PROBABILISTIC SEISMIC RESPONSE OF SIMPLE INELASTIC SYSTEMS

by

E. H. Vanmarcke[I] and D. Veneziano[II]

SYNOPSIS

The paper presents a probabilistic dynamic analysis of the relative displacement response to earthquake-like excitation of simple elasto-plastic systems and of bilinear hysteretic systems affected by gravity. Nonlinear response measures are expressed in terms of the statistics of the response of an associated linear oscillator which is characterized by the initial properties of the hysteretic system. For elasto-plastic systems, new approximate analytical results are presented for the probability distribution of the plastic drift, the ductility factor and the time to first exceedance of a specified level of plastic deformation. A Markov model is proposed to study the response of bilinear hysteretic systems. For gravity-affected systems, results are presented for the probability of collapse and for the expected number of cycles to failure. These results are compared with those obtained by Husid(1).

INTRODUCTION

For many structural and mechanical systems it is permissible to allow for plastic deformation during severe but infrequent random vibratory motions such as those due to earthquakes. To take advantage of their plastic capacity often provides an efficient means of absorbing energy and limiting the oscillation amplitudes. A problem of considerable interest in the safety analysis of structures during strong-motion earthquakes, therefore, is to determine the probability distribution of important measures of inelastic behavior, e.g., the ductility factor and the time to collapse. The type of force-deformation relationship considered in this paper is a bilinear hysteretic system characterized by the initial stiffness k_1, the slope ratio k_2/k_1, and the yield displacement as shown in Figure 1. The effect of gravity on the inelastic response measures is given particular attention.

Several investigators (Refs. 1, 2, 3, among others) obtained response statistics of simple nonlinear hysteretic systems through time history analyses, using real and computer-generated ground motions. Other work (e.g., Ref. 4) in which it is attempted to obtain rigorous random vibration solutions, attests to the fact that the mathematical complexity of the problem is formidable.

The approach outlined in this paper is based on the idea that, at

[I] Assistant Professor of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

[II] Research Assistant, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

times when no plastic action occurs, i.e., between response excursions into the inelastic domain, the inelastic system behaves like an elastic oscillator. The total displacement then consists of (i) an oscillatory zero-mean linear elastic component, and (ii) an inelastic deformation, d, which remains constant as long as the elastic motion does not cross over into the domain bounded by the positive and negative yield levels (which may be different for different levels of plastic deformation, d). Of course, for elasto-plastic (E-P) systems, the positive and negative yield levels have the same absolute value and do not depend on the level of inelastic deformation, d.

The inelastic response is modeled as a continuous-time random process with as "state" variable the plastic deformation, d. Changes or transitions from one value of d to another can occur only at times when the elastic response component exceeds the yield level. The transition probabilities (derived from linear probabilistic analysis) are state-dependent, except in the case of E-P systems. They can be expressed in terms of the characteristics of the ground motion (intensity, parameters of the power spectrum) and of an associated linear system (period and damping ratio) decribed below. It is convenient to analyze E-P system response first, and then to extend the results of this analysis to bilinear systems.

THE ASSOCIATED LINEAR SYSTEM

Let Y(t) represent the displacement response of an E-P structure (Fig. 1b) with a yield level Y=a to a random excitation $\ddot{u}_o(t)$. At the start of the motion and until Y(t) crosses the yield level for the first time, the response of the E-P system is identical to that of an associated linear system, such as that shown in Fig. 1c. It is characterized by a spring with stiffness k_1 and by a dashpot with damping coefficient c. Problems surrounding the onset of plastic deformation are equivalent to a first-crossing problem for the associated linear oscillator. Also, before plastic deformations occur, the inelastic response Y(t) is described by the linear differential equation

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = -\ddot{u}_o(t) \tag{1}$$

where $\omega_n = (k_1/m)^{1/2}$ is the undamped natural frequency and $\zeta = c/2m\omega_n$ is the damping ratio. In between yield level crossings, the E-P system also behaves like a linear oscillator. Suppose that at some known time t, the most recent yield level crossing brought the total plastic deformation up to the value D(t) = d. The total displacement at t will then consist of a permanent set d and a linear elastic component X(t), i.e., Y(t) = d + X(t). The process D(t) changes rather abruptly when plastic action occurs. For d = 0, i.e., before any plastic yield occurred, we have Y(t) = X(t). The differential equation describing the elastic part, X(t), of the total displacement of the E-P system, in between plastic excursions, has the form

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = -\ddot{u}_o(t) \tag{2}$$

A sample function of the process X(t) is shown in Fig. 2b. It is obtained by subtracting the plastic set, shown in Fig. 2c, from the total E-P response, seen in Fig. 2a. The permanent set D(t) remains constant when the absolute value of X(t) is smaller than the yield level a. Each time S(t) exceeds the yield displacement, however, inelastic action is known to occur. The total permanent set developed in the time interval 0 to t is the sum of a number of individual contributions, each associated with a single crossing of $|X(t)| = a$. Much can be learned about the sizes of these contributions and about the length of the time intervals between yield level crossings by examining the response of the associated linear system shown in Fig. 1c. In particular, it will be useful to focus attention on the excursions of X(t) outside the range (a,-a).

SOME PERTINENT RESULTS FOR SIMPLE LINEAR SYSTEMS

Let the linear elastic system shown in Fig. 1c be subjected to a stochastic support motion $\ddot{u}_o(t)$. The response quantity of interest is the relative displacement $x = u-u_o$ and the equation of motion is given by Eq. 2. Assume that the support motion can be modeled as a zero-mean stationary random process characterized by a wide-band spectral density function $G(\omega)$ and an equivalent duration s. The (one-sided) spectral density function $G_X(\omega)$ of the relative displacement may then be simply expressed in terms of the input spectrum $G(\omega)$ and the transfer function of the system. We have[5]

$$G_X(\omega) = G(\omega)/[(\omega^2 - \omega_n^2)^2 - 4\zeta^2\omega_n^2\omega^2] \tag{3}$$

A typical sample function of the response of a lightly damped linear structure to wide-band random excitation is shown in Fig. 3. It has the appearance of a modulated sinusoid with a period equal to the structure's natural period. Focusing attention on a pair of fixed relatively high threshold levels $x = \pm a$, it is of interest to observe that the peaks of X(t) whose values are outside of the range (a,-a) tend to occur in groups or "clumps", i.e., successive peak values of X(t) tend to be significantly correlated.

In Fig. 3, N_a denotes the random number of consecutive peaks whose values lie outside the range (a,-a). The senior writer has recently shown[6] that degree of correlation among successive peaks importantly depends on a factor $q = (1 - \lambda_1^2/\lambda_0\lambda_2)^{1/2}$, where $\lambda_i = \int_0^\infty \omega^i G_X(\omega)d\omega$ = i^{th} moment of the spectral density function $G_X(\omega)$.
It can be shown that q lies between 0 and 1 and that it is a measure of the spread of $G_X(\omega)$ about a center frequency. It is well-known that $\lambda_0 = \sigma_x^2$, i.e., the area under the power spectrum, equals the variance of X(t). For stationary Gaussian processes, the expected value of N_a takes approximately the following form[6]

$$E[N_a] = (1 - \exp\{-\sqrt{\pi/2}\, rq\})^{-1} \tag{4}$$

where $r = a/\sigma_x$ = the threshold level normalized with respect to the standard deviation of X(t). $E[N_a]$ is referred to as the average clump size. Note that for large values of rq, $E[N_a] \to 1$ and for $rq \ll 1$,

$E[N_a] \simeq (2/\pi)^{1/2}(rq)^{-1}$.

A well-known result in random vibration analysis, due to S. O. Rice[7] is that the mean ν_a of crossings with positive slope of a level $X=\underline{a}$ by a stationary Gaussian process $X(t)$, is

$$\nu_a = \frac{1}{2\pi}(\lambda_2/\lambda_0)^{1/2} \exp\{-a^2/2\sigma_x^2\} = \nu_0 \exp\{-r^2/2\} \tag{5}$$

in which $\nu_0 = (\lambda_2/\lambda_0)^{1/2}/2\pi$ = the mean rate of zero crossings with positive slope. Also, the mean rate of excursions outside the range (a,-a) equals $2\nu_a$. Finally, the average number, μ_a, of "clumps" per time unit is approximately

$$\mu_a = 2\nu_a/E[N_a] = 2\nu_0(1 - \exp\{-\sqrt{\pi/2}\, rq\}) \exp\{-r^2/2\} \tag{6}$$

If the excitation is a Gaussian white noise, then $G_X(\omega) = G_0|H(\omega)|^2$, then[5,6]

$$\lambda_0 = \sigma_x^2 = \frac{\pi G_o}{4\zeta\omega_n^3} \qquad \nu_0 = \frac{1}{2\pi}\omega_n \qquad q \simeq \frac{2}{\sqrt{\pi}}\zeta^{1/2} \quad \text{if } \zeta \text{ small} \tag{7}$$

The factor q is plotted as a function of the damping ratio ζ in Fig. 4. It may be noted that the parameters λ_0, ν_0 and q are relatively easily obtainable for an arbitrary spectral density function $G_X(\omega)$.

THE BASIC MODEL

In the previous section it has been argued that for simple elastic systems the crossings of a relatively high threshold level are likely to occur in clumps when the viscous damping ratio ζ is small. Yield level crossings of the elasto-plastic response also tend to occur in clumps. For elastic systems, the average rate at which clumps of crossings of the level $\underline{a}$ occur equals μ_a and the average time between clumps is $1/\mu_a$. The larger the mean clump size, $E[N_a]$, the larger the average time between successive clumps. For E-P systems, $1/\mu_a$ is approximately equal to the average time between clumps of inelastic excursions. It is useful to treat these clumps as "points in time" at which inelastic action occurs, i.e., at which the permanent set D(t) jumps to a different value. In fact, the total plastic deformation D(s) developed in the time interval 0 to s, may be thought of as a sum of individual contributions D_i, $i = 1,2,\ldots,N(s)$, each of which is the result of a single clump of yield level crossings, i.e., $D(s) = \sum_i D_i$. N(s) is the random number of contributions during the time interval (0,s). For relatively high yield levels the time between clumps may be expected to have an exponential distribution with mean $1/\mu_a$. The approximate validity of this hypothesis was checked by Yanev[8] through analysis of E-P system response to simulated white noise and Tajimi-filtered white noise[9]. Some typical results are shown in Fig. 5 for a white noise excited E-P system characterized by an initial natural period = 0.2 sec., a viscous damping ratio = 0.02, and a yield displacement $\underline{a} = r\sigma_x$, where r is allowed to vary between 1.5 and

3.25 in increments of 0.25. The white noise intensity is chosen so that σ_x, the r.m.s. response of the associated linear system, equals 0.04 in. Fig. 5a shows that there is good agreement between the sample averages (denoted by m) of the time between clumps of yield level crossings and the elastic inter-clump times μ_a^{-1} (see Eq. 6). Also shown in Fig. 5a is a plot of $(2\nu_a)^{-1}$, ν_a being the mean rate of threshold up-crossings (see Eq. 5). Estimates of the standard deviation (denoted by s) of the time between clumps are very close to the corresponding sample means. This does suggest that the adoption of an exponential distribution (for which mean and standard deviation are theoretically identical) is reasonable. Other tests, described in Refs. 8 and 10, for different E-P systems subjected to excitation with both white and non-white spectra, lead to the same conclusion.

Karnopp and Scharton[11] derived a simple approximate expression for δ, the average amount of inelastic deformation (in absolute value) resulting from a single isolated crossing of the yield level a, i.e., $\delta = \sigma_x^2/2a = \sigma_x/2r$. This result follows from the argument that all the kinetic energy, $m\dot{X}^2/2$ (where m is the mass of the system; $\dot{X}$ is the impact velocity), will be released by yielding action. The average value of the kinetic energy at impact is approximately equal to $m\omega_n^2\sigma_x^2/2 = k_1\sigma_x^2/2$. The expected plastic deformation, δ, may be obtained from the energy equation, $F_y\delta = k_1\sigma_x^2/2$, where F_y denotes the yield force (see Fig. 1b). Typical sample values, obtained by numerical simulation[8,10], of the mean and standard deviation of $|D_i|$, the absolute value of the total plastic set during a clump of yield level crossings, are plotted in Fig. 5b. The smooth curve in Fig. 5b corresponds to Karnopp and Scharton's estimate δ. The fact that the sample means and standard deviations are nearly equal suggests that the probability distribution of $|D_i|$ is also approximately exponential (with mean value δ). Furthermore, the contributions D_i are equally likely to be positive or negative, and their probability density function is symmetrical with respect to zero. Therefore, the mean and variance of D_i will be approximately equal to 0 and $2\delta^2$, respectively.

ELASTO-PLASTIC SYSTEM RESPONSE MEASURES

Plastic Drift or Permanent Set

The total deformation D(s) developed during s seconds of "steady" (constant σ_x and μ_a) elasto-plastic (E-P) response is viewed as the sum of individual (positive and negative) contributions D_i, $i = 1,2,\ldots,N(s)$. Each contribution results from a single clump of yield level crossings. On the basis of the theoretical arguments, backed up by computer simulation, presented in the preceding section, we assume that the random number of contributions N(s) has a Poisson distribution with mean $\mu_a s$. Also, successive contributions D_1, D_2, etc., are assumed to be mutually independent of the clump occurrence process N(t). They are identically distributed, with a common probability density function $f_D(d) = (2\delta)^{-1}\exp(-|d|/\delta)$, $-\infty \leq d \leq +\infty$ which is symmetrical about d=0. Recall that $\delta = \sigma_x^2/2a$.

Under these assumptions, the expected value and the variance of $D(s) = \sum_i D_i$, where i goes from 0 to a random number N(s), become[12]

$$E[D(s)] = \mu_a \, s \, E[D_i] = 0 \tag{8}$$

$$Var[D(s)] = \mu_a \, s \, (Var[D_i] + E^2[D_i]) = 2\mu_a \, s \, \delta^2 \tag{9}$$

The moment-generating function and the probability density function of D(s) can also be obtained[10]. If interest focuses on the case when considerable plastic action occurs, the Central Limit Theorem could be invoked to rationalize adopting the assumption that D(s) has a Gaussian distribution.

Ductility Factor

If plastic action occurs during the time interval (0,s) then the ductility factor F (a random variable!) can be expressed as follows

$$F = \frac{1}{a} \left(\max_{0 \leq t \leq s} |D(t)| \right) + 1 = \frac{M_s}{a} + 1 \tag{10}$$

where M_s is the peak inelastic deformation. If no plastic action occurs, it equals the ratio of the maximum elastic deformation to the yield displacement. Eq. 9 can be used "unconditionally" when the probability of no plastic action is negligibly small.

Peak Inelastic Deformation

The peak inelastic deformation is the absolute maximum value of D(t) in the time interval 0 to s. Its probability distribution can be approximated by viewing the crossings, at positive slope, by $|D(t)|$, of a fixed threshold d, as a nonstationary Poisson process with mean rate $\nu_d(t)$. The peak inelastic deformation M_s will be less than or equal to d if no crossing of the level d occurs in (0,s). Hence,

$$P[M_s \leq d] = \exp\{- \int_0^s \nu_d(t)dt\} \tag{11}$$

$\nu_d(t)$ is proportional to μ_a, the mean rate of occurrence of jumps in the process D(t), i.e., $\nu_d(t) = \mu_a p_d(t)$, where $p_d(t)$ is the probability that a plastic set contribution at time t results in an upcrossing of the level d. If such a contribution, D, is positive then an upcrossing will occur if $D < -d-D(t)$ and $D(t) \geq -d$. We have

$$p_d(t) = 2 \int_{-\infty}^{d} P[D > d-x] f_{D(t)}(x)dx \simeq e^{-d/\delta} \, e^{\mu_a t} \tag{12}$$

The expression on the right side of Eq. 12 is approximately valid only if $\mu_a t < d/\delta$. It results from inserting $P[D > d-x] = (1/2)\exp\{-(d-x)/\delta\}$ into the integrand, expanding $\exp\{x/\delta\}$, and replacing the upper limit d by ∞. Finally, inserting Eq. 12 into Eq. 11 yields an approximate expression for the probability distribution of the peak inelastic deformation

$$P[M_s \leq d] = \exp\{- \int_0^s e^{-d/\delta} \, e^{\mu_a t} dt\} = \exp\{-(e^{\mu_a s} - 1)e^{-d/\delta}\} \; ; d \geq 0 \tag{13}$$

There is a finite probability, $P[M_s=0]$, that no plastic action will occur. By taking d=0 in Eq. 13, one finds this probability to be about $\exp\{-\mu_a s\}$ when $\mu_a s \ll 1$; it becomes negligibly small for large values of $\mu_a s$. Notice that Eq. 13 has the form of a Type I Extreme Value Distribution[13]. A characteristic value of M_s, found by setting $P[M_s \leqslant d] = e^{-1}$, equals $M_s^* = \delta \ln(e^{\mu_a s} - 1)$. Recall that $\delta = \sigma_x/2r$ and that μ_a is given by Eq. 6.

Time to First Crossing of a Given Level of Plastic Deformation

The probability distribution of the time, T_d, to first crossing of a given level of plastic deformation is intimately related to the distribution of M_s. One can write

$$P[T_d > s] = P[M_s \leqslant d] \tag{14}$$

It suffices to view the expression for $P[M_s \leqslant d]$ in Eq. 13 as a function of s, with d as a known parameter.

ANALYSIS OF BILINEAR HYSTERETIC SYSTEMS

The basic model which views the total inelastic deformation D(t) as a cumulative process with increments made at random "points" in time, continues to hold when the force-deformation relationship is not of the elasto-plastic type. But the statistical properties of the sizes of the increments and of the time intervals between these "points" now vary depending upon the state of the system, i.e., the level of inelastic deformation at which the system operates. In particular, at any given time t, they depend upon the values of the positive and negative yield levels corresponding to the plastic deformation D(t). (For E-P systems these yield levels remain constant regardless of the value of D(t).) As an example, Fig. 1a shows the force-displacement diagram of a simple frame with rigid girder and columns for which gravity loads have the effect of making the second slope k_2 negative[1]. During the process of drift accumulation the smallest yield level ranges from zero (at $D(t) = d_m$) to a (at $D(t) = 0$).

The particular kind of "memory" and the state-dependent nature of the cumulative damage suggest a Markov process continuous in both state and time to be a suitable stochastic model. It is computationally convenient, however, to discretize the range of possible permanent deformations. Fig. 6 shows the displacement axis divided into 13 states, with states 1 and 13 signifying collapse; state 7 is the initial state. In this case, the probability of being in state i at time t is $p_i(t)$, i = 1 to 13, and $p_1(t) + p_{13}(t)$ is the probability of collapse. These probabilities depend upon: (i) the probabilities of transition from one state to another given the occurrence of a clump of yield level crossings, and (ii) for each state, the average value of the exponentially distributed time between clumps of yield level crossings. The first set of parameters, the transition probabilities, can be evaluated using an argument similar to that which earlier led to the distribution of the plastic set increment D. The mean times between clumps have the form of μ_a^{-1}, with μ_a given by Eq. 6. Modifications are required, however, to account for the unequal and sometimes very low yield levels[14].

The results of a Markov analysis of the response to Gaussian white noise of three gravity-affected hysteretic systems whose properties are listed on Fig. 7 are presented in Figures 6 and 7. Fig. 6 shows the state probabilities $p_i(t)$ as a function of the duration t of stationary motion for structure No. 2. The failure probability is $p_1(t) + p_{13}(t)$. The white noise intensity is adjusted so that the r.m.s. response σ_x of the associated linear system is equal to a, i.e., $r = a/\sigma_x = 1$. The solid lines in Fig. 7 give the expected number of cycles (expected time divided by natural period) to failure for the three structures, as a function of the ratio $r = a/\sigma_x$ (note that σ_x is a measure of the excitation intensity). Husid[1] used both recorded earthquakes and artificial stationary motions with Tajimi-type spectral density to develop an empirical relationship for the expected number of cycles to failure of systems of the type shown in Fig. 1. Husid's best estimates are represented by the dotted lines in Fig. 7. For purposes of comparison, an "equivalent" white noise excitation was considered by substituting G_o by $G(\omega_n)$ in Eq. 7. The comparison appears to be quite satisfactory.

CONCLUSION

The approach outlined in this paper leads to a set of new approximate analytical results for the statistical properties of several important inelastic response measures for elasto-plastic systems undergoing steady earthquake-like random motion. The probability distributions of the peak inelastic deformation and of the time required for the inelastic deformation to cross a specified value, are expressed in terms of the yield level, the ground motion spectral characteristics and the properties (period and damping ratio) of the associated linear system. These results can be used to predict earthquake response of E-P systems when the associated linear system response, particularly σ_x, is in an approximate steady state during the intense part of the ground motion. An equivalent duration, s, of stationary associated linear system response needs to be used.

A continuous-time Markov model is also described which is used to study the response of bilinear hysteretic systems affected by gravity. Results for the expected number of cycles to failure are compared with those obtained by Husid, and they are found to be in reasonably good agreement.

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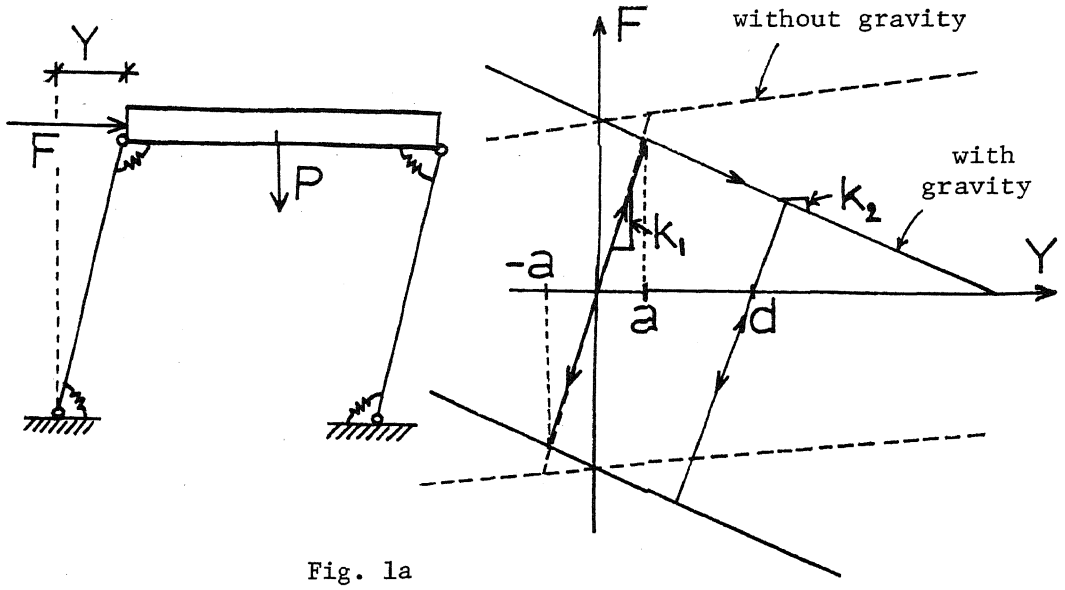


Fig. 1a

Bilinear System

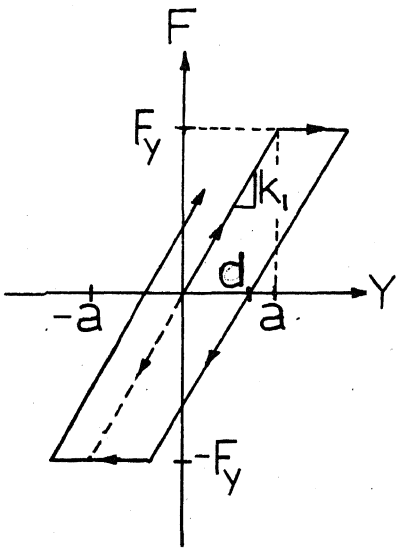


Fig. 1b

Elasto-Plastic System

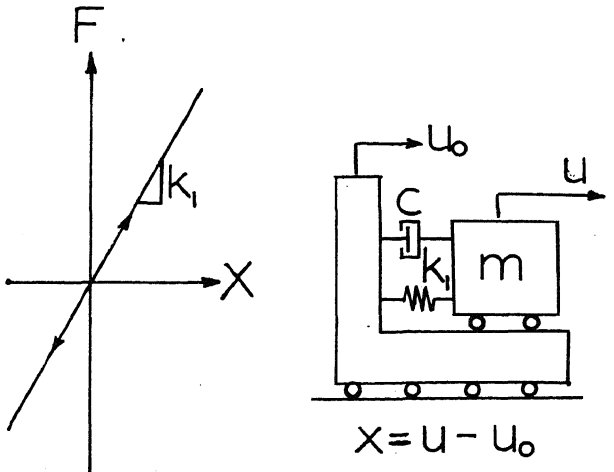


Fig. 1c

Associated Linear System

Figure 1

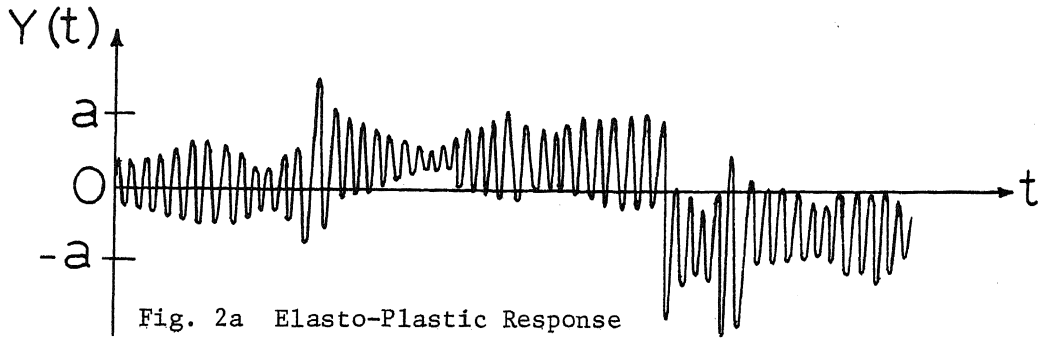


Fig. 2a Elasto-Plastic Response

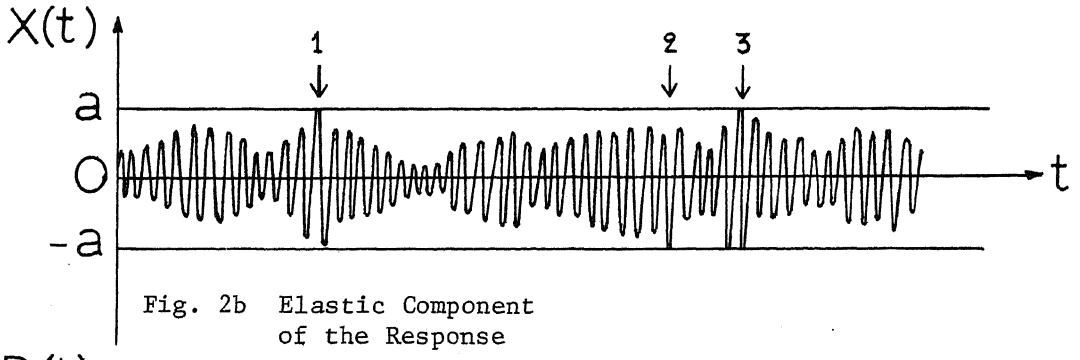


Fig. 2b Elastic Component of the Response

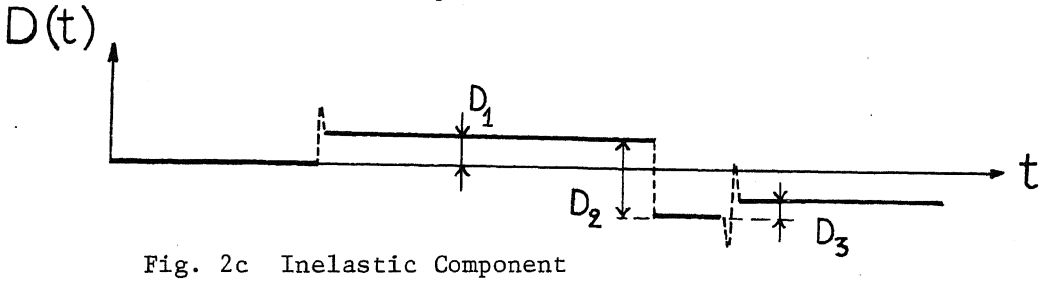


Fig. 2c Inelastic Component of the Response

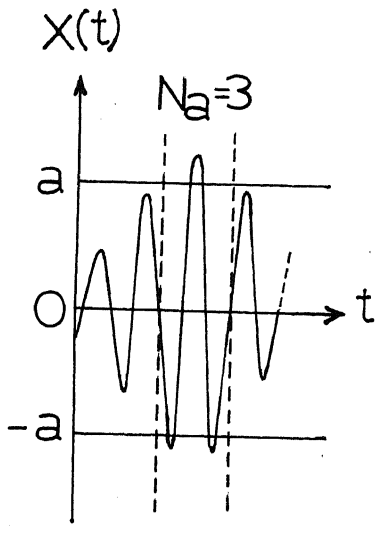


Fig. 3 A clump of Crossings Outside the Range (-a,a)

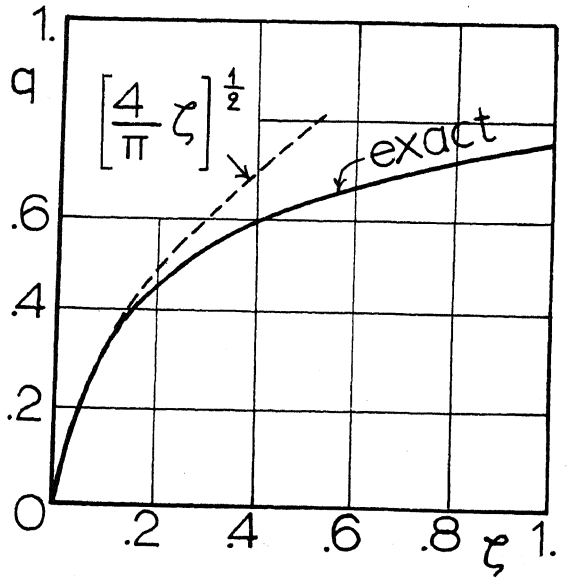


Fig. 4 The Factor q as a Function of the Damping ζ

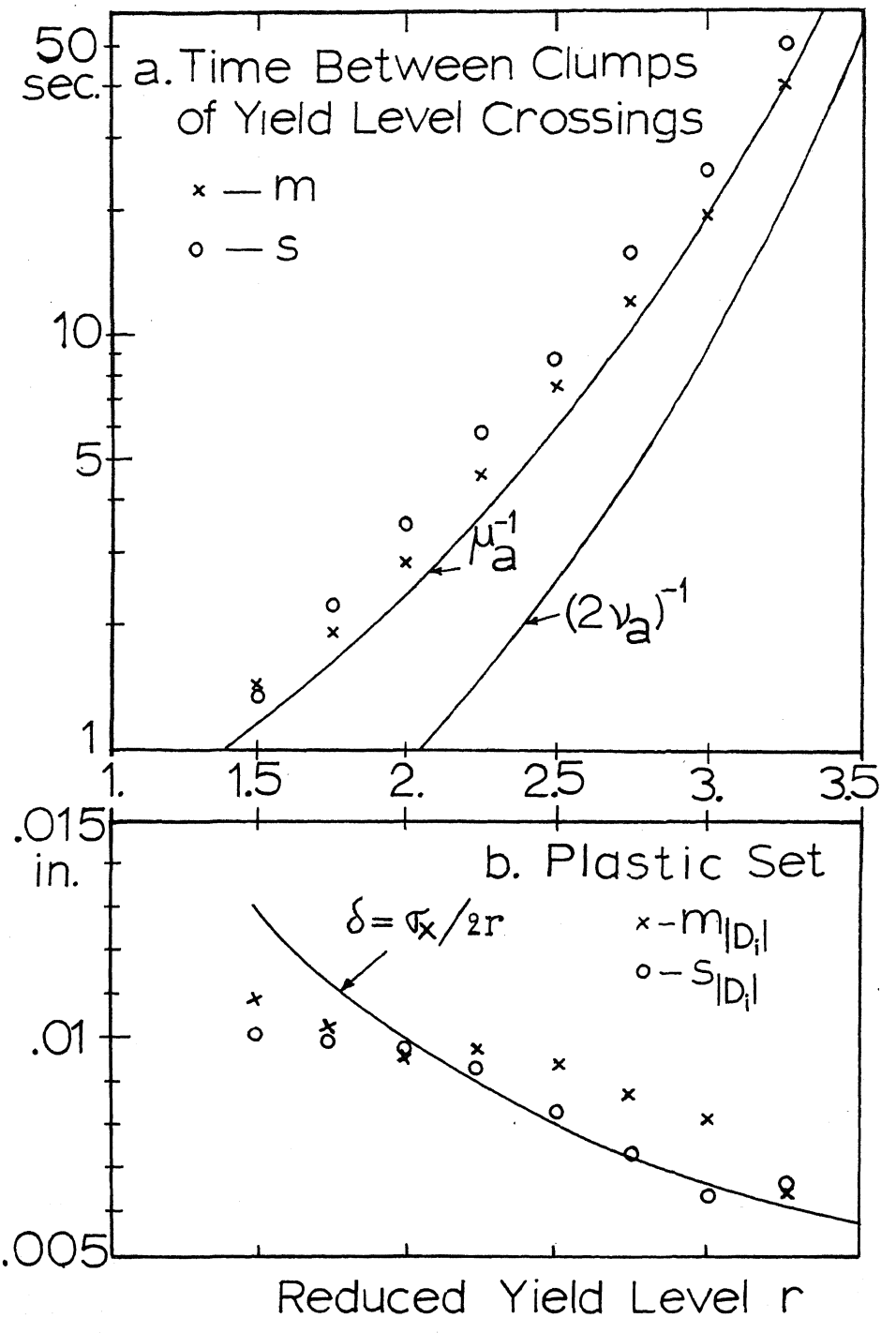


Fig. 5 Comparison of Simulated and Theoretical Values for the Mean and the Standard Deviation of (a) the Time Between Clumps of Yield Level Crossings, and (b) the Plastic Set for a Single Clump.

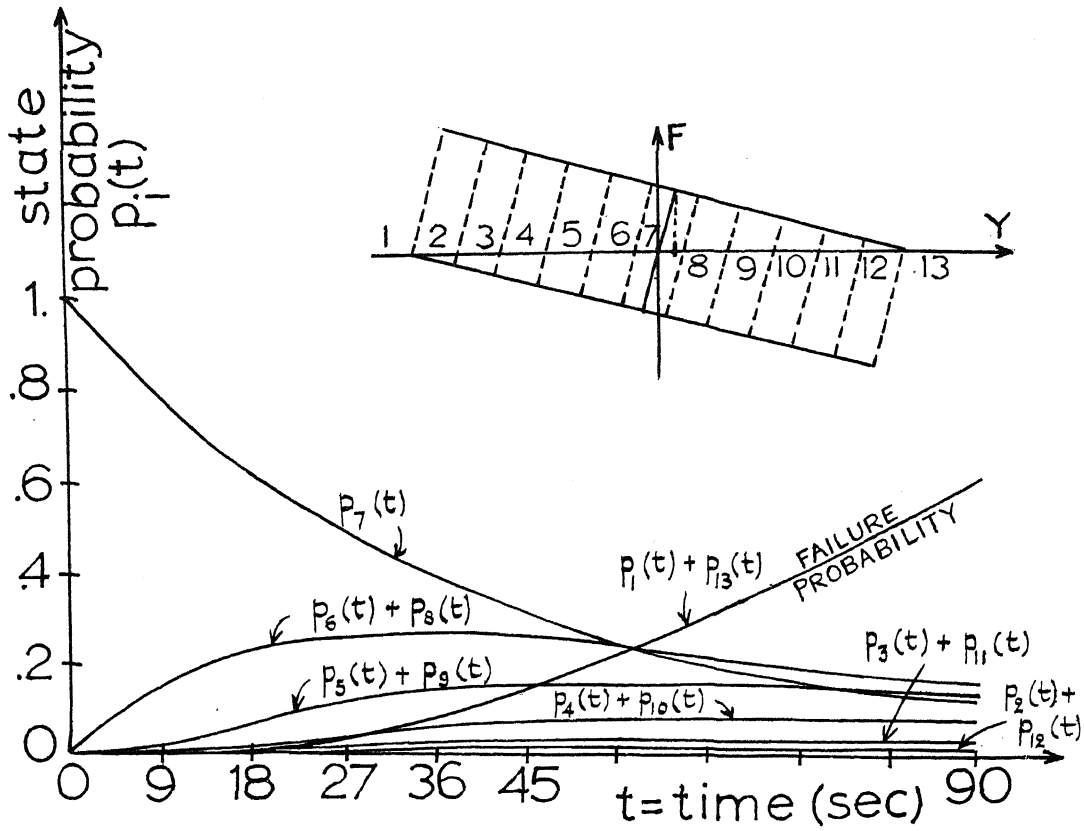
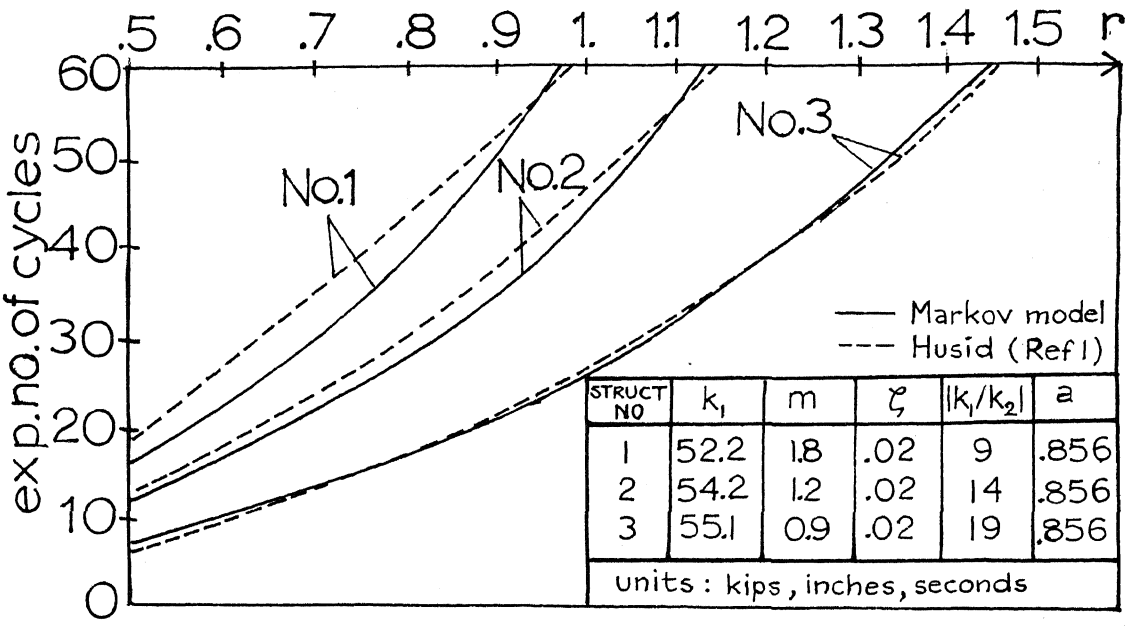


Fig. 6 The State Probabilities for Structure No. 2 (whose properties are given below).



STRUCT NO	k_1	m	ζ	$\lvert k_1/k_2 \rvert$	a
1	52.2	1.8	.02	9	.856
2	54.2	1.2	.02	14	.856
3	55.1	0.9	.02	19	.856

Fig. 7 The Expected Number of Cycles to Failure