EXPERIMENTAL STUDY ON VIBRATIONAL CHARACTERISTICS OF THE STRUCTURES WITH EMBEDDED FOUNDATION

By

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SYNOPSIS

In recent years, engineers have become increasingly aware of the need of evaluating the effect of soil and foundation for seismic design. However, the evaluation of the interaction effect is very difficult because of complicated conditions beneath the foundation; soil layers, pile foundation and embedded foundation. This paper discusses the interaction effect of superstructures and embedded foundation on the basis of a model experiment. The experimental analysis was made by two methods; one the finite element method and the other theory of spring mass system. Mainly the relation of damping values and frequencies in each mode of soil-structure system was studied.

EXPERIMENTAL PROCEDURE

First of all, model experiment of soil structure system should satisfy the law of similitude corresponding real phenomena. We adopted dimensionless frequency$\left(\frac{d\omega}{V_s}\right)$, with foundation characteristic(d), shear wave velocity(v_s), and frequency of excitation(ω), which is a familiar one used by Arnold, et al(1955), Tajimi(1959) in their model experiments.
The dimensionless frequency is generally not more than the value of 2.0. As reference with above dimensionless frequency, we decided characteristic foundation length, shear wave velocity and exciting frequencies.
A test frame consists of phosphor columns and aluminum plate slabs. An embedded foundation is framed also with phosphor plates. The model ground is made piling up the several sheets of geratine-filled foam rubber, which simulates real ground. The experiment was performed on four test frames; 1 to 4 story frames and three different depths of embedment. The experimental system was so made that any of the four test frames could be connected to any of the three embedments. Unit story is 15cm high with a slab of 15cm square. Each of embedment is 18cm square in plane with three different depths; 10cm, 20cm and 30cm.
The whole system is excited by a vibration generator set on the slab. The entire experimental system is shown in Fig. 1. While the physical constants of the foam rubber itself are Young's modulus E of 700 g/cm^2 and density ρ of 0.12 g/cm^3. After dipping foam rubber into geratine, we obtained a shear wave velocity Vs of 7.0m/sec and density ρ of 0.504 g/cm^3.

The vibrational characteristics of a coupled system combining structure and embedment in series were measured by means of accelerometers laid on slabs. We obtained the natural frequencies and damping ratios of the coupled systems using the resonance curves resulting from the vibration tests. Photo.1 shows measurement system. The procedure of vibration test

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is the following.
Vibration Generator → Accelerometer → Dynamic Strainmeter → Recorder → Resonance Curve → Natural Frequency, Damping Ratio

EXPERIMENTAL RESULTS

Experimental results are shown in Table 1. The followings are some of the results found in the tests.

1) The first natural frequency of model ground which was assumed to be infinite half space was nearly 8.0 cycles per second. It was found that there was no distinct difference between the natural frequencies of model ground with and without embedment.

2) The vibrational characteristics of superstructures on a rigid base are as follows.

Structure A(two-story) : $f_1 = 5.58c/s$ ($h_1=0.48\%$), $f_2=15.33c/s$ ($h_2=0.39\%$)

Structure B(three-story): $f_1 = 3.76c/s$ ($h_1=0.25\%$), $f_2=11.60c/s$ ($h_2=0.27\%$)
$f_3=17.66c/s$ ($h_3=0.34\%$)

Structure C(four-story) : $f_1 = 3.40c/s$ ($h_1=0.55\%$), $f_2= 9.56c/s$ ($h_2=0.48\%$)
$f_3=15.48c/s$ ($h_3=0.33\%$), $f_4=20.65c/s$ ($h_4=0.34\%$)

where f_j and h_j are the natural frequency and the fraction of critical damping for j-th mode of vibration, respectively. The superstructures are those of a shear-type considering the natural frequencies. The damping properties of the structures are not of nature with a resistance proportional to the velocity but probably hysteretic, and the damping ratios are appoximately less than 0.5% of critical damping in any mode.

3) Vibrational characteristics of the coupled system are as follows. The natural frequencies in each mode of vibration are decreased in the order of not more than 20% of those of structures alone on a rigid base.

4) As to the variation ratios of frequencies for each mode of vibration, it can be expressed as follows.

$$\Delta f_1 > \Delta f_2, \ \Delta f_2 < \Delta f_3, \ \text{and} \ \Delta f_3 > \Delta f_4 \tag{1}$$

where the variation ratio of frequency for j-th mode of vibration (Δf_j) is defined as follows.

$$\Delta f_j = \left(1 - \frac{f_{SG}}{f_S} \right) \times 100 \ (\%) \tag{2}$$

in which f_S, and f_{SG} are the natural frequencies for j-th mode of the superstructure and the coupled system, respectively. The values of Δf_j were found to become larger as the depth of the embedment became smaller. Generally, the value for second mode of vibration were rather small in any mode.

5) The damping ratios of each mode of vibration of the coupled system were found to have the following relations.

$$h_1 > h_2, \quad h_2 < h_3, \quad \text{and} \quad h_3 > h_4 \tag{3}$$

The phenomena mentioned above were pronounced when the depth of embedment was small, and the damping ratio also large.

6) It is interesting to notice that the nature of damping for each mode of the coupled system was the same as that of the variation ratios of frequencies.

EXPERIMENTAL ANALYSIS

Eigenvalue analysis was made by two methods;
A) Method 1: analysis by the finite element method, B) Method 2: analysis by the theory of spring mass system.
A) Method 1: In the first place, the coupled system was divided into some elements as shown in Fig. 2 so that eigenvalue problem could be solved by the finite element method, and then Young's modulus, Poisson's ratio, mass density and thickness were decided for each element. Followings were taken into account to represent structure by finite elements.

1) Frames are connected in series to imaginary elements and these element have rollers at the boundary so that only a lateral displacement can be released. A frame consists of two elements in every unit story.

2) Dimension of unit element is 7.5cm wide,15cm high and 18 cm deep. An appropriate Poisson's ratio was assumed, and equivalent Young's modulus was determined by a trial and error method under the condition that the natural frequencies agree with those of experiments.

3) It was assumed that an embedment was rigid body, and was divided the whole weight of the embedment uniformly into each element. As a result of analysis using the physical constants of model ground which were obtained by the test, the natural frequency of the ground was different from that of experiment. This is probably because of not taking into account the boundary conditions of out-of-plane restrain. We determined modified shear velocity under condition that first natural frequency of the ground agreed with that of experiment.

4) As for ground, it was assumed that the elements near the boundary had higher elastic modulus. Through the above procedure, the equation of undamped motion of the coupled system is expressed as,

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{4}$$

in which [M] and [K] are mass and stiffness matrices, respectively. Substituting,

$$x = U\, e^{i\omega t} \tag{5}$$

into Eq. (4), the expression,

$$-\omega^2 [M]\{U\} + [K]\{U\} = \{0\} \tag{6}$$

is obtained. In order for Eq. (6) to have nontrivial solution,

$$\left| [K] - \omega^2 [M] \right| = 0 \qquad (7)$$

Solving Eqs. (6) and (7), we obtain the natural frequencies and normal modes of the coupled system. The participation factor for j-th mode of vibration (β_j) is

$$(\beta_j)_{xy} = \frac{\sum m_i (_jU_i)_{xy}}{\sum m_i \{(_jU_i)_x^2 + (_jU_i)_y^2\}} \qquad (8)$$

where, m and U are the mass of nodal point and the displacement for j-th mode, respectively. Suffix x and y indicate the direction of displacement. Figure. 3 shows the vibrational characteristics of the coupled system.

B) Method 2: We also studied the experimental results by the theory of spring mass system. We applied some formula based upon the elastic half-space theory for spring constants and virtual masses.
The equation of motion of the coupled system is the same as Eq. (1), in which mass and stiffness matrices are presented by,

$$
M = \begin{vmatrix}
m_0 & 0 & \cdot & 0 & 0 \\
0 & m_1 & 0 & & m_1 H_1 \\
\vdots & & \ddots & \vdots & \vdots \\
0 & 0 & \cdot & m_n & \vdots \\
0 & m_1 H_1 & \cdot & \cdot & I_0 + \Sigma m_i H_i^2
\end{vmatrix}
\qquad
K = \begin{vmatrix}
\alpha_{ij} + \dfrac{1}{k_H} & & 0 \\
\rule{0pt}{12pt} & & \\
0 & & \dfrac{1}{k_R}
\end{vmatrix}^{-1}
\qquad (9)
$$

where m_0= mass of the foundation, m_i=mass of structure,
I_0= moment of inertia of embedment with respect to rocking axis,
k_H= spring constant of swaying, k_R= spring constant of rocking,
α_{ij}= influence coefficient, H_i= height of the floor of the structure
from foundation base.

The formulae for spring constants, virtual mass, and moment of inertia of embedment are as follows,
1) Horizontal spring constant is obtained by assuming that the distribution of stress over the contact area between the foundation is rigid.

$$k_H = \frac{8\ b\ G}{2-\nu} \cdot Ax \qquad (10)$$

2) Rocking spring constant is assumed to be triangular.

$$k_R = \frac{\pi\ G}{1-\nu} \cdot \frac{Jy}{b} \cdot \frac{1}{2} \cdot Bx \qquad (11)$$

3) The resistance of embedment of the foundation for rocking is,

$$k_R' = \frac{2\ \pi\ G}{1-\nu} \cdot \frac{Jy'}{h} \cdot Bx \qquad (12)$$

4) The effective mass and mass moment of inertia of model ground is,

$$m_0 = \frac{8\ \rho\ a_e^3}{2-\nu}\ \Gamma_2{}^2 \left(\frac{4}{\pi^4} - \frac{1}{4}\right) \qquad I_0 = \frac{\pi\ \rho\ a_e^5}{12(1-\nu)}\ \Gamma_1{}^2 \qquad (13)$$

where b and h are a half length of rectangular foundation along axis of rotation, and depth of embedment, respectively. A_x and B_x are shape factors of spring constant. J_y and J_y' are sectional moment of inertia of rectangular foundation, and that of embedment with depth (h) about the rotational axis, respectively. Γ_1 and Γ_2 are chosen appropriately for the dimensionless frequency and a_e is equivalent radius of the foundation.

The results by analysis are shown in Table 1 with those of experiment. Concerning the natural frequencies of structure alone on a rigid base, the both values of analysis are in fairly good agreement with those of experiments. The variation ratio of j-th mode of vibration are generally large for first mode. The trend is more remarkable as the depth of embedment becomes smaller. Above all it is noticed that the value of variation ratio for second mode are the smallest among all modes and it is common for both experiments and the analysis by the finite element method. One the other hand the variation ratio by the spring mass system became smaller for higher modes. It seemed that numerical results by the finite element method simulated better the complicated vibrational behavior of the coupled system, judging from the experimental results.

DAMPING CHARACTERISTIC(METHOD 1)

There are many questions of energy dissipating mechanism of structure. According to the results of vibration tests of practical structure, it is seen that the damping in each mode is not always of viscous nature. One of the purpose of this experiment was to study the higher mode vibration characteristics of the coupled system. We obtained damping properties of beyond 4-th mode of vibration and also the relation between the variation ratio of the frequency and the damping.

In this section we discuss damping properties of the coupled system using the results of the analysis by the finite element method. Comparing the foregoing experimental results with those of analysis, it could be seen that the variation ratios of frequencies were similar to those of the damping. Hence, the expression of h_j as a functions of Δf_j was suggested. As the variation ratio of frequencies Δf_j would imply the degree which the system was affected by the soil foundation, it changed by transformation of ground. It was assumed that damping was proportional to the strain rate of the ground. We defined "excited-zone" near the embedment as the zone where the strain of ground was comparatively large because of the existence of the embedment. We then calculated the strain within each element using each mode shapes by analysis and then summed up the strain within the excited-zone in every mode. Figure.4 shows how to calculate the above defined strain of the element and Table 2 shows the results. We then propose a rule to calculate the damping ratio of a coupled system. The damping ratio of j-th mode is assumed twofold; one the damping of the superstructure and other of the ground. An interesting expression found in the course of experiment is as follows,

$$ h_j = h_{sj} + \frac{h_H \, \Delta\beta U_{Hj} + h_v \, \Delta\beta U_{vj}}{\Delta\beta U_H' + \Delta\beta U_v'} \qquad (14) $$

where h_j is the damping ratio of j-th mode or vibration of a coupled system, h_{sj} , h_H, and h_v are damping ratios of the superstructure, and of the

horizontal and vertical ground motion, respectively.
$\Delta \beta U_{HJ}$ and $\Delta \beta U_{VJ}$ are horizontal and vertical strains of the excited-zone of the coupled system. $\Delta \beta U_H'$ and $\Delta \beta U_V'$ are that of the soil foundation system.

Generally, the damping of soil foundation in experiment is a coupled of horizontal and vertical motions, therefore it is necessary to divide the damping into individual elements. The damping ratio is assumed to be,

$$h' = \frac{h_H \, \Delta \beta U_H' + h_V \, \Delta \beta U_V'}{\Delta \beta U_H' + \Delta \beta U_V'} \tag{15}$$

Substituting the relation dependent upon the theory of the wave energy loss $h_H / h_V = 0.87/1.33$, into Eq. (15), we obtain,

$$h' = \frac{h_H (\Delta \beta U_H' + 1.53 \, \Delta \beta U_V')}{\Delta \beta U_H' + \Delta \beta U_V'} \tag{16}$$

Using the experimental results h' and $\Delta \beta U_H'$ and $\Delta \beta U_V'$ by the analysis, we can calculate the damping of ground h_H and h_V. Table 3 shows the values of the strain of ground. The damping ratio of superstructure h_{SJ} was about 0.50% and that of ground h_H and h_V were 3.30% and 5.15%, respectively. Using the values resulting from the experiment and analysis, we calculated the damping ratios and the results were shown in Fig.1 with that of experiment. As a result, it seems that the damping properties for each mode of the coupled system depend on the extent of contribution of the damping of each component to the modal value.

DAMPING CHARACTERISTIC(METHOD 2)

In the previous section, we proposed a rule to calculate the damping ratios based on the experimental evidence.
Next, we studied the damping using an orthogonal damping matrix derived by Caughey. As an orthogonal damping matrix, on the uncoupled damped mode of vibration, the following form of matrix is derived by Caughey

$$C = M \sum_b a_b \{M^{-1}K\}^b \tag{17}$$

where a_b is damping constant, and b is arbitrary integer.
Using the normal coordinate, we obtain the expression,

$$h_r = \frac{1}{2} \left(\frac{a_0}{\omega_r} + a_1 \omega_r + \cdots + a_{n-1} \omega_r^{2n-3} \right), \quad r = 1, 2, .., N \tag{18}$$

Taking the Rayleigh damping given by the first two terms of Eq. (18),

$$h_r = \frac{1}{2} \left(\frac{a_0}{\omega_r} + a_1 \omega_r \right) \tag{19}$$

As it was noticed by experiment that a damping ratio of second mode of vibration of the coupled system was smallest of all mode, was set the conditions that the 2-nd mode damping became the smallest, and we obtained,

$$\frac{dh}{d\omega}\bigg|_{r=2} = \left(-\frac{a_0}{\omega_r^2} + a_1 \right) \quad \omega_r \big|_{r=2} = \sqrt{\frac{a_0}{a_1}} \tag{20}$$

where, the damping constants a_0 and a_1 can be determined using 2-nd mode natural frequency and damping ratio by experiment.

Then desired damping ratio can be calculated by Eq. (19).
The numerical evaluation by Rayleigh damping matrix is shown in Fig. 7,
with that of experiment. Also the damping constants a_0 and a_1 are shown
in Table 4. Figure. 5 shows a ratio of the damping constants of the coup-
led system to that of superstructure fixed on a rigid base

It was found that Rayleigh damping matrix could verify the experimen-
tal damping ratios of higher modes of the coupled system. As to the damp-
ing constants of the coupled system, the constant a_0 is 4 to 6 times lar-
ger in the case of 0 cm depth of embedment and 1.5 to 3 times larger in
the case of 10 cm to 30 cm depth than that of structure fixed on the base.
On the other hand, a_1 is 1.5 to 3.0 times larger in the former case and
0.5 to 1.5 times larger in the latter case than that of structure on the
fixed on the base. After all it is said that the effect of ground on a
damping ratio depends greatly on the damping a_0. Consequently, it might
be said that this model experiment was more sensitive to the mass matrix.

CONCLUSIONS

1) Eigenvalue analysis was made by means of two methods: one the fin-
ite element method, the other the theory of spring mass system. It was
found the finite element method could simulate better the complicated vib-
rational behaivior of the coupled system.

2) It was confirmed that the variation ratios of frequencies for each
mode vibration obtained by the finite element method showed better agree-
ment with that found in the experiment than that by the spring mass system.

3) A rule to calculate the damping ratios of the coupled system is
established by using the finite element method. The damping ratios of j-th
mode of vibration of the coupled system is twofold: the damping of a sup-
erstructure and the damping of the ground. The damping ratios obtained
by the proposed rule were reasonable as compared those by experiments.

4) As to the damping calculated by the Caughey series method, it was
proved that a mass matrix in a vibration system had great influence on the
experimental results.

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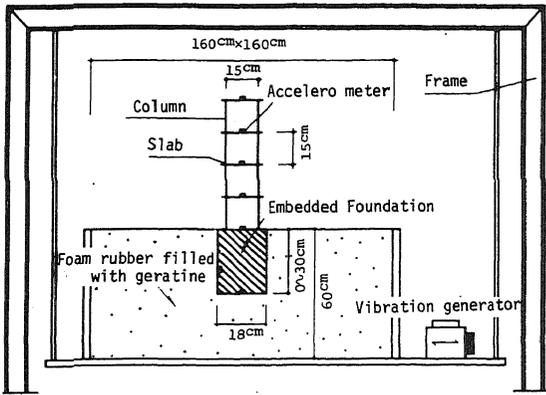Report of The Institute of Industrial Science University of Tokyo March
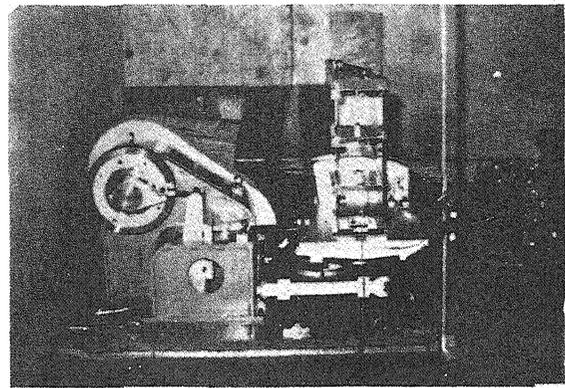1959.
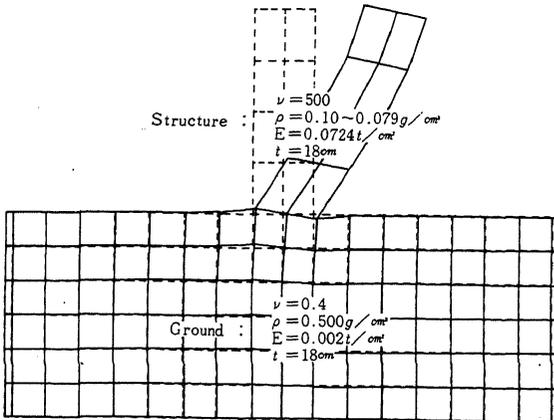
FIG. 1.—EXPERIMENTAL SYSTEM

FIG. 2.—DIVIDED ELEMENT OF A
COUPLED SYSTEM

PHOTO. 1.—VIEW OF MEASUREMEMENT
SYSTEM
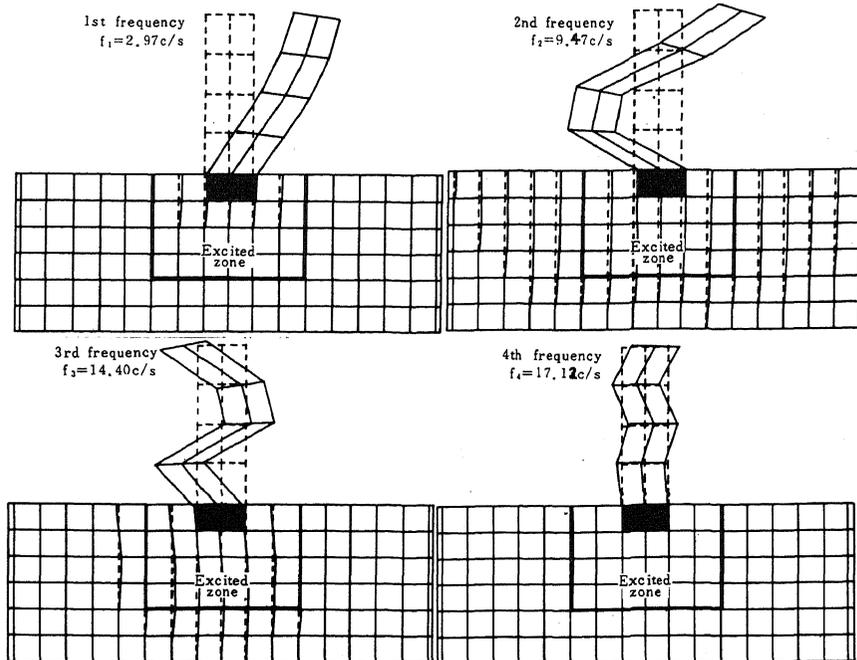
FIG. 3.—NATURAL FREQUENCIES AND MODE SHAPES OF A
COUPLED SYSTEM (BT THE FINITE ELEMENT METHOD)

$d\beta u_H\frac{1}{2} = \beta u_{H2} - \beta u_{H1}$

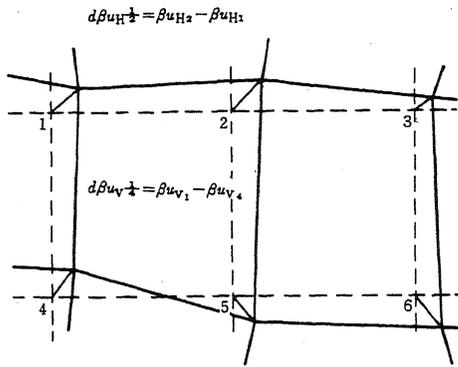$d\beta u_V\frac{1}{2} = \beta u_{V1} - \beta u_{V4}$
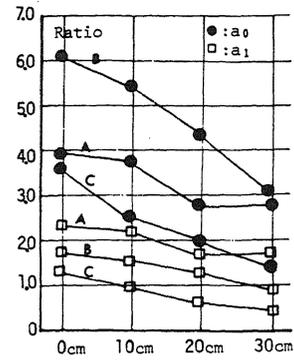
FIG. 4.—DEFINED STRAIN
OF AN ELEMENT

FIG. 5.—RATIOS OF DAMPING
CONSTANT OF THE COUPLED
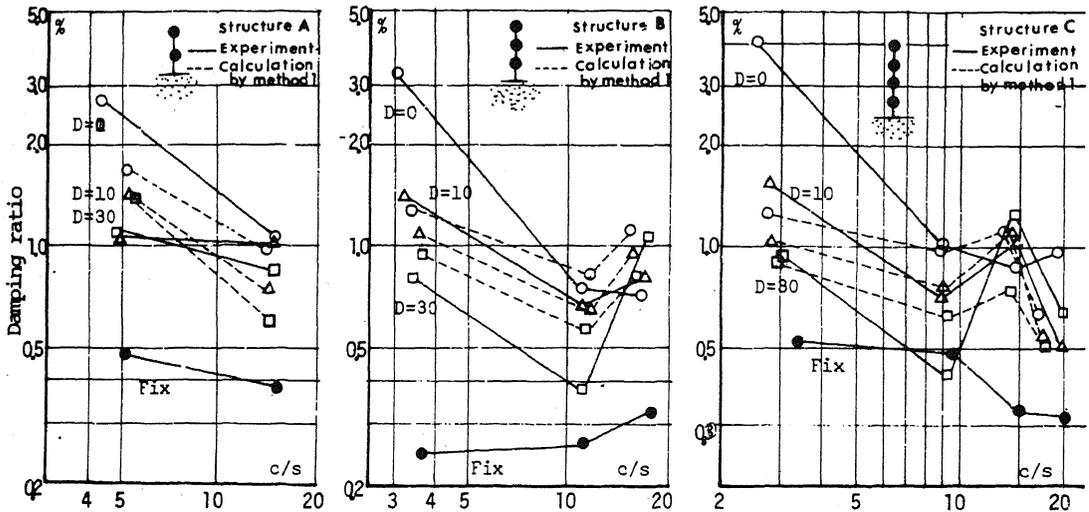SYSTEM TO THAT OF THE
STRUCTURE ON A RIGID BASE

FIG. 6.—DAMPING RATIOS OF EXPERIMENT AND CALCULATION BY METHOD 1
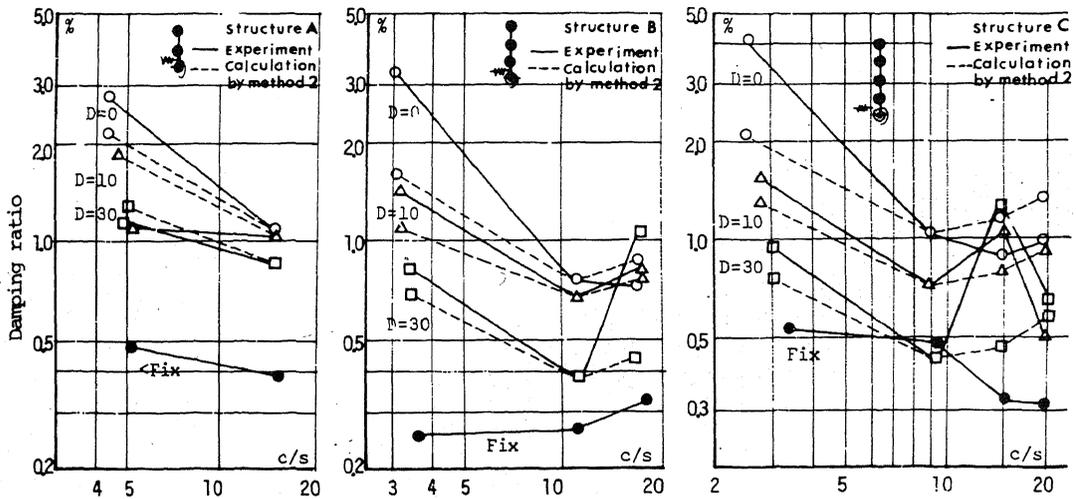
FIG. 7.—DAMPING RATIOS OF EXPERIMENT AND CALCULATION BY METHOD 2

TABLE 1.-RESULTS OF EXPERIMENTS AND ANALYSES

D: Depth of Embedment (cm)

Embedded depth / Structure		Fix Experiment f c/s h %	Method 1 f c/s	Method 2 f c/s	D=30 cm Experiment f c/s h %	Method 1 f c/s	Method 2 f c/s	D=10 cm Experiment f c/s h %	Method 1 f c/s	Method 2 f c/s	D=0 cm Experiment f c/s h %	Method 1 f c/s	Method 2 f c/s
A	1st	5.58 0.48	5.80	6.08	5.09 (9.50) 1.18	5.46 (5.85)	5.69 (6.60)	4.81 (13.80) 1.09	5.30 (8.10)	5.16 (15.10)	4.54 (18.60) 2.79	5.12 (11.72)	4.54 (34.20)
	2nd	15.33 0.39	14.24	15.26	15.28 (0.33) 0.82	14.24 (0.07)	14.92 (2.22)	15.37 (-0.26) 1.09	14.17 (0.28)	14.43 (5.45)	15.28 (0.33) 1.14	14.01 (1.62)	13.74 (10.90)
B	1st	3.76 0.25	3.93	4.37	3.57 (5.05) 0.82	3.74 (5.10)	3.95 (9.60)	3.33 (11.40) 1.44	3.59 (8.67)	3.29 (24.70)	3.02 (19.37) 3.20	3.42 (13.12)	2.62 (40.00)
	2nd	11.60 0.27	11.54	12.16	11.39 (1.80) 0.40	11.56 (-0.17)	11.61 (4.52)	11.44 (1.38) 0.69	11.53 (0.09)	10.78 (11.30)	11.30 (2.58) 0.77	11.39 (1.33)	10.00 (17.50)
	3rd	17.75 0.34	15.51	16.93	17.25 (2.82) 1.15	15.39 (0.78)	16.79 (0.84)	17.24 (2.87) 0.84	15.68 (-1.11)	16.71 (-1.30)	17.65 (0.56) 0.73	15.04 (3.04)	16.65 (1.71)
C	1st	3.39 0.55	3.28	3.79	3.03 (9.01) 0.99	3.17 (3.36)	3.30 (12.90)	2.84 (16.20) 1.57	2.97 (9.45)	2.54 (33.00)	2.57 (24.20) 4.18	2.80 (14.73)	1.88 (50.00)
	2nd	9.56 0.48	9.42	10.27	9.40 (1.67) 0.43	9.51 (-0.95)	9.51 (7.40)	9.45 (0.74) 0.73	9.47 (-0.51)	8.53 (16.90)	9.26 (3.10) 1.06	9.37 (0.49)	7.88 (23.20)
	3rd	15.47 0.33	14.60	15.32	14.87 (3.88) 1.32	14.52 (0.48)	15.07 (1.63)	14.89 (3.75) 1.13	14.40 (1.37)	14.91 (2.67)	15.01 (3.00) 0.87	14.34 (1.75)	14.83 (3.20)
	4th	20.65 0.32	17.15	19.39	20.65 (0.00) 0.68	17.14 (0.06)	19.13 (1.34)	20.59 (0.29) 0.50	17.12 (0.17)	18.89 (22.83)	20.57 (0.38) 0.92	17.09 (0.35)	18.75 (3.30)

The value within the symbol ( ) is the variation ratios of frequencies Δf

TABLE 2.-STRAINS OF THE EXCITED ZONE OF THE COUPLED SYSTEM

Structure		D = 0 cm f c/s	$\Delta\beta_{uH}$ ($\times 10^{-1}$)	$\Delta\beta_{uv}$ ($\times 10^{-1}$)	D = 10 cm f c/s	$\Delta\beta_{uH}$ ($\times 10^{-1}$)	$\Delta\beta_{uv}$ ($\times 10^{-1}$)	D = 30 cm f c/s	$\Delta\beta_{uH}$ ($\times 10^{-1}$)	$\Delta\beta_{uv}$ ($\times 10^{-1}$)
A	1 st	5.12	1.895	1.805	5.30	1.684	1.335	5.46	1.489	0.772
	2 nd	14.01	1.096	0.714	14.17	0.459	0.310	14.24	0.144	0.043
B	1 st	3.42	0.984	1.106	3.59	0.906	0.751	3.74	0.702	0.413
	2 nd	11.39	0.680	0.367	11.53	0.306	0.165	11.56	0.213	0.058
	3 nd	15.04	1.260	1.100	15.68	1.421	0.546	15.39	0.440	0.330
C	1 st	2.80	0.838	0.955	2.97	0.753	0.628	3.17	0.402	0.350
	2 nd	9.37	0.850	0.681	9.47	0.425	0.256	9.51	0.332	0.179
	3 rd	14.34	1.203	0.530	14.40	1.014	0.700	14.52	0.171	0.191
	4 th	17.09	0.144	0.114	17.12	0.112	0.081	17.14	0.024	0.019

TABLE 3.-STRAINS AND DAMPING RATIOS OF THE EXCITED ZONE OF THE SOIL FOUNDATION SYSTEM

——	1st freq.	$\Delta\beta_{uH}$	$\Delta\beta_{uv}$	h_H (%)	h_v (%)
D = 0 cm	7.74	0.900	0.362	3.36	5.15
D = 10 cm	7.77	0.827	0.362	3.30	5.10
D = 30 cm	8.20	0.617	0.226	3.36	5.15

TABLE 4.-DAMPING CONSTANTS

Structure	Damping Constant	Fix	D=30	D=10	D=0
A	$a_0 \times 10^2$	0.27	0.79	1.06	1.09
	$a_1 \times 10^2$	0.51	0.86	1.13	1.18
B	$a_0 \times 10^2$	0.09	0.28	0.50	0.55
	$a_1 \times 10^2$	0.62	0.56	0.96	1.10
	$a_2 \times 10^6$	-0.07			
C	$a_0 \times 10^2$	0.17	0.26	0.43	0.62
	$a_1 \times 10^2$	1.39	0.73	1.25	1.86
	$a_2 \times 10^6$	-0.73	——	——	——
	$a_3 \times 10^9$	0.15	——	——	——

D: Depth of Embedment (cm)