OPTIMUM SEISMIC DESIGN OF AUDITORIUMS

by

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SYNOPSIS

We develop criteria for the optimum choice of design parameters for simple structures that can fail or be damaged under gravity loads or under their combination with earthquakes. Structural properties and most of failure depend on the live load present when the earthquake occurs. We assume that the structure is either full or empty, which is appropriate for auditoriums. The case of a continuous gradation of damaged states is examined.

THE GENERAL PROBLEM

Assuming additivity of utilities, optimization in structural design amounts to maximization of the objective function

$$\Omega = B - C - L \tag{1}$$

where B is benefits derived from the structure's existence, C its initial cost, and L losses due to failure (damage or collapse). These quantities are expected present values. They are functions of the vector of design parameters $\underline{x}$. The problem consists in finding the optimal $\underline{x}$, $\underline{x}_0$. We shall treat Ω as a continuous function of $\underline{x}$, but the basic concepts can be extended to a discrete-variable approach.

We shall discount future values through the actualization function $g(t) = \exp(-\gamma t)$, where t is time, counted from the instant when the structure is completed, and γ is the actualization rate.

Let a specific design be defined by the vector of (random) structural properties $\underline{X}$ and denote the vector of the corresponding nominal or design values by $\underline{x}$ ($\underline{x}$ is a functional of the probability distribution of $\underline{X}$).

As long as the structure does not fail, benefits derived from its existence usually decrease slowly with the nominal uniform load x_g it can carry and with x because of a decrease in rentable area and other secondary effects (which can be substantial in tall buildings). Given a type of failure or level of damage the reliability function increases with these two parameters.

L is given by the sum of losses H_{ij} incurred in transition from state i into j (state 0 corresponds to the structure as built), multiplied by the corresponding values of g. In general, H_{ij} depends on the time when the transition takes place and on the live load acting at that instant; there

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are several transitions from i to j, for the structure can be repaired and rebuilt. If H is a continuous function of structural responses, we replace the sum with an integral with respect to these responses.

B is the sum of terms of the form $\int b_i g\, dt$, where b_i is the benefit per unit time derived from existence of the structure in state i, and each integral covers the interval during which the structure is in a given state. If the structure is systematically restored to its initial state and we treat the reduction in benefits during restoration as a contribution to L, B equals approximately $\int_0^\infty b_0 g\, dt$, the benefits that would be produced by the structure if it never entered a limit state.

These expressions can be put in terms of transition probabilities, which depend on the live load present at the instant of passage into a new limit state. In what follows we shall consider special cases that fulfill certain stationarity conditions.

BENEFITS AND LOSSES: COLLAPSE THE ONLY LIMIT STATE

Suppose that, under the sole action of gravity, the structure can fail upon completion or shortly thereafter, and that it can also fail under earthquake action; that the only type of failure it admits is collapse; that live load is either zero or some finite value W_p; and that the structure has one degree of freedom under gravity load and one under earthquake. This is an idealization of some auditoriums. W_p is the weight of people filling the auditorium.

Suppose further that the structure is rebuilt systematically after failure and that C, H_{01}, and the failure probabilities are independent of k -- the number of times that the structure fails and is rebuilt.

Failure due to gravity loads occurs if $U_f \geq X_g$, where U_f is the load effect with full auditorium and X_g is the resistance to vertical loads. U_f and X_g are random variables. $U_f = U_e + U_p$, where subscript e refers to empty auditorium and p to live load. In linear systems, $U_e = \int w_e I\, dA$ and $U_p = \int \alpha w_p I dA$, where w is load per unit area, I is influence ordinate, A is area, and α is a dynamic magnification factor. We assume that α eventually reaches some maximum value whenever the auditorium is full. In plastic structures I is the ordinate of the failure mechanism. We assume that the space distribution of live load is the same in all load applications and that $\alpha = 1$ while the structure is responding to earthquake.

Under the circumstances the structure, either as built or after one or more collapses and reconstructions, survives until the occurrence of a sufficiently strong earthquake.

H_e will denote the loss in case the auditorium fails while empty, H_f if it fails when full, and $H_p = H_f - H_e$ will be taken proportional to W_p.

The failure probability under gravity loads, with empty auditorium, is

$$F_e = P(U_e \geq X_g)$$

$$= \int_0^{\infty} P_{X_g}(u)\ p_{U_e}(u)\ du \tag{2}$$

where P(·) without subscript is the probability that the event in parenthesis be true, and with subscript it is a probability distribution function; p is the corresponding probability density function.

The failure probability under gravity loads, with full auditorium, given that the structure did not fail while empty, is

$$P(U_e < X_g \leq U_f) = F_f - F_e \tag{3}$$

where $F_f = P(U_f \geq X_g)$. The expected loss caused by failure due to gravity is $F_eH_e + (F_f - F_e)H_f = F_fH_f - F_eH_p$. It follows from eq 3 that the expected loss due to gravity is

$$L_g = (F_fH_f - F_eH_p)(1 - F_f) \sum_{k=1}^{\infty} kF_f^{k-1}$$

$$= \frac{F_fH_f - F_eH_p}{1 - F_f} \tag{4}$$

Now idealize earthquakes as a generalized Poisson process. Let X denote the resistance to these disturbances and Y the maximum response to one such event, so $Y \geq X$ implies failure. In general, X and Y are functions of X_g and W_p. Suppose that the auditorium has a weight W_e a fraction q_e of the time, and $W_f = W_e + W_p$ a fraction $q_f = 1 - q_e$ of the time.

To compute the probability distribution of the failure rate under earthquake action, write

$$z \equiv (z/Z)(Z/Y)(Y/X)(X/x)x \tag{5}$$

where x is the nominal or design value of X and z is the computed value of the random variable Z, which is some parameter of ground motion (say, maximum ground acceleration or velocity). We can treat factors in eq 5 as stochastically independent, while x is deterministic. Probability distributions of z/Z have been proposed for the case when Z is computed from earthquake magnitude and focal distance (1). Distributions are also available for Z/Y, given Z (ref 2, for example, derives the distributions of responses of damped single-degree systems normalized with respect to the expected undamped response; we can treat this expectation as Z; the literature contains results for other types of structures). The distribution of X/x can be assigned on the basis of the distributions of material properties, geometrical discrepancies between drawings and the actual structure, and errors in formulas of analysis.

Given z we can obtain the rate $\nu(z)$ at which it is exceeded. We can use a Poisson model of seismicity (3), according to which, over a wide range of z,

$$\nu(z) = az^{-r} \tag{6}$$

where a and r are constants that depend on the meaning of z and on the site

the structure is to be erected; r lies between 2 and 4.

The failure rate is the mean rate at which $z \geq \phi x$, where $\phi = (X/x)(Z/Y)(z/Z)$. Given ϕ we can find the failure rate from

$$\lambda(x) = \nu(\phi x)$$

Since ϕ is a random variable, so is λ. Usually it is reasonable to assign $\ln \phi$ a normal distribution. Its expectation and variance equal the sum of those of the logarithm of X/x, Z/Y, and z/Z. This allows computing the required statistics.

If the structure is not rebuilt, the expected present value of the loss due to failure is $L_1 = E(H\int_0^\infty fg\, dt)$, but since $f = \lambda \exp(-\lambda t)$,

$$L_1 = E(\lambda H)\int_0^\infty e^{-(\lambda+\gamma)t}\, dt$$

$$= E\left(\frac{\lambda H}{\lambda + \gamma}\right) \tag{7}$$

Ordinarily the intervals during which the auditorium is full or empty are short relative to the most significant values of λ^{-1}. Then we can write, with good accuracy,

$$L_1 = q_e H_e E_e + q_f H_f E_f \tag{8}$$

where $$E_{e,f} = E\left(\frac{\lambda_{e,f}}{\lambda_{e,f} + \gamma}\right)$$

and subscripts e and f refer to empty and full auditorium.

If the structure is rebuilt systematically, we get the expected present value of losses due to seismic failures of the empty auditorium:

$$L_e = q_e H_e E_e \sum_{k=0}^{\infty} E_e^k = \frac{q_e H_e E_e}{1 - E_e}$$

and a similar expression for failures of the full auditorium. Hence, the expected present value of the losses due to earthquake is $L_s = L_e + L_f$, or

$$L_s = \frac{q_e H_e E_e}{1 - E_e} + \frac{q_f H_f E_f}{1 - E_f} \tag{9}$$

Combining with eq 4 we find

$$L = L_g\left(1 + \frac{q_e E_e}{1 - E_e} + \frac{q_f E_f}{1 - E_f}\right) + L_s$$

$$= L_g\left(\frac{q_e}{1 - E_e} + \frac{q_f}{1 - E_f}\right) + L_s \tag{10}$$

Here $E_{e,f}$ must be computed subjected to the condition that the structure has not failed under gravity loads. At the same time, $B = \int_0^\infty b_0 g\, dt$.

BENEFITS AND LOSSES: NONSTRUCTURAL DAMAGE

Under more general conditions damages wrought by the first earthquake are functions $H_{e,f}$ of Y. Curve in fig 1 is a schematic version of contractor's estimates of the corresponding losses in rentability and costs of repair, based on photographs of laboratory-tested masonry panels. If the structure is repaired or rebuilt systematically and left in a state probabilistically identical with the structure as originally built, and if it can be damaged by earthquake only but can undergo collapse under earthquake or gravity load, eq 10 still applies but E_e and E_f correspond to Y_1, the deformation causing collapse, while

$$L_s = q_e \int_0^{Y_1} \frac{H'_e e_e}{1 - e_e} dY + q_f\left(\int_0^{Y_1} \frac{H'_e e_f}{1 - e_f} dY + \frac{\Delta H_f E_f}{1 - E_f}\right) \qquad (11)$$

where $H'_e = dH_e/dY$, ΔH_f is the loss increment due to collapse, and $e_{e,f}$ are the values of $E_{e,f}$ associated with deformation Y. This solution is consistent with the assumption that the probability distributions of structural parameters for either empty or full auditorium are always the same before the occurrence of an earthquake. If the structure does not collapse, another acceptable assumption may be that it is restored to its condition as built.

Neither hypothesis is altogether realistic. Since the dispersion of ϕ is large, results obtained from the two assumptions differ substantially; moreover, the true answer does not necessarily lie between them. The matter deserves detailed study in representative cases. For the sake of brevity we shall adopt the first hypothesis.

Systematic repairing does not always constitute the best policy. For example, if H is proportional to Y, eqs 6 and 10 lead to infinite L. Given H(Y), there is some value of H below which the structure should be left in the damaged state. While the structure remains damaged there is a reduction in benefits per unit time. If a sufficiently strong earthquake occurs, the damage is large enough to justify repair, and benefits per unit time resume their original value. For the model we have postulated, though, the value of H above which repair is justified is higher when gravity loads can cause failure, for rebuilding after one such failure restores the benefits per unit time to b_0. The opposite may hold when earthquake damage weakens the structure under gravity loads.

OPTIMIZATION

Once B, C, and L are expressed in terms of x_g and x, the pair of values of these parameters must be found which maximizes Ω. Trial and error, mathematical programming, or other methods can be used for the purpose.

When B is insensitive to x_g and x it suffices to minimize C + L.

EXAMPLES

The following examples are deemed typical of conditions in the city of Acapulco, Guerrero, in Mexico. In the first one the only possible limit

state is collapse. In the second, earthquake damage is a function of maximum seismic response, as shown by the dashed line in fig 1. The pertinent parameters for the first example are, in units of tons and dollars, $W_e = 150$, variable W_p, $\alpha = 2.4$, $H_e = 50\ 000$, $H_p = 30\ 000\ W_p$, $\gamma = 0.08/yr$, $C = 45\ 000 + 1000\ x_g/W_e + 45\ 000\ x/W_e$. (This expression for C is based on analyses of typical frames.) Here x denotes the nominal resistance to base shear. The structure rests on firm ground and its natural period is sufficiently short that the base shear can be taken equal to the ground acceleration times the auditorium weight. Also, $\ln X_g/x_g \overset{d}{=} \ln \phi_x \overset{d}{=} N(0,0.2)$,* $\ln \phi_y \overset{d}{=} N(1.20, 0.20)$, $\ln \phi_z \overset{d}{=} N(-0.14, 1.02)$ (1), and $\nu = 30\ 000\ z^{-2.70}\ yr^{-1}$ (4).

The computed optimal x and x_g are shown in fig 2 for $q_f = 0.2$. The dashed line corresponds to $H_p = 300\ 000\ W_p$. The optimal central safety factors for gravity loads are displayed in fig 3. The optimal safety factor for W_e is 1.88. That for W_p can be taken as 2.6 without serious error, as it only differs significantly from this value when $W_p << W_e$.

We can define an equivalent live load, say W_p', to be used in earthquake resistant design, such that the base shear coefficient that, when multiplied by $1.88\ W_e$, gives the design base shear for $W_p = 0$, should when multiplied by $1.88\ W_e + W_p'$, produce the proper base shear for $W_p > 0$. From figs 2 and 3 it follows that $W_p'/\alpha W_p$ lies between 1.79 and 2.05.

In this case the live load that is used for design against gravity loads should be approximately doubled in seismic design.

Parameters assumed for the second example are the cause as for the first. The computed optimum x and x_g are shown by dot-dash lines in fig 2.

CONCLUDING REMARKS

We examined reliability optimization in the seismic design of auditoriums regarded as structures whose properties are time dependent. Systematic repair and reconstruction is not always the best policy. There is a level of damage below which the structure should not be repaired. This policy must be defined at the design stage.

Seismic design can be formulated in a traditional format using an equivalent live load, which is not necessarily smaller than in design for gravity loads. The approach is awkward in that design live loads for roofs depend on the expected number of people on the floor below. Explicit optimization seems more desirable.

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* meaning that the probability distributions of these variables are normal with expectation zero and standard deviation 0.2.

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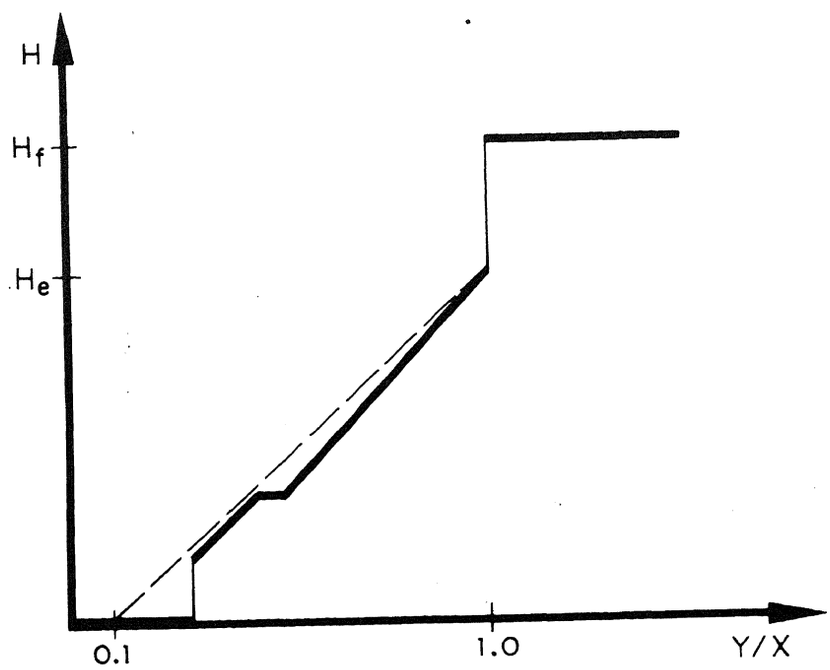

Fig 1 Loss function for structural members supporting or enclosing wall or partition panels (based on ref 5)

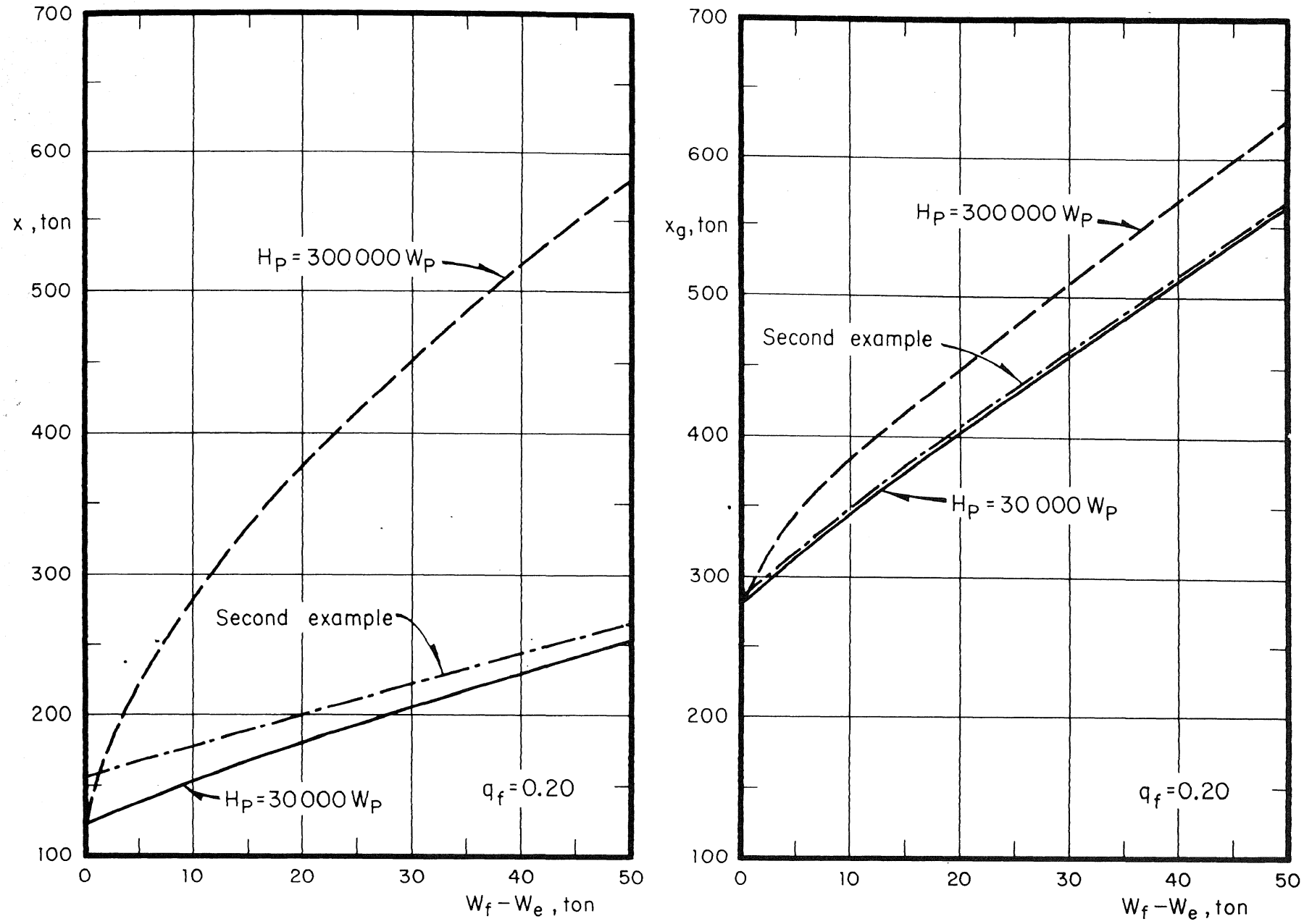

Fig 2 Optimal values of design parameters in auditorium-type structure

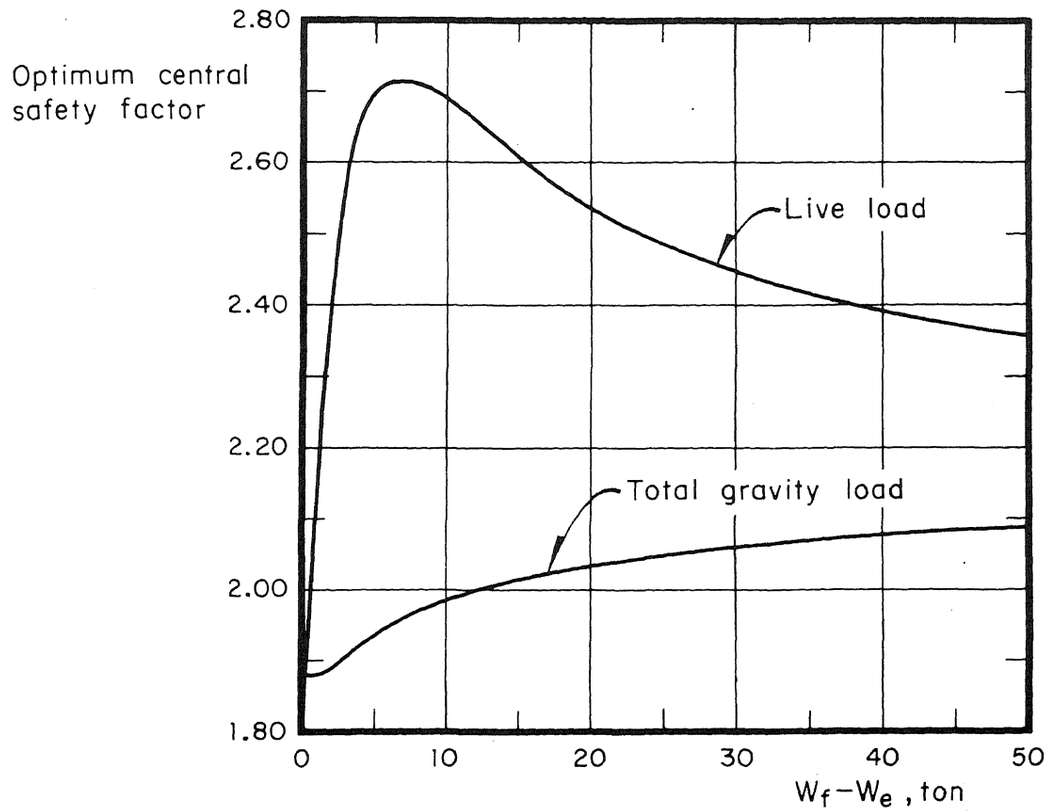

Fig 3 Optimal central safety factors for gravity loads