ESTIMATING STRUCTURAL PARAMETERS FROM RESPONSE DATA

by

A. J. Schiff[I], P. J. Feil[II], and J. L. Bogdanoff[III]

ABSTRACT

A general procedure is suggested for improving the ability to estimate the damping and natural frequency of low frequency structures from earthquake induced response data. Using this procedure parametric estimates of system parameters are developed. The new estimation procedure is compared with the method of moments and auto-regressive estimation techniques using digital and analog simulation techniques.

INTRODUCTION

The ultimate goal in analysing structures is to obtain information which will help the designer produce, in an economical manner, structures which are safer and better serve their intended function. Sources of such information include the testing and analysis of components, models or real structures. It is important to note that if earthquake response data is available it provides a unique opportunity to evaluate structural parameters from data obtained from potentially damaging inputs. This paper is concerned with evaluating large full-scale structures from response data obtained from earthquake excitations. The structural evaluation as it is addressed in this paper primarily consists of system identification in which modal frequencies and damping are identified.

The next section will review the problems and restrictions associated with the use of earthquake excitations and the characteristics of the structures to be analyzed. The statistical character of the identification process and the inherent limitations imposed by earthquake data are noted. The method of parametric curve fitting (PCF) is formulated. The auto-regressive model used by Akaike and the methods of moment used by Vanmarcke are introduced. Finally, the different methods are compared.

CHARACTERISTICS OF THE IDENTIFICATION PROBLEM

The realistic characterization of the systems to be identified from earthquake response data would present significant problems under controlled laboratory tests. In the field, the situation is indeed difficult: the structure and foundation are nonlinear and time dependent even under moderate service loads; these characteristics can become highly localised. Complete records of ground motion and structural response are not easy to get in many cases because of instrumentation problems: the first part of the records are customarily missing; a common local time for synchronizing records within a structure is also usually missing; the records stop after

I Associate Professor, School of Aeronautics, Astronautics, and Engineering Sciences, Purdue University, Lafayette, Indiana.

II Doctoral Candidate, School of Aeronautics, Astronautics, and Engineering Sciences, Purdue University, Lafayette, Indiana.

III Professor, School of Aeronautics, Astronautics, and Engineering Sciences, Purdue University, Lafayette, Indiana.

a preset length which may omit the decay of the structural motion; further, inherent short record length places severe restrictions on the possibility of reducing statistical variability of estimates obtained from the records. Finally, it is difficult to relate a record of ground motion or of motion at one place in the foundation to excitation of the entire foundation. Yet, in spite of these difficulties (elaborated upon in[1]), the information to be gained about a real structure under service condition which it must survive makes the analysis worth the required effort. The start in this effort requires simplifying assumptions.

Several assumptions will be made, some of which are at variance with the reality of the situation. Even with these assumptions, the results of analysis will be severely limited. The rigid adherence to the realities of the situation would make a difficult situation impossible. It will be assumed that the system to be identified is linear with constant coefficients and that the response is stationary and ergodic. While the initial development also assumes that the system to be identified is a single degree-of-freedom oscillator this requirement can be relaxed for the parametric curve fitting (PCF) method. The effects of nonstationarity of the excitation will also be evaluated.

All of the methods to be considered here will use estimates of the system's frequency response to estimate system parameters. The fundamental relationship which enables spectral estimates to be used to estimate a system's frequency response is given by

$$P_o(\omega) = |H(i\omega)|^2 P_i(\omega) \qquad (1)$$

where $P_i(\omega)$ is the power spectral density of the input to the system,
$H(i\omega)$ is the frequency response of the system, and
$P_o(\omega)$ is the power spectral density of the output of the system.

It will be assumed that the input power spectral density is flat in the region of the system's natural frequencies. Under these assumptions the power spectral density of the system response is an estimate of the square of the modulus of the frequency response. The estimate for the system resonant frequency is the frequency at which the response peaks and typically the damping is estimated by the frequency interval $\Delta\omega$ between the half power points of the response as indicated in Figure 1. The value for the natural frequency, ω_n, is obtained from the expression for the acceleration resonant frequency ω_a.

$$\zeta = \frac{\Delta\omega}{2\omega_n} \qquad \omega_n = \omega_a(1 - 2\zeta^2)^{1/2} \qquad (2)$$

Two methods of obtaining response spectra will be utilized. The first method uses the fast Fourier transform (FFT) to obtain unsmoothed spectral estimates. The second approach is to use the final predictor error method to estimate an auto-regressive sequence as developed by Akaike. Spectral estimates are then obtained from the auto-regressive coefficients. There are some fundamental limitations in obtaining spectral estimates which effect the identification process. While these limitations apply to both methods of spectral estimation, FFT and auto-regressive modeling, they are easier to visualize in reference to the FFT method. The highest resolution of statistically independent spectral estimates is determined by

record length, T, of the data and is expressed by equation (3).

$$\Delta f = \frac{1}{T} \tag{3}$$

Thus, for an earthquake of 10 seconds duration the resolution of the spectral estimate is .1 Hz. Figure 2 shows the scaling in the interval from .3 Hz to 6 Hz. The implications of this limitation are indicated by the response curves for .5, 1 and 5 Hz oscillators with 4% critical damping. As the natural frequency of an oscillator decreases, fewer points are available to define its frequency response curve. While an integral representation of the Fourier transform enables the spectrum to be evaluated at intermediate points, these points are not statistically or algebraically independent. Thus, no information is obtained which is not obtainable from the FFT spectral estimates. The conclusion to be drawn here is that if the record length is limited, as it is for earthquakes, severe limitations are imposed in estimating parameters for structures with periods longer than 1/10 that of the record length.

The PCF method for parameter estimation will use the unsmoothed FFT of the system response to estimate its power spectral density. The advantages of this procedure is that maximum spectral resolution is obtained. The typical smoothing of FFT spectral estimates or the use of the Blackman-Tukey approach to spectral estimation yields spectral estimates with reduced resolution and introduces bias errors. That is, a sharp spectral peak which is narrower than the spectral smoothing window will be underestimated. The disadvantage in using the FFT is that it does not provide a consistent estimate of the power spectral density. That is, as longer records are analyzed the variability of the estimate does not converge to zero. The wild fluctuations of unsmoothed spectral estimates will be seen later. The estimation of system damping using the half power method on unsmoothed spectral estimates yield unreliable estimates. The use of PCF or the method of moments yields relatively stable parametric estimates with unsmoothed spectral estimates.

METHOD OF PARAMETRIC CURVE FITTING

The difficulties associated with spectral resolution and estimate variability for low natural frequencies suggests the use of parametric estimation techniques. The objective is to reduce the variability of estimates of structural parameters by utilizing knowledge of the situation. In particular, the statistical properties of the spectral estimates are known and the form of the system's frequency response in the region around its natural frequency is assumed to be that of a linear single degree-of-freedom system. The distribution function of unsmoothed spectral estimates is proportional to a chi-squared distribution with 2 degrees-of-freedom. The weighting function, f_j, which transforms the spectral estimates to identically distributed random variables is the inverse of the squared frequency response. The particular form of the frequency response depends on the response variable measured. The form considered here is for acceleration response to a white noise acceleration excitation at the base of the structure. Thus

$$f_j = \frac{1}{|H_a(i\omega)|^2} = \frac{(\omega_n^2 - \omega_j^2)^2 + 4\,\zeta^2\,\omega_n^2\,\omega_j^2}{\omega_n^4 + 4\,\zeta^2\,\omega_n^2\,\omega_j^2} \tag{4}$$

where ω_j corresponds to the frequency of the spectral estimate. The normalized spectral estimate is given by

$$y_j = A\, x_j\, f_j \tag{5}$$

where A is a gain factor,

x_j is the unsmoothed spectral estimate at ω_j, and

f_j is the weighting function.

Due to the skew character of the distribution function a least squares fit is inappropriate. The procedure adopted here is to adjust the parameters ζ, ω_n and A in the weighting function so that

$$\sum_j^N (F(y_j) - \frac{1}{2}) = 0 \tag{6}$$

where $F(y_j)$ is the distribution function of y_j. This is equivalent to minimizing E with respect to ζ, ω_n and A, where

$$E = \sum_j (y_j - \frac{1}{2})\, e^{-1} + e^{-\frac{1}{2}} \tag{7}$$

If the Jacobian is positive definite, the Newton-Rapson method is used. Otherwise, a simple gradient method is used. More details on these methods are contained in[2,3].

If the spectral estimate is obtained from a linear oscillator subjected to band limited white noise and if it is assumed that there are no measurement or round off errors, the spectrum will have the assumed form. Figure 3 shows an unsmoothed spectral estimation and the square of the frequency response for a 3.75 Hz oscillator with 4% of critical damping. This is a log plot with 10 DB per division. It can be seen that fluctuations of ± 20 DB are not uncommon. It can also be seen that the estimated spectrum at its high frequencies is above the true frequency response. This is 90 DB down from the peak and the small distortion comes from computer round off error. The details of how these curves have been obtained are contained in the last section.

In general, only the data in an interval, the analysis interval (AI), surrounding the peak is used to obtain the estimates. As the AI is increased to include more points the variability of the resulting parameter estimates is reduced if the form of the spectral estimate is undistorted from the assumed form. This is the case in Figure 3. Figure 4 shows a spectral estimate with measurement noise. Distortion in the estimated spectra can be seen at the higher frequencies when the amplitude of the response falls below that of the measurement noise. In this case, as the AI increases to include badly distorted portions of the frequency response, the fitted curve will be distorted, introducing bias to the estimated parameters. The parameter most sensitive to the bias errors is the damping. A method for determining the width of the AI is to plot ζ as the AI increases as shown in Figure 5. The PCF method can utilize the frequency response for a two degree-of-freedom system and thus yield parametric estimates for closely spaces natural frequencies.

OTHER TYPES OF SPECTRAL AND PARAMETRIC ESTIMATION

In the previous section, unsmoothed spectral estimates obtained through the FFT were used to get estimates for system damping and natural frequency. This section will describe the method of moments for estimating structural parameters from spectral estimates, and an alternative method for getting spectral estimates.

Vanmarcke[4,5] has developed a method which utilizes the moments of the frequency response function. The method used here has been slightly modified to eliminate bias errors present in the original scheme, details of which are contained in[1]. The advantages of this technique are that the entire frequency response in the AI is utilized thus reducing the statistical variability of the estimates of ω_n and ζ. This method is also based on the response being from a single degree-of-freedom oscillator. The method cannot be applied directly if modal frequencies are too close. There is also no criteria for selecting the size of the analysis interval. This method as with the PCF method allows unsmoothed spectral estimates to be used. The statistical characteristics of spectral estimates are not utilized in this method so that other types of spectral estimates, such as the following, can also be used.

An alternative method for obtaining spectral estimates has been developed by Akaike[6,7] which utilizes an auto-regressive representation of a time series. Using this method smooth spectral estimates are always obtained, however, the statistical variability of resulting spectral estimates is difficult to estimate. The application of Akaike's method is straightforward and is summarized in[1]. Once the spectral estimate is obtained, the damping can be estimated by any of the other methods described. It is suggested, however, that the method of moments described earlier be used. Note that this method only uses a single time series, that of the response. Thus, the estimate of the frequency response amplitude assumes that the spectra of the excitation is flat in the region around the natural frequency being investigated.

COMPARISON OF METHODS

Three methods of generating data for the evaluation of the different estimation techniques were used. The first used a digital computer to solve the system differential equations using a fourth order Runga-Kutta integration. The spectral estimates shown in Figures 3 and 4 were obtained from time series generated in this manner. The system response was allowed to reach its stationary value before response data was collected. Figure 3 shows the effects of round off error, some 90 DB down from the response peak.

An analog computer was also used to simulate the response of a system to excitation noise. Figure 6 shows the response without intensionally added measurement noise. In this case the computer noise is seen to be some 70 DB below the peak response as compared to 90 DB for digitally generated data. Analog generated response data was used to evaluate transient and nonstationary effects.

A third simulation method was used to generate spectral estimates directly. Figure 7 shows a spectra generated by using equation (5). That is, a sequence of numbers distributed as a chi-squared with two degrees-of-

freedom was scaled with the frequency response given in equation (4). The simularity with Figure 3 should be noted. This technique was used to evaluate PCF and the method of moments for low natural frequencies. Figure 8 shows the spectra of the system with the lowest natural frequency for which reasonable damping estimates could be obtained which was for a .78 Hz oscillator. The PCF and MM method yielded estimates of ζ of .030 and .21, respectively. The true value was .04. It should be noted that these estimates were obtained from simulated data free of measurement errors. Thus the dynamic range of the data is 90 DB, an unrealistic range for real data. When the analysis interval is restricted to data no lower than 40 DB below the peak realistic spectral estimates could not be obtained with this oscillator.

Figure 9 shows the spectral estimates using the auto-regressive method without and with measurement noise corresponding to Figure 3 and 4, respectively. In this case, the fit to the spectrum is very good. Note that with measurement noise the skirts of the spectral estimate are broader. Comparison of the results using the MM, PCF and AR methods for data shown in Figure 3, 4 and 9 is summarized in Table 1. The analysis interval for these values was determined from Figure 5. Figure 10 shows the true frequency response (curve A) for a 12.5 Hz, 4% critically damped oscillator. The estimates with no noise and with a 21 Hz sine added are depicted in curves B and C, respectively. The differences in curves B and C illustrates that a poor signal to noise ratio in one portion of the spectrum can influence the entire spectral estimate when using the auto-regressive (AR) approach.

All of the response data analysed above has been obtained from stationary excitations and indeed the response was allowed to reach stationarity before data was collected. Thus the only real simularity to earthquake response data is that record lengths were restructed to 10 seconds duration. The effects of response and excitation nonstationarity on the estimation of damping are evaluated. Three tests were run using the analog system. For the first test the system was allowed to reach stationarity before digitization of the response was initiated as above. Ten seconds of both the response and excitation were retained, the excitation being shown in Figure 11. In the second test this excitation was again applied to the system which was started from rest, thus the excitation was stationary but the response was nonstationary. In the third test the amplitude of the excitation was modified as shown in Figure 12 and the structure was started from rest. Thus the excitation takes on a nonstationary form in this case. The resulting spectra were similar in form although the signal to noise ratio at high frequencies for the final test was markedly poored than for the first two tests. This is a result of the generally lower excitation as indicated in Figure 12. The damping estimates for the three tests as obtained by PCF and MM are shown in Table 2. The same analysis interval of 3.7 bandwidths was used throughout. It can be seen that nonstationary effects can significantly effect the results.

CONCLUSIONS

The unsmoothed FFT estimates of power spectral density can be used to estimate the damping and natural frequency of a structure from short response records. The PCF and MM methods of estimating damping give similar results with the MM typically yielding smaller estimates. The PCF can obtain estimates of damping for lower frequency systems than the MM. The auto-

regressive generated spectral estimates can be entirely distorted by a poor signal to noise ratio in limited region of the spectrum. Neglecting the effects of nonstationarity of the excitation or of the response can cause significant estimation errors. For the limited data analysed the nonstationarity of the structural response (zero initial conditions) has a larger influence than the nonstationarity of the excitation (time envelope of the excitation).

ACKNOWLEDGMENT

The preparation of this paper was supported in part by the National Science Foundation. The assistance of Mr. Gregory Guthrie in data collection and analysis is gratefully acknowledged.

REFERENCES

1. Schiff, A. J., "Identification of Large Structures Using Data From Ambient and Low Level Excitations," System Identification of Vibrating Structures, Mathematical Models From Test Data, Pilkey and Cohen, Ed. ASME, 1972.

2. Schiff, A. J. and Feil, P., "Considerations in Estimating Structural Parameters From Response Data," Proceedings of U.S.-Japan Joint Seminar on Stochastic Methods in Dynamical Problems, Kyoto, Japan, Oct. 1971.

3. Schiff, A. J., "Identification of Structures From Response Data," To be published.

4. Vanmarcke, E. H., "Parameters of the Spectral Density Function: Their Significance in the Time and Frequency Domains," Report 70-58, Dept. of Civil Engineering, Mass. Institute of Technology.

5. Vanmarcke, E. H., "Estimation of the Dynamic Properties of Off Shore Structures," Third Annual Off Shore Technology Conference, Houston, Texas, April, 1971.

6. Akaike, H., "Power Spectrum Estimation Through Auto-Regressive Model Fitting," Ann. Institute Statist. Math., Vol. 21, No. 3, 1969.

7. Akaike, H., "A Method of Statistical Identification of Discrete Time Parameter Linear Systems," Ann. Institute Statist. Math., Vol. 21, No. 2, 1969.

COMPARISON OF PARAMETRIC ESTIMATES

		True Value	Moment Method	Parametric Curve Fitting	Auto-Regressive
With No Measurement Noise (Figure 3)	ω_n	23.56	23.74	23.84	23.81
	ζ	.04	.0279	.0395	.0304
With Measurement Noise (Figure 4)	ω_n	23.56	23.76	23.87	23.90
	ζ	.04	.0341	.0436	.0599

TABLE 2

EVALUATION OF NONSTATIONARY EFFECTS ON ESTIMATES OF DAMPING

	True	Test 1	Test 2	Test 3
PCF	.04	.034	.060	.071
MM	.04	.029	.049	.065
Stationary Excitation		Yes	Yes	No
Stationary Response		Yes	No	No

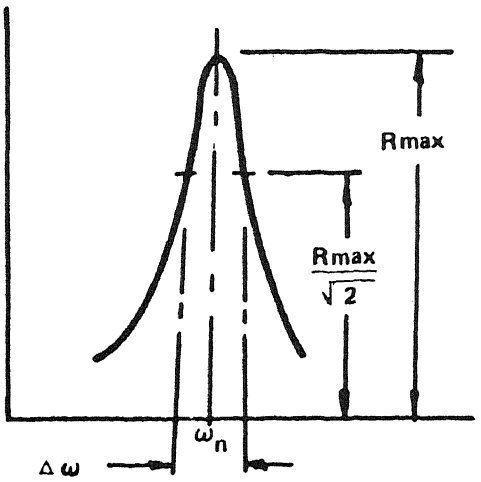

Fig. 1 Use of half-power method

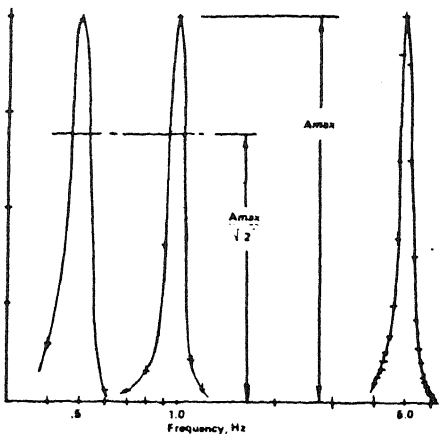

Fig. 2 Spacing of spectral estimates

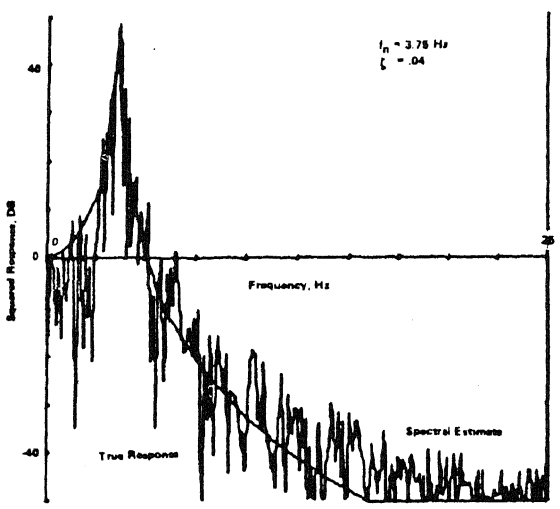

Fig. 3 Spectral estimate and true frequency response

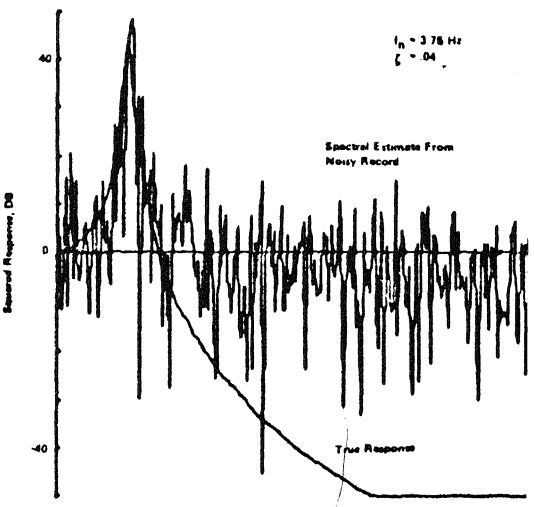

Fig. 4 Estimated spectrum with noise and the true frequency response

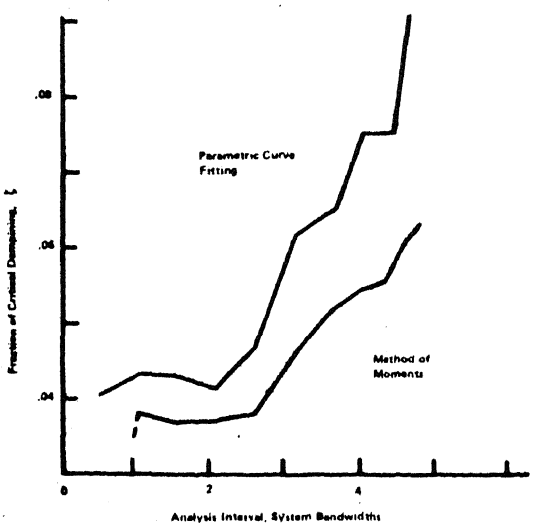

Fig. 5 Estimates of ζ for increasing analysis interval

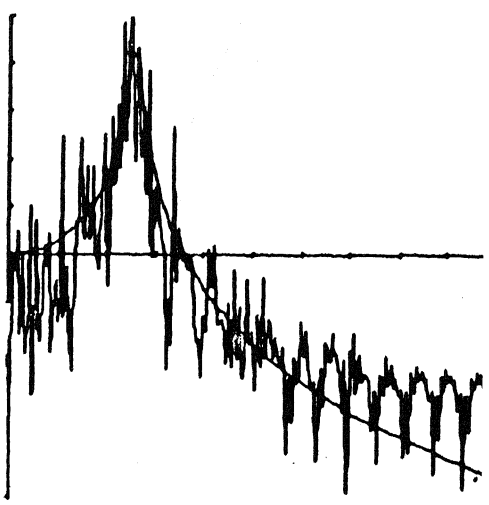

Fig. 6 Spectral estimate from analog computer simulation

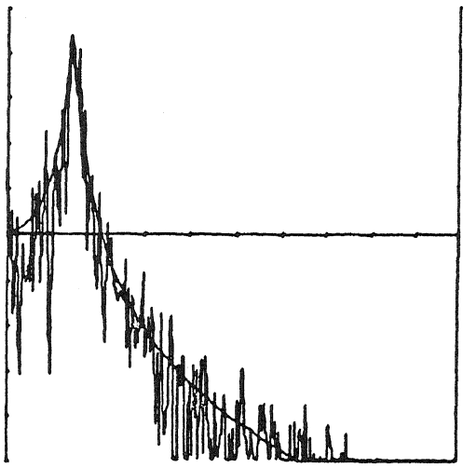

Fig. 7 Simulated spectrum

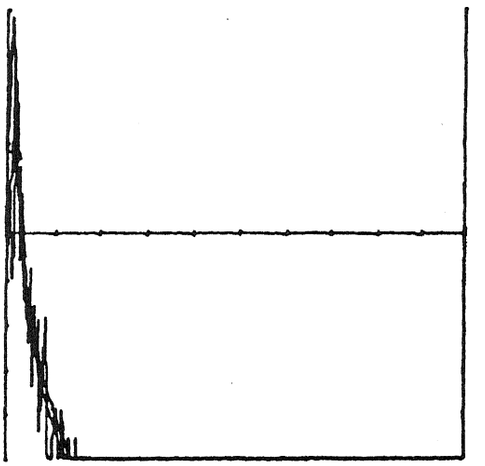

Fig. 8 Simulated spectrum

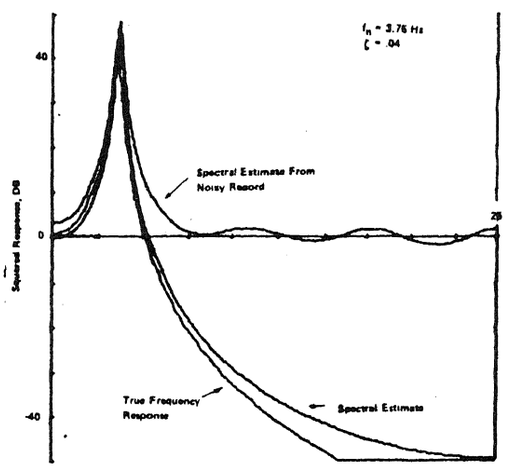

Fig. 9 Auto regressive spectral estimates

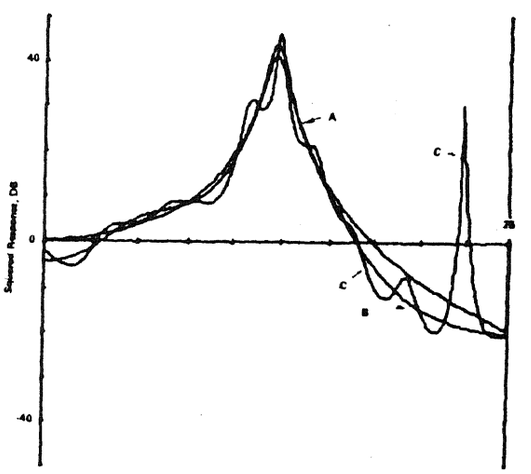

Fig. 10 Auto regressive spectral estimates with sinusoidal noise

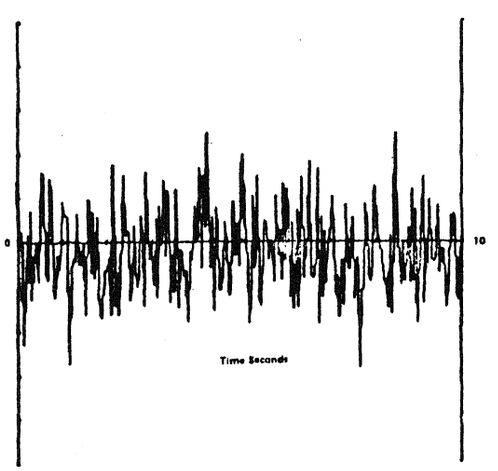

Fig 11 Stationary simulated acceleration

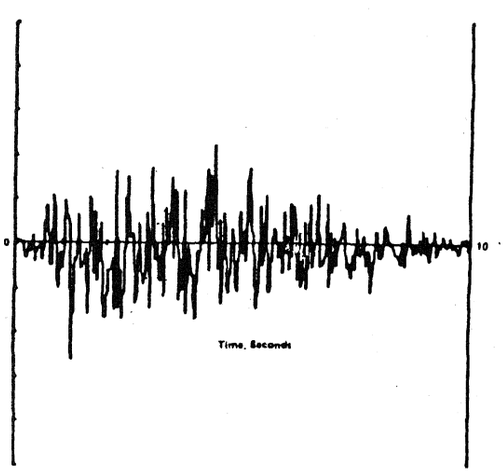

Fig 12 Nonstationary simulated acceleration