\zeta_i/\pi}}{p_i^2} \bar{S}_\tau^2(\omega_i, \zeta_i) \quad (17)$$

$$\lambda_{2,ii} = \frac{\omega_i^2}{p_i^2} \bar{S}_\tau^2(\omega_i, \zeta_i) \quad (18)$$

Thus, substituting Eqs. 14, 17, and 18, together with the previously given expressions for $\rho_{m,ij}$, in Eq. 5, the moments of the response power spectral density are obtained directly in terms of the response spectrum ordinates and the modal properties of the structure. With these moments known, the statistical quantities of response are evaluated as described in the preceding section. In particular, if $\bar{R}_{i\tau} = \Psi_i \bar{S}_\tau(\omega_i, \zeta_i)$ denotes the maximum response in mode i , this development yields

$$\sigma_R = \left(\sum_i \sum_j \frac{1}{p_i p_j} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right)^{\frac{1}{2}} \quad (19)$$

$$\sigma_{\dot{R}} = \left(\sum_i \sum_j \frac{\omega_i \omega_j}{p_i p_j} \rho_{2,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right)^{\frac{1}{2}} \quad (20)$$

$$\bar{R}_\tau = p \sigma_R = \left(\sum_i \sum_j \frac{p^2}{p_i p_j} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right)^{\frac{1}{2}} \quad (21)$$

$$\sigma_{R_\tau} = q \sigma_R = \left(\sum_i \sum_j \frac{q^2}{p_i p_j} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right)^{\frac{1}{2}} \quad (22)$$

where p and q are the peak factors for the response process, $R(t)$, and are obtained from Eqs. 11-13 using $\nu = \sqrt{\lambda_2/\lambda_0}/\pi$ and $\delta = \sqrt{1-\lambda_1^2/\lambda_0\lambda_2}$. Another useful response quantity is the mean response frequency, $\bar{\omega} = \pi\nu$, which is of interest in problems of structural fatigue. Using Eqs. 19 and 20,

$$\bar{\omega} = \frac{\sigma_{\dot{R}}}{\sigma_R} = \left(\frac{\sum_i \sum_j \frac{\omega_i \omega_j}{p_i p_j} \rho_{2,ij} \bar{R}_{i\tau} \bar{R}_{j\tau}}{\sum_i \sum_j \frac{1}{p_i p_j} \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau}} \right)^{\frac{1}{2}} \quad (23)$$

Observe that this frequency is a weighted average of the modal frequencies.

It is important to note in the preceding expressions that, since $\bar{S}_\tau(\omega_i, \zeta_i)$ by definition is positive, the sign of $\bar{R}_{i\tau}$ is always the same as that of Ψ_i . This sign could be positive or negative, depending on the modal characteristics of the structure and on the direction of input. It follows, then, that the cross terms in Eqs. 19-23 would have negative values when the effective participation factors for the two modes assume opposite signs.

In many practical applications, the mean of the maximum response is all that is needed. A simplification of Eq. 21 is, therefore, of special interest. It is noted that the ratios p/p_i

in this equation are all near unity. (This ratio is nearest to unity for the mode which has the closest frequency to the average frequency, $\bar{\omega}$, and it decreases with increasing mode number.) This is because of the slow variation of the peak factor with the parameter $\nu\tau$; see Fig. 2. Neglecting these ratios in Eq. 21, which is equivalent to assuming a constant peak factor, is therefore a possible simplification. This yields

$$\bar{R}_\tau = \left(\sum_i \sum_j \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau} \right)^{\frac{1}{2}} \quad (24)$$

Note that with this simplification, the mean response is directly given in terms of the modal responses and the coefficients $\rho_{0,ij}$, i.e., there is no need to compute the spectral moments from Eq. 5. Also note that this expression for the mean response is independent of the duration, except that which is implicit in the specified response spectrum. A corresponding simplification for the other response quantities is not possible except for $\bar{\omega}$, which after multiplying the numerator and the denominator in Eq. 23 by ρ , and neglecting the ratios ρ/ρ_i , yields

$$\bar{\omega} = \left(\frac{\sum_i \sum_j \omega_i \omega_j \rho_{2,ij} \bar{R}_{i\tau} \bar{R}_{j\tau}}{\sum_i \sum_j \rho_{0,ij} \bar{R}_{i\tau} \bar{R}_{j\tau}} \right)^{\frac{1}{2}} \quad (25)$$

Observe that with this simplification, $\bar{\omega}$ becomes the average of the modal frequencies as weighted by the maximum modal responses.

Another simplification in the response expressions is possible when the structural frequencies are well separated. As shown in Fig. 1, the coefficients $\rho_{m,ij}$ diminish in such cases. Therefore, all cross terms in the expressions for the response, i.e. Eqs. 5 and 19-25, can be dropped. In particular, Eq. 24 in this case reduces to

$$\bar{R}_\tau = \left(\sum_i \bar{R}_{i\tau}^2 \right)^{\frac{1}{2}} \quad (26)$$

This is the well known square-root-of-sum-of-squares (SRSS) rule for modal combination. It is clear from this derivation that the SRSS rule for the mean response is only adequate for structures with well spaced frequencies. When modal frequencies are closely spaced, this rule may lead to erroneous results and should not be used.

APPLICATION TO EARTHQUAKE LOADING

In applying the above procedure to earthquake loading, the validity of several assumptions inherent in the derivation of the method must be examined. These assumptions are: (a) the input is stationary; (b) the input is Gaussian; (c) the input is wide banded; and (d) the response is stationary. Whereas earthquake-induced ground motions are inherently nonstationary, the strong phase of such motions is often nearly stationary. Since the peak response usually occurs during this phase, it is reasonable, at least for the purpose of a response spectrum method, to assume a stationary process. This assumption would clearly become less accurate for short-duration, impulsive earthquakes. The assumption of Gaussian input is acceptable on the basis of the central limit theorem, since the earthquake ground motion is the accumulation of a large number of randomly arriving pulses [1]. The wide-band assumption for the earthquake motion is acceptable based on investigations in Refs. 3 and 5. Finally, for the assumption of stationary response, it is well known (e.g., Ref. 6) that the response of a not-too-lightly damped oscillator to a wide-band input reaches stationarity in just a few cycles. Thus, this assumption should be acceptable for structures whose fundamental periods are several times shorter than the strong-phase duration of the ground motion. These considerations also suggest that the duration of the strong phase of the ground motion is the appropriate value for the parameter τ in the response spectrum method.

It is clear from the above discussion that the response spectrum method for earthquake loadings will be most accurate for earthquakes with long, stationary phases of strong shaking and for not-too-lightly damped structures whose fundamental periods are much longer than the duration of earthquake. Through a large number of example studies, it has been found that the procedure is quite accurate for typical structures and earthquakes (see the example below). It has also been found in these studies that Eq. 24 for the mean response closely approximates the maximum response for a deterministic ground motion with a non-smooth response spectrum. Maximum errors in such applications are expected to be within 10 to 30 percent, depending on the response frequency.

As was indicated before, several formulations for the mean of the peak response have previously been given [7,8,10]. These are generally similar to Eq. 24 of the present formulation with different expressions given for $\rho_{0,ij}$. In the method of Rosenblueth et al. [7], which is the most widely known, this coefficient is given as a function of the modal frequencies and damping ratios as well as the duration of input. Unfortunately, no specific definition of the duration (i.e., total duration or strong-phase duration) was given in their development. This ambiguity remains to be a shortcoming of their formulation. (Note that in the present formulation $\rho_{0,ij}$ is independent of duration.) In the methods of Refs. 8 and 10, no closed-form expressions for this coefficient were given. These methods require much more computational effort and, therefore, are less desirable.

EXAMPLE APPLICATION

As an example application of the proposed procedure, the responses of a 5-story building structure to a set of 20 simulated ground motions are studied. The building has uniform floor masses and story stiffnesses with the typical floor plan and properties as shown in Fig. 3. It is subjected to ground motions in the x direction only; however, because of asymmetry about the x axis, the center of mass at each floor has a rotational as well as a translational degree of freedom. As a consequence of this, the structure has closely spaced frequencies, as shown in Fig. 3. The ground motions were simulated using a computer program by Ruiz et al. [9]. These were generated as samples of filtered, Gaussian shot noise with a Kanai-Tajimi [5] power spectral density. An intensity function similar to that of a type-B earthquake, as defined by Jennings et al. [4], was used for this purpose. It includes a stationary strong-motion phase of 11 seconds and is scaled to produce a mean peak ground acceleration of $0.5g$. A sample of the simulated ground motions is illustrated in Fig. 4.

Using numerical integration, the response spectrum associated with each individual ground motion was computed. These were averaged to obtain the mean spectra shown in Fig. 5. These spectra were used with the proposed method to compute the various responses of the structure. To examine these results, time-history analyses were made of the building responses to each individual ground motion. Samples of such results were used to compute simulated values of the means and standard deviations of peak responses. Table 1 summarizes these results for several selected responses of the building. Numbers inside parenthesis in this table denote percent errors relative to the simulated values. As can be observed, the response spectrum method for the mean (Eqs. 21 or 24) and the standard deviation (Eq. 22) of peak responses closely predicts the simulated values. For the mean response, Eq. 24 appears to give results nearly as good as Eq. 21. However, Eq. 26, which is equivalent to the SRSS method, is in gross error. This is clearly due to the closeness of frequencies for the structure under consideration.

ACKNOWLEDGEMENT

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Table 1. Summary of Results for Example Structure

Response Description	R_r						σ_{R_r}		
	simul.	Eq. 21	Eq. 24	Eq. 26	simul.	Eq. 22			
Roof displ., ft.	0.264	0.261 (-1)	0.256 (-2)	0.196 (-26)	0.052	0.048 (-8)			
Roof rot. $\times 10^2$, rad.	0.231	0.245 (+6)	0.263 (+14)	0.536 (+132)	0.044	0.044 (+1)			
Roof accel., g.	1.430	1.416 (-1)	1.387 (-3)	1.044 (-27)	0.278	0.250 (-10)			
Roof ang. accel., rad/sec ² .	0.402	0.430 (+7)	0.446 (+11)	0.929 (+131)	0.068	0.074 (+9)			
Base shear, kip.	1848	1840 (-0)	1830 (-1)	1386 (-25)	352	318 (-10)			
Base torque, kip-ft.	5454	5781 (+6)	6163 (+13)	12599 (+131)	1116	1037 (-6)			

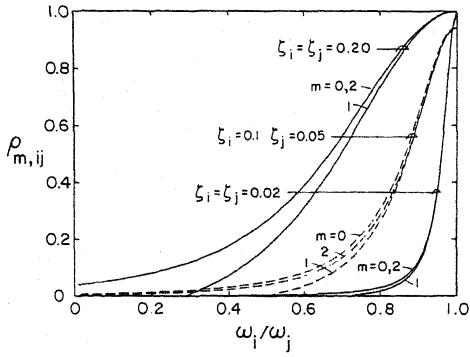


Fig. 1. Coefficients $\rho_{m,ij}$ for Response to White Noise

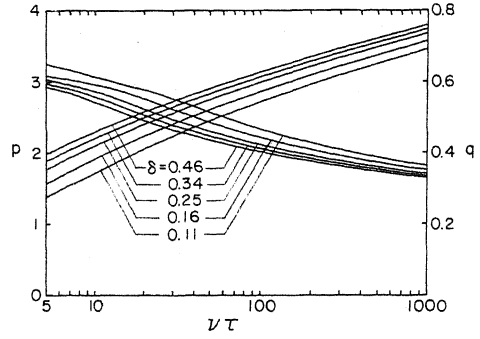


Fig. 2. Peak Factors for Stationary Gaussian Process

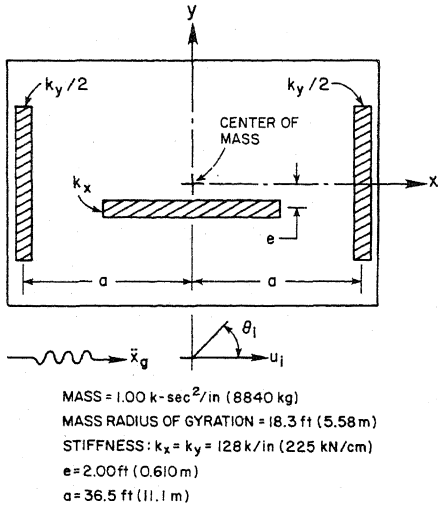


Fig. 3. Typical Floor Plan and Properties of Example Structure

Modal Properties		
Mode	Freq., cps	Damp. ratio
1	2.00	0.05
2	2.11	0.05
3	5.84	0.05
4	6.17	0.05
5	9.20	0.05
6	9.72	0.05
7	11.80	0.05
8	12.50	0.05
9	13.50	0.05
10	14.20	0.05

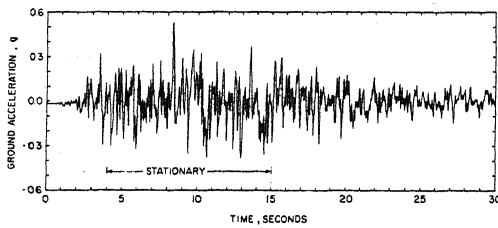


Fig. 4. Sample of Simulated Ground Motion

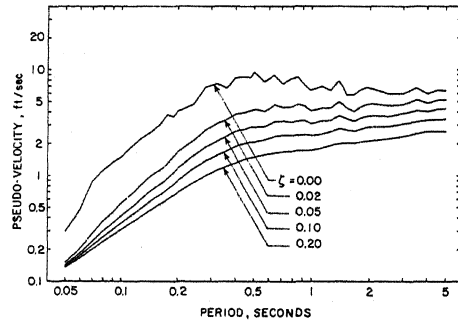


Fig. 5. Mean Response Spectra for Simulated Ground Motions