STATISTICALLY EFFICIENT AND COMPUTATIONALLY EFFICIENT IDENTIFICATION OF STRUCTURAL PARAMETERS FROM RANDOM VIBRATION RECORDS

by

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SYNOPSIS

A new statistically efficient and computationally efficient maximum likelihood computational procedure for determining the period and damping values of linear multi-degree-of-freedom structures and estimates of their statistical reliability from records of their response to random winds or earthquake excitation is described. Damping and period estimates computed from a modern nine-story steel frame building are analyzed and compared with the results obtained from steady-state forced vibration tests.

INTRODUCTION

A new statistically efficient and computationally efficient maximum likelihood computational procedure for determining the period and damping values of lumped mass-spring-damper models of multi-degree-of-freedom systems and estimates of their statistical reliability from records of their response to random winds or earthquake excitation is described.

The problem we treat is known in statistical inference in time series as the problem of estimating the parameters of a stochastic differential equation using regularly sampled observations [1]. The novelty in our approach has been (1) in developing the relationship between the mixed autoregressive-moving average (AR-MA) time series that represents the regularly sampled continuous time vibration data that we analyze and the underlying differential equation structural system model [2]-[4] and (2) in developing a computationally efficient maximum likelihood AR-MA time series parameter estimation procedure that uses only correlation function data [5]. In contrast with other known methods of determining periods and damping in structures [6]-[7], our method yields both the structural system parameters and an evaluation of the statistical reliability of those estimates.

In this paper only computations from wind excited structural responses are discussed. The extension to input-output earthquake response type data is a straightforward extension of the procedures discussed here and is deferred to a future publication. Damping and period estimates from a modern nine-story steel frame building are obtained using our method and compared with the results obtained from steady state forced vibration tests [8].

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THE PROBLEM

The building structure under consideration is represented by the n degree-of-freedom second order coupled stochastic differential equation model.

$$M\ddot{z}(t) + C\dot{z}(t) + Kz(t) = f(t) \tag{1}$$

The input f(t), an approximation to a random wind, is assumed to be an n component white noise with covariance $E[f(t)f'(t')] = D\delta(t-t')$. The vector of displacements (velocity or acceleration may be used) is $z'(t) = [z_1(t),\ldots z_n(t)]$ where $z_i(t)$ is the relative displacement of the i th mass to the ground.

The top story building data $z_n(t)$ is assumed to be sampled regularly with sampling interval T_s. The resultant scalar discrete time series represented by z_t t=1,2,... can be represented by the mixed AR-MA time series model [3],[4]

$$\sum_{i=0}^{2n} \alpha_i z_{t-i} = \sum_{i=1}^{2n} \tilde{\beta}_i x_{t-i} \; . \tag{2}$$

The $\{x_t\}$ in (2) is an uncorrelated sequence with variance σ^2. The AR parameters $\{\alpha_i\}$ i=1,...2n, computed from the M,C,K parameters in (1) can be expressed in the characteristic equation form

$$\sum_{i=0}^{2n} \alpha_i \mu^{2n-i} = \prod_{i=1}^{n} (\mu - e^{-\xi_j w_{nj} T_s + i w_{nj}\sqrt{(1-\xi_j^2)}\, T_s})(\mu - e^{-\xi_j w_{nj} T_s - i\, w_{nj}\sqrt{(1-\xi_j^2)}\, T_s}) \tag{3}$$

where $\xi_j w_n \pm i w_{nj}\sqrt{1-\xi_j^2}$ j=1,...n are the eigenvalues of the system in (1).

On the assumption that the regularly sampled observation process consists of the regularly sampled building vibration data $\{z(t)\}$ plus an uncorrelated zero mean additive noise process which may be due to instrument error, digital conversion error or other sources, the observed process, $\{y_t\}$ can be represented by the mixed AR-MA time series

$$\sum_{i=0}^{2n} \alpha_i y_{t-i} = \sum_{i=0}^{2n} \beta_i x_{t-i}, \quad \alpha_0 = \beta_0 = 1 \tag{4}$$

We assume that a finite discrete time data sequence y_t, t=1,...N is observed. The problem is to obtain statistically efficient estimates of the structural system parameters ξ_j, w_{nj} j=1,...n using the y_t, t=1,...N data. We compute maximum likelihood estimates of the $\{\alpha_i\}$ $\{\beta_i\}$ i=1,...2n AR-MA parameters in (4), then via (3) transform the $\{\alpha_i\}$ AR parameters to the ξ_j, w_{nj} j=1,...n structural system parameters.

In the maximum likelihood procedure, the correlation function of the

{Yt} data is operated on by the inverse of the filter in (4) that produced {Yt}. The result is cxx(0) the correlation function of the innovation sequence (discrete white noise) input to the system. The quantity V=cxx(0) is the maximum likelihood functional. The parameters $\{\alpha_i\}, \{\beta_i\}$ i=1,...2n are iteratively adjusted in a Newton-Raphson procedure until V is minimized. The hessian matrix H of the second order partial derivatives of V with respect to the AR-MA parameters used in the Newton-Raphson procedure is the Fisher information matrix [9].

The inverse of H is the covariance matrix of the AR-MA parameter estimates. The maximum likelihood parameter estimates achieves the Cramer-Rao theoretical lower bound on the covariance of the parameter estimates [9]. A covariance matrix for the structural system parameter estimates can be computed from the terms of the covariance error matrix of the AR parameter estimates and the transformation relating the structural parameters to the AR parameters $\{\alpha_i\}$ in (3). Because the mapping between the AR parameters and the structural parameters is one to one and unique and the AR estimates are efficient, the structural parameters are efficient [10].

APPLICATION TO PRACTICAL DATA

Initial spectral analysis of the Cherry-Brady [6] digitally sampled correlation function data revealed the contaminating influence of aliased air-conditioner motor vibrations and vibrations from other mechanical devices in the building. (See fig. 1). Subsequent correlation function data filtering and data rate reduction (with a corresponding decreased Nyquist frequency) isolated the first two modes.

The maximum likelihood computational procedure applied to the reduced correlation function data with N-300 and T_S=.06 seconds yielded the following results, the corresponding AR-MA estimate of the spectrum is shown in fig. 2.

ξ_1 = .036 (1 ± 0.70)	[.0055]
w_{n1} = 1.11 (1 ± 0.04) Hz	[1.10 Hz]
ξ_2 = .064 (1 ± 0.28)	[.0085]
w_{n2} = 3.87 (1 ± 0.02) Hz	[3.40 Hz]

The ± entries are the coefficient of variations with the associated parameter estimates. The coefficient of variation is the ratio of the standard deviation to the mean of the parameter estimate. The bracketed entries are the estimates obtained in steady-state forced vibration tests [8]. As can be seen from the coefficient of variation the damping estimates are very unreliable, probably due to the very high noise level in the original data and the very short record (18 sec.). Analysis of additional wind vibration and San Fernando earthquake data taken on the same building which is less noisy, which expose a larger number of degrees of freedom and with larger numbers of data points (N) are in progress.

The results reported here are the first structural system parameter estimates known to be performed whose statistical reliability has been evaluated. They are also the first known statistically efficient structural system parameter estimates.

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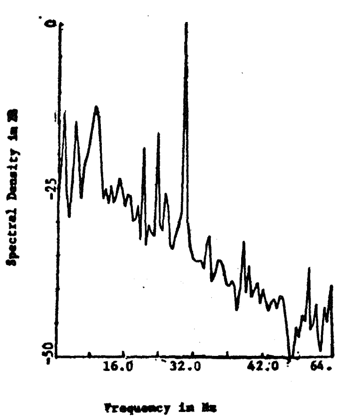

FIG.1. Spectral Density of Cherry-Brandy Data. Nyquist Frequency = 1/.015 Hz.

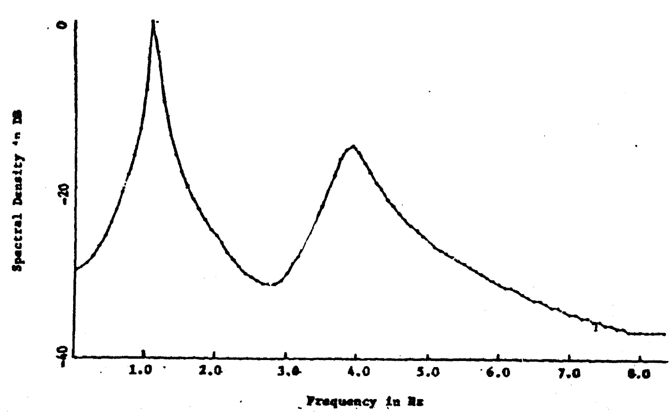

FIG.2. AR-MA Estimate of Spectral Density Filtered and Reduced Cherry-Brady Data. Nyquist Frequency, 1/.12 Hz