A MEASURE OF DURATION OF STRONG GROUND MOTION

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SUMMARY

A simple procedure is proposed for estimating the strong-motion duration s_o and the r.m.s. strong-motion acceleration σ_o of earthquake ground motion records. The strong-motion duration is found to be nearly proportional to the quantity I_o/a_{max}^2, where a_{max} is the maximum ground acceleration and I_o is the Arias Intensity (i.e., the integral of the squared accelerations). A less important factor influencing the relationship between s_o and I_o/a_{max}^2 is the predominant period T_o of the strong phase of the accelerogram. According to a simplified form of the definition, duration is directly proportional to I_o/a_{max}^2; the proportionality factor is about 7.5. The strong-motion duration, as proposed, provides an important link between the major spectral representations of earthquake ground motion: the Fourier amplitude of acceleration, the acceleration spectral density function and the response spectrum.

INTRODUCTION

The duration of strong shaking may significantly affect the damage caused by an earthquake and plays an important role in many problems in earthquake engineering. The response of lightly damped linear systems and of yielding or strength-degrading nonlinear systems is known to depend importantly on the duration of shaking. Duration is also a key parameter affecting the likelihood of occurrence of the phenomena of low-cycle fatigue of structures or liquefaction of soil during earthquakes.

No single quantitative measure of the duration of strong shaking is in common usage in earthquake engineering. Studies of the dependence of duration on magnitude (Housner, 1965) and on distance and magnitude (Esteva and Rosenblueth, 1964) are not based on formal, quantitative definitions of duration. Two crude but simple measures of duration have been mentioned in the engineering literature. The first defines duration as the time interval between the first and last peaks equal to or greater than a given level, usually 0.05 g, on the accelerogram (Page et al., 1975). The second definition is based on the concept of cumulative energy obtained by integrating squared accelerations: duration is the time interval required to accumulate a prescribed fraction of the total energy, for example, 95 percent (Husid et al., 1969) or 90 percent (Trifunac and Brady, 1975). Bolt (1973) suggested that both these definitions could also be applied to sinusoidal components of the earthquake motion occurring within different narrow frequency bands.

Geophysical models of faulting mechanisms suggest that duration is re-

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lated to the time interval of fault rupture (i.e., fault dimension divided by average velocity of rupture propagation) which is in turn related to the "corner period" of the far-field Fourier amplitude spectrum of ground displacement (Brune, 1976).

BACKGROUND

In this section, we examine two well-known and closely related functions which describe the frequency content of earthquake ground acceleration. They are needed to understand the relationship between various measures of intensity of earthquake motion and to permit introduction of the proposed definition of the duration of strong shaking.

The Fourier amplitude spectrum (F.a.s.) of the ground acceleration a(t) is the absolute value of the Fourier transform of a(t).

$$A(\omega) = \left| \int_{-\infty}^{\infty} a(t)\ e^{-i\omega t}\ dt \right| = \left| \int_{0}^{t_o} a(t)\ e^{-i\omega t}\ dt \right| \qquad (1)$$

in which ω = frequency of vibration (rad/sec), $i = \sqrt{-1}$, and t_o = length of the digitized accelerogram (in seconds). The squared Fourier amplitude spectrum $A^2(\omega)$, indicates how the total energy in the earthquake motion is distributed over the frequency axis. Its integral over all frequencies is directly related to the total motion "energy" I_o, or the Arias Intensity (1970), as follows:

$$I_o = \int_0^{t_o} a^2(t)\ dt = \int_{-\infty}^{\infty} a^2(t)\ dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} A^2(\omega)\ d\omega = \frac{1}{\pi} \int_0^{\infty} A^2(\omega)\ d\omega \qquad (2)$$

The equality in the center of Eq. 2 is Parseval's relation, while the equality on the right side results from the fact that since a(t) is real, F.a.s. is an even function of frequency (i.e., $A(-\omega) = A(\omega)$).

Due to the randomness of the phasing of contributing sinusoids, the F.a.s. of recorded ground motions appears highly variable; the size and location of peaks and valleys are quite sensitive to details of the computation (e.g., choice of time and frequency intervals). Only the expected value of $A^2(\omega)$ (obtained by appropriate local averaging or smoothing over frequency) is of interest. In what follows, $A(\omega)$ will refer to the r.m.s. (root-mean-square) Fourier amplitude of acceleration, i.e., the square root of the mean of $A^2(\omega)$.

Closely related to $A^2(\omega)$ is the power spectral density function $G(\omega)$ which indicates how the ground motion "power", or energy per unit time, is distributed over all the frequencies. The function is:

$$G(\omega) \propto \frac{1}{s_o} A^2(\omega) \qquad (3)$$

in which s_o is the yet to be determined strong-motion duration. $G(\omega)$ is the one-sided ($\omega \geq 0$) spectral density function (s.d.f.). Implied by Eq. 3 is an idealization of the earthquake as a segment of limited duration of a random process with constant spectral density function (i.e., a random process that is stationary in the wide sense). A basic property of the s.d.f. is that the mean square acceleration σ_o^2 is obtained by integrating $G(\omega)$ over all frequencies.

$$\sigma_o^2 = \int_0^\infty G(\omega)\ d\omega \tag{4}$$

in which σ_o = r.m.s. strong-motion acceleration. Also,

$$I_o = s_o \sigma_o^2 \tag{5}$$

In words, the total ground motion intensity I_o is distributed uniformly, at constant average power σ_o^2 over the strong-motion interval s_o. Combining Eqs. 2-5 yields the value π^{-1} for the proportionality constant in Eq. 3.

$$G(\omega) = \frac{1}{\pi s_o} A^2(\omega) \tag{6}$$

A relationship of this form is given by Bendat (1958).

STRONG-MOTION DURATION

The question at hand is how to determine quantitatively the strong-motion duration s_o and the corresponding strong-motion r.m.s. acceleration σ_o from earthquake records. Eq. 5 suggests that large values of s_o imply small r.m.s. values σ_o. At one extreme, when s_o equals the record length t_o, we obtain $\sigma_o = [I_o/t_o]^{\frac{1}{2}}$. For very small values of s_o, the r.m.s. acceleration can become as large as the maximum amplitude of acceleration a_{max}; this happens when $s_o = I_o/a_{max}^2$. These extreme choices for s_o are undesirable because they imply values of σ_o which are not related in a consistent manner to the actual maximum acceleration a_{max}. This observation is the key to the proposed method for evaluating strong-motion durations of earthquake records. The idea is that an approximate relationship must exist between σ_o and a_{max}. The relationship is of course probabilistic in nature. The theory of stationary Gaussian random functions provides a prediction of the most probable value of the peak factor $r = a_{max}/\sigma_o$ during a known time interval s_o of steady strong shaking. Specifically, the value of a_{max}/σ_o, which is exceeded <u>once on the average</u> during the interval s_o (or which has a probability of e^{-1} of not being exceeded during s_o) is approximately:

$$r = \frac{a_{max}}{\sigma_o} = \begin{cases} \sqrt{2\ \ell n(2s_o/T_o)} & s_o \geq 1.36\ T_o \\ \sqrt{2} & s_o \leq 1.36\ T_o \end{cases} \tag{7}$$

in which T_o = predominant period of the earthquake motion, and s_o/T_o = number of cycles during the time interval s_o. Eq. 7 is derived on the basis of the common assumption that the crossings of a specified, relatively high threshold occur as a Poisson arrival process. For stationary Gaussian random processes, this assumption is asymptotically correct for very high thresholds (Cramer, 1966), but the formula (Eq. 7) is inappropriate for very low thresholds (Vanmarcke, 1975). The lowest peak factor permitted is $\sqrt{2}$, the ratio between the amplitude and the r.m.s. value for a simple sinusoid. The condition $a_{max} = \sqrt{2}\ \sigma_o$ implies $s_o = (e/2)T_o \simeq 1.36\ T_o$. The value of r predicted by Eq. 7 is relatively insensitive to the choice of T_o within the range of most common values (0.2 sec - 0.6 sec).

Assuming T_o is known, Eqs. 5 and 7 can be viewed as a system of two equations and two unknowns, s_o and σ_o. The solution for s_o is implicit in the following equation:

$$s_o = r^2 \frac{I_o}{a^2_{max}} = \begin{cases} [2\ \ln(2s_o/T_o)](I_o/a^2_{max}) & s_o \geq 1.36\ T_o \\ 2\ I_o/a^2_{max} & s_o \leq 1.36\ T_o \end{cases} \quad (8)$$

The ratio (I_o/a^2_{max}) can easily be computed for any strong-motion earthquake record.

The solution implied in Eq. 8 is plotted in Fig. 1. Note that the duration s_o varies nearly as a linear function of I_o/a^2_{max} for any given value of T_o. The value T_o is varied within the range 0.2 sec - 0.6 sec.

It would be fair to criticize the choice of the specific exceedance probability made in determining the theoretical peak factor (Eq. 7). It is to some extent arbitrary. Clearly, exceedance probabilities are different for different accelerograms, but it is impractical to attempt to estimate these probabilities. Any reasonable alternative to Eq. 7 (involving a different exceedance probability or a somewhat different expression for the peak factor) will lead to analogous results for the parameters s_o and σ_o and the basic format of the solution (Eq. 8) will not change.

Theoretical peak factors are not very sensitive to the exact choice of the exceedance probability, assuming it is the central range. For example, if a 50% exceedance probability is selected (instead of $e^{-1} = 0.37$), the corresponding median peak factor is given by $\sqrt{2\ \ln(2.8\ s_o/T_o)}$ (provided it is not smaller than $\sqrt{2}$). Within the range of values of practical interest for the ratio s_o/T_o (between 1.3 and 500), the ratio of the peak factors $r_{0.5}/r_{e-1}$ is about 1.05. This would lead to a 10% increase in the predicted strong-motion duration if $p = 0.50$ is selected.

SOME NUMERICAL RESULTS

As an example, consider in Fig. 2 the E-W component of an accelerogram recorded on bedrock at San Rocco, Friuli (Italy) in 1976. The peak acceleration is $a_{max} = 0.085\ g = 83.4\ cm/sec^2$ and the Arias Intensity $I_o = 2734\ cm^2/sec^3$; this leads to $I_o/a^2_{max} = 0.393$. The dominant period within the interval of strong shaking is obtained by counting zero-crossings; we estimate $T_o \simeq 0.20$ seconds. The trial-and-error solution based on Eq. 8 is: $s_o = 2.6$ sec and $\sigma_o = a_{max}/2.55 = 0.033$ g.

Statistical studies were carried out based on a data set of 140 horizontal components of 70 western United States strong-motion records corresponding to different event/site pairs. It is the same data set selected by McGuire and Barnard (1977) and used by Vanmarcke (1979). Eleven sites (22 records) were classified as "rock" sites, and 59 sites (118 records) as "soil" sites.

The mean duration for all records is 9.3 sec, the median is 9.0 sec, and the standard deviation is 8.7 sec. For the 32 records with a "near-field" designation, the mean and the standard deviation are 6.3 and 5.5 sec, respectively. For the "far-field" records, the mean is 10.2 sec, and the standard deviation is 9.4 sec. For the records on "rock", the mean duration is 5.1 sec, and for the records on "soil", it is 10.1 sec.

Histograms of the strong-motion durations (s_o) peak factors (r) and predominant periods (T_o) are shown in Figures 3, 4 and 5 respectively. The mean, the meadian and the standard deviation are 2.67, 2.75 and 0.38, respectively for the peak factor and 0.34 sec, 0.32 sec and 0.18 sec, respectively, for the predominant period. Further information, including various regressions on distance and magnitude, may be found in Ref. 14.

SIMPLIFIED DEFINITION OF DURATION

The arbitrariness of the choice of exceedance probability (e.g., e^{-1} versus 0.5) and the near-linearity in the relationship between s_o and I_o/a^2_{max} motivates the proposal for a simplified version of the definition of strong-motion duration. It is based on the assumption that the peak factor is constant. A reasonable value is r = 2.75, the median value obtained in the afore-mentioned statistical study. This leads to the following 'preliminary estimate' for s_o:

$$s_o = (2.75)^2 \frac{I_o}{a^2_{max}} \simeq 7.5 \frac{I_o}{a^2_{max}} \qquad (9)$$

This preliminary solution may be used to replace s_o in the expression on the right side of Eq. 8, leading to an approximate explicit formula for s_o as a function of I_o/a^2_{max} and T_o. We have:

$$s_o = \left[2\ \ell n\left(\frac{15\ I_o}{T_o\ a^2_{max}}\right)\right]\frac{I_o}{a^2_{max}} = \left[5.42 - 2\ \ell n\ T_o + 2\ \ell n\left(\frac{I_o}{a^2_{max}}\right)\right]\frac{I_o}{a^2_{max}} \qquad (10)$$

For example, for the 1976 San Rocco accelerogram in Fig. 2, Eq. 9 gives s_o = 7.5 x 0.393 = 2.95 sec, and based on T_o = 0.2 sec, Eq. 10 yields s_o = 2.65 Fig. 6 shows a scattergram of the durations obtained (a) from Eq. 8, and (b) on the basis of the simplified "definition" (Eq. 9) for the set of 140 accelerograms.

CONCLUSIONS

The main features of the proposed definition of strong-motion duration s_o are the guarantees that (i) the total energy I_o is preserved, (ii) a consistent relationship exists between a_{max} and the strong-motion r.m.s. acceleration σ_o, and (iii) it links the earthquake parameters I_o, a_{max}, σ_o, s_o and T_o by simple relationships (Eqs. 6 and 7-10).

The characterization of strong earthquake motion as a limited-duration segment of stationary stochastic process permits effective use of methods of random vibration to predict seismic response of many structures (Vanmarcke, 1976). The limited duration captures the essential transient character of earthquakes, while the spectral density function represents the frequency content during the strong phase of the earthquake. Random vibration analysis also yields analytical predictions of earthquake response spectra. The strong-motion duration, as proposed, provides a crucial link between the major spectral representations of earthquake ground motion: the Fourier amplitude of acceleration, the acceleration spectral density function (see Eq. 6) and the response spectrum.

The proposed procedure can also be used to obtain durations for different frequency-filtered components of earthquake records (as suggested by Bolt, 1973), as well as for time histories of ground velocity or ground displacement. In each case, the key quantity is the ratio of the "total energy" to the square of the maximum amplitude.

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Fig. 1 - Relationship Between Strong Motion Duration and I_o/a^2_{max}

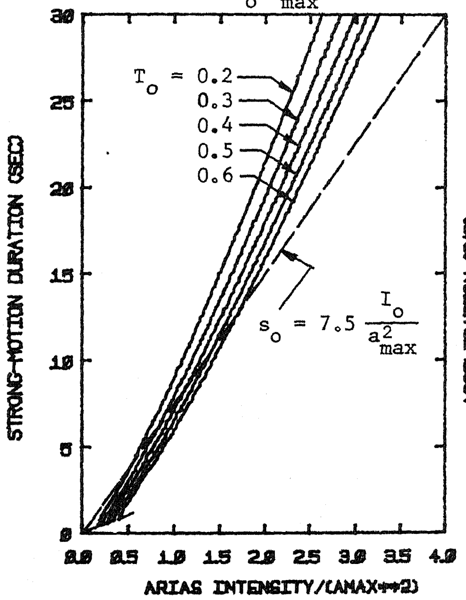
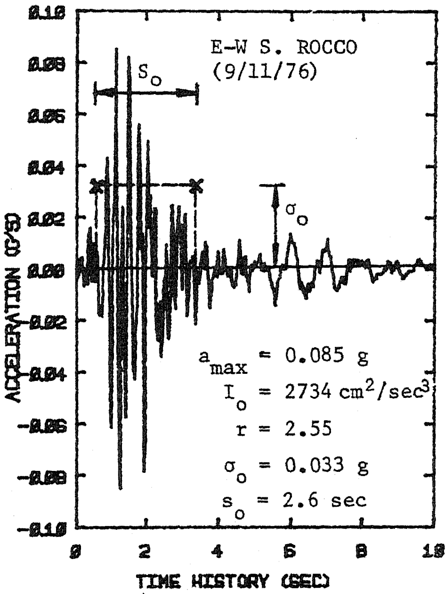


Fig. 2 - Accelerogram of S. Rocco Friuli Earthquake



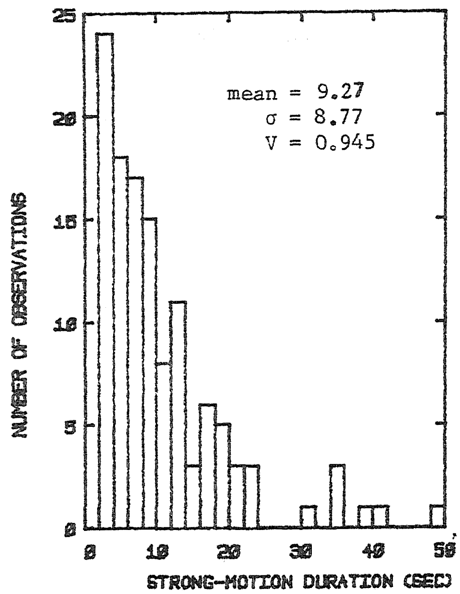


Fig. 3 - Histogram for Strong-Motion Duration

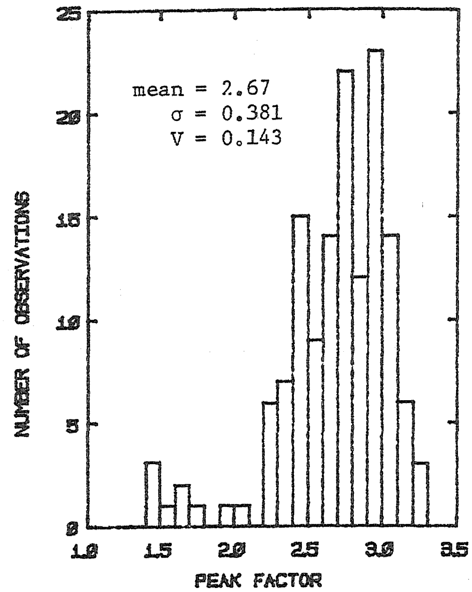


Fig. 4 - Histogram for Peak Factor

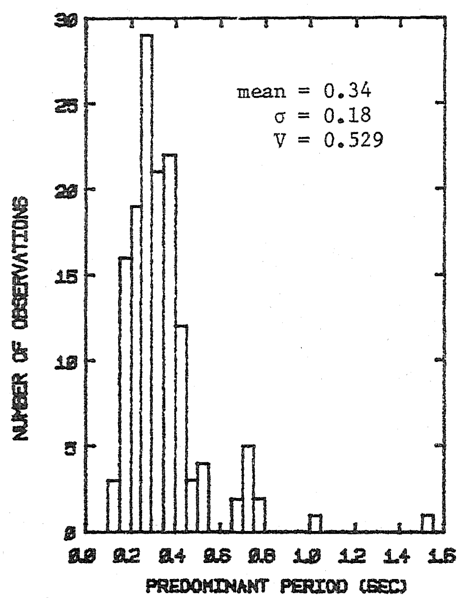


Fig. 5 - Histogram for Predominant Period

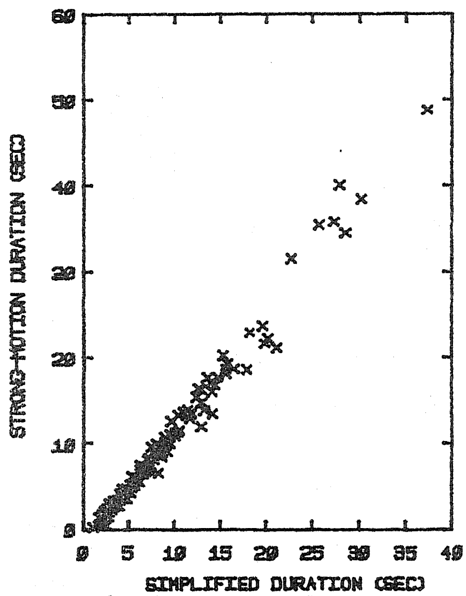


Fig. 6 - Scattergram for s_o versus Simplified s_o