STRESS - STRAIN ANALYSIS METHOD FOR DAMS UNDER SEISMIC ACTIONS

by

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The main difficulty which arises in solving stress-strain state problems for dams under seismic actions is the lack of information on the seismic sources. The only available information is provided by seismograms of displacements on the surface of the foundation which can be measured on the site of the future dam footing.

When the same seismic actions are repeated after the erection of the dam, the mentioned seismograms cannot be used as boundary conditions along the footing of the dam because they get altered under the influence of the dam. Nevertheless, the knowledge of these seismograms turns out to be sufficient for the solution of the problem. Below we show haw to do this.

A plane dynamic problem of the theory of elasticity and hydroelasticity is discussed for the dam-foundation-liquid system. The solution is carried out in finite differences by a method described in [1, 2].

I. Let the unknown source generate a wave motion in the foundation which is assumed to be elastic halfplane. This wave motion manifests itself on the surface $y=0$ in the form of horisontal and vertical displacements $u(x,y,t)$ resp. $v(x,y,t)$ which are known /measured/. In the presence of the dam the wave motion in the half-plane gets altered under the influence of reactions $\sigma(x,t), \tau(x,t)$ which arise along the footing of the dam /see fig. 1 /. Therefore the motion in the half-plane will be a superposition of displacements $u(x,y,t)$, $v(x,y,t)$ caused by the seismic source, and of an additional displacements $\Delta u(x,y,t)$, $\Delta v(x,y,t)$ caused by the mentioned reactions. So the displacements in the half-plane /including the dam footing/ will be

$$U(x,y,t)=u(x,y,t)+\Delta u(x,y,t)\,, \quad V(x,y,t)=v(x,y,t)+\Delta v(x,y,t) \quad (1)$$

These displacements related to the points on the footing $U(x,0,t), V(x,0,t)$ are the boundary conditions for the dam which determine the motion in it.

2. Let us give here some formulae from [2] which are the fi-

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nite-difference analogues of the Lamé motion equations. These formulae permit to calculate the displacements U, V at the moment $t+\Delta t$ at each node i of the domain under consideration if these displacements are known at the moments t and $t-\Delta t$ /the explicit three-layers difference scheme/. The boundary conditions for the domain and the initial conditions must be known.

All the inner nodes of the network domain /see fig. 1 / the finite-difference motion equations are of the form

$$\begin{aligned} U_{\substack{i\\t+\Delta t}} &= -2\frac{C_2^2}{C_1^2 m^2} U_{\substack{i\\t}} + \left(U_{i+\Delta x}+U_{i-\Delta x}\right)_t + \frac{C_2^2}{C_1^2 m^2}\left(U_{i+\Delta y}+U_{i-\Delta y}\right)_t + \\ &+\left(1-\frac{C_2^2}{C_1^2}\right)\frac{1}{4m}\left(V_{\substack{i+\Delta x\\i+\Delta y}}+V_{\substack{i-\Delta x\\i-\Delta y}}-V_{\substack{i+\Delta x\\i-\Delta y}}-V_{\substack{i-\Delta x\\i+\Delta y}}\right)-U_{\substack{i\\t-\Delta t}} = \mathcal{K}_1(U,V)_{t,t-\Delta t}\,; \\ V_{\substack{i\\t+\Delta t}} &= 2\left(1-\frac{1}{m^2}-\frac{C_2^2}{C_1^2}\right)V_{\substack{i\\t}} + \frac{1}{m^2}\left(V_{i+\Delta y}+V_{i-\Delta y}\right)_t + \frac{C_2^2}{C_1^2}\left(V_{i+\Delta x}+V_{i-\Delta x}\right)_t + \\ &+\left(1-\frac{C_2^2}{C_1^2}\right)\frac{1}{4m}\left(U_{\substack{i+\Delta x\\i+\Delta y}}+U_{\substack{i-\Delta x\\i-\Delta y}}-U_{\substack{i+\Delta x\\i-\Delta y}}-U_{\substack{i-\Delta x\\i+\Delta y}}\right)_t - V_{\substack{i\\t-\Delta t}} = \mathcal{L}_1(U,V)_{t,t-\Delta t}\,. \end{aligned} \tag{2}$$

Here U and V are the x - and y -axes components of the displacements, $C_1=\sqrt{\frac{E}{\rho(1-\mu^2)}}$, $C_2=\sqrt{\frac{E}{2\rho(1+\mu)}}$ are the longitudinal and transversal velocities of the elastic waves, $m={}^{\Delta y}/_{\Delta x}$ is the mesh ratio of the network, $\Delta t=\frac{\Delta x}{C_1}$ /for $m>1$ / is the time step, E, μ and ρ are the elastic modulus, Poisson's coefficient and the density of the material, $\mathcal{K}$ and $\mathcal{L}$ are the abridged notations for the finite-difference operators. The indeces notation of the nodes in the vicinity of i is clear from the figure 2 / i being the abridged notation of the node $(x_i\,,\,y_i)$ /.

On these segments of the boundary, where the load is given, the motion equations /transformed in a special way [1] / are written out in finite differences as follows, taking into account the boundary conditions:

For the nodes i on the line parallel to the x -axis

$$\begin{aligned} U_{\substack{i\\t+\Delta t}} &= 2\left[1-c_2^2\Delta t^2\left(\frac{2+\mu}{\Delta x^2}+\frac{1}{\Delta y^2}\right)\right]U_{\substack{i\\t}} + \frac{(2+\mu)c_2^2\Delta t^2}{\Delta x^2}\left(U_{i+\Delta x}+U_{i-\Delta x}\right)_t \\ &+\frac{2c_2^2\Delta t^2}{\Delta y^2}U_{\substack{i-\Delta y\\t}} - \frac{c_2^2\Delta t^2}{\Delta x\Delta y}\left(V_{i+\Delta x}-V_{i-\Delta x}\right)_t + \frac{2\Delta t^2}{\rho}\left[\frac{q_{yx}}{\Delta y}+\frac{1+\mu}{4}\left(\frac{\partial q_y}{\partial x}\right)\right]_{\substack{i\\t}} - U_{\substack{i\\t-\Delta t}} = \mathcal{K}_2(U,V)_{t,t-\Delta t}\,; \end{aligned}$$

$$V_{\substack{i\\t+\Delta t}} = 2\left[1+c_1^2\Delta t^2\left(\frac{\mu}{\Delta x^2}-\frac{1}{\Delta y^2}\right)\right]V_{\substack{i\\t}} - \frac{\mu c_1^2\Delta t^2}{\Delta x^2}\left(V_{i+\Delta x}+V_{i-\Delta x}\right)_t + \frac{2c_1^2\Delta t^2}{\Delta y^2}V_{\substack{i-\Delta y\\t}} - \tag{3}$$

$$-\frac{\mu C_1^2 \Delta t^2}{\Delta x \Delta y}\left(U_{i+\Delta x}-U_{i-\Delta x}\right)_t+\frac{2\Delta t^2}{\rho}\left[\frac{q_y}{\Delta y}+\frac{1+\mu}{2(1-\mu)}\left(\frac{\partial q_{yx}}{\partial x}\right)\right]_{\substack{i\\t}}-\underset{t-\Delta t}{V_i}=\mathcal{L}(U,V)_{t,t-\Delta t}\,.$$

$q(x,t)$ being the given load on the boundary.

For the nodes on the line parallel to the y -axis x and y, U and V must be interchanged in / 3 /. Analogous formulae are obtained [1, 2] for nodes on the oblique segments of the boundary, for vertices of different kinds, etc.

To ensure the stability of this explicit difference scheme an error correction of the computed values $\underset{t+\Delta t}{U_i}, \underset{t+\Delta t}{V_i}$ must be carried out [2] on each step Δt according to the formulae

$$\underset{t+\Delta t}{\tilde{U}_i}=k\underset{t+\Delta t}{U_i}+\frac{1-k}{2}\left(\underset{t}{\tilde{U}_i}+\underset{t+2\Delta t}{U_i}\right), \tag{4}$$

where $\underset{t+\Delta t}{\tilde{U}_i}, \underset{t}{\tilde{U}_i}$ being the corrected values of the displacements $\underset{t+\Delta t}{U_i}=\mathcal{K}(U,V)_{t,t-\Delta t}$, $\underset{t+2\Delta t}{U_i}=\mathcal{K}(U,V)_{t+\Delta t,t}$ and analogously for V.

Here k is the correction coefficient which lies in the interval $0{,}78\geqslant k\geqslant 0{,}57$ for $\mu=0{,}167$.

Some additional methods to ensure the stability of the difference scheme /on the boundary parallel to the x -axis for $m>1{,}4$ / are given in [2].

Under known displacements the following formulae serve to determine the stresses at the nodes i:

$$\sigma_{x,\underset{t}{i}}=\frac{E}{2(1-\mu^2)}\left(\frac{U_{i+\Delta x}-U_{i-\Delta x}}{\Delta x}+\mu\frac{V_{i+\Delta y}-V_{i-\Delta y}}{\Delta y}\right)_t;$$

$$\sigma_{y,\underset{t}{i}}=\frac{E}{2(1-\mu^2)}\left(\frac{V_{i+\Delta y}-V_{i-\Delta y}}{\Delta y}+\mu\frac{U_{i+\Delta x}-U_{i-\Delta x}}{\Delta x}\right)_t; \tag{5}$$

$$\tau_{xy,\underset{t}{i}}=\frac{E}{4(1+\mu)}\left(\frac{U_{i+\Delta y}-U_{i-\Delta y}}{\Delta y}+\frac{V_{i+\Delta x}-V_{i-\Delta x}}{\Delta x}\right)_t.$$

3. The following remark is important for the solution of the problem under consideration. The displacements $u(x,0,t)$, $v(x,0,t)$ on the boundary being known, the displacements $u(x,-\Delta y,t)$ and $v(x,-\Delta y,t)$ on the pre-contour line /see fig. 1, line ef / are easily determined. In fact, on the ground of / 3 / and taking into account the absence of load on the border of the half-plane, the displacements at the pre-contour nodes $\underset{t}{u_{i-\Delta y}}$, $\underset{t}{v_{i-\Delta y}}$ are expressed by

$$\underset{t}{u_{i-\Delta y}}=\frac{m^2}{1-\mu}\left(\underset{t+\Delta t}{u_i}+\underset{t-\Delta t}{u_i}\right)-\left[\frac{m^2\mu(1+\mu)}{1-\mu}-1\right]\underset{t}{u_i}-\frac{m^2(2+\mu)}{2}\left(u_{i+\Delta x}+u_{i-\Delta x}\right)_t+$$
$$+\frac{1}{2}\frac{\Delta y}{\Delta x}\left(v_{i+\Delta x}-v_{i-\Delta x}\right)_t; \tag{6}$$

$$\mathcal{V}_{\substack{i-\Delta y\\ t}} = \frac{m^2}{2}\left(\mathcal{V}_{\substack{i\\ t+\Delta t}} + \mathcal{V}_{\substack{i\\ t-\Delta t}}\right) - \left[m^2(1+\mu)-1\right]\mathcal{V}_{\substack{i\\ t}} + \frac{\mu m^2}{2}\left(\mathcal{V}_{i+\Delta x} + \mathcal{V}_{i-\Delta x}\right)_t + \frac{\mu}{2}\frac{\Delta y}{\Delta x}\left(\mathcal{U}_{i+\Delta x} - \mathcal{U}_{i-\Delta x}\right)_t .$$

Here the right-hand members contain only values of $\mathcal{U}$ and $\mathcal{V}$ known on the border of the half-plane.

Thus the displacements $\mathcal{U}$ and $\mathcal{V}$ at each moment t are at our disposal not only on the border of the half-plane but also on the adjacent pre-contour line of nodes under the footing. All this holds when there is no dam.

4. In the presence of the dam reaction arises along its footing from the moment t_0 of the arrival of the displacement wave front at the footing /see fig. 3 /. At $t \leqslant t_0$ the displacements on the footing line cd as well as in the dam body are still zero, but at the adjacent nodes of the half-plane on the line ef they are already different from zero at t_0 and are determined by / 6 /. Therefore, at the moment t_0 stresses /reactions σ, τ / arise on the footing line which can be computed by / 5 /. Reactions σ and τ which change in the course of time are sources of the displacement propagation of $\Delta\mathcal{U}, \Delta\mathcal{V}$ in the half-plane.

As all the displacements on the footing line cd and in its vicinity are known at the moments t_0 and $t_0+\Delta t$, it is possible to compute immediately by formulae / 2 / the displacements $U_{\substack{i\\ t_0+\Delta t}}$, $V_{\substack{i\\ t_0+\Delta t}}$ for the moment $t_0+\Delta t$ at the nodes i on the footing line, and as follows from / 1 / also the quantities

$$\Delta\mathcal{U}_{\substack{i\\ t_0+\Delta t}} = U_{\substack{i\\ t_0+\Delta t}} - \mathcal{U}_{\substack{i\\ t_0+\Delta t}} , \qquad \Delta\mathcal{V}_{\substack{i\\ t_0+\Delta t}} = V_{\substack{i\\ t_0+\Delta t}} - \mathcal{V}_{\substack{i\\ t_0+\Delta t}} , \tag{7}$$

which are generated by the reactions. Obviously, the expressions / 7 / /where t_0 is substituted for t / may be used to compute the quantities $\Delta\mathcal{U}_{\substack{i\\ t+\Delta t}}$, $\Delta\mathcal{V}_{\substack{i\\ t+\Delta t}}$ on the footing line at each t, if only the displacements[I] $U_{\substack{i\\ t+\Delta t}}$, $V_{\substack{i\\ t+\Delta t}}$ are already determined on the footing line /see below, / 8 / /.

5. In accordance with the above the following algorithm may be proposed for the solution of the problem.

Let the displacements U_i, V_i be known at the two consecutive

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I The quantities $\Delta\mathcal{U}_{\substack{i\\ t+\Delta t}}, \Delta\mathcal{V}_{\substack{i\\ t+\Delta t}}$ at the nodes i on the footing line may be determined by the use of / 3 / and by treating the reactions σ, τ as outside loads on the half-plane boundary. The identity of $\Delta\mathcal{U}$ and $\Delta\mathcal{V}$ determined both by / 3 / and by / 7 / may be easily established. To do this it is necessary to replace the reactions in / 3 / by their expressions in displacements / 5 /, having in view that the quantities U, V referring to the half-plane must be written in the form $\mathcal{U}+\Delta\mathcal{U}$, $\mathcal{V}+\Delta\mathcal{V}$

moments $t_0, t_0+\Delta t$ in the dam body and on the line of nodes in the half-plane adjacent to its footing /see fig. 3 , domain $\mathcal{D}$ bounded below by the line $aef\ell$ /. Besides, let the quantities Δu_i, Δv_i in the half-plane be known at the same moments /see fig. 3 , domain $\mathcal{F}$; $\mathcal{D}$ - and $\mathcal{F}$ - domains overloap/. In this case the values of $U_{i\atop t+\Delta t}$, $V_{i\atop t+\Delta t}$ in the domain $\mathcal{D}$, except on the line $aef\ell$, can be computed according to the formulae / 2 / - / 4 / above /and others taken from [1,2]/. The same for the values $\Delta u_{i\atop t+\Delta t}$, $\Delta v_{i\atop t+\Delta t}$ in the domain $\mathcal{F}$ except on the line cd . To compute the values $U_{i\atop t+\Delta t}$, $V_{i\atop t+\Delta t}$ on the line $aef\ell$ using the same formulae, as well as the values $\Delta u_{i\atop t+\Delta t}$, $\Delta v_{i\atop t+\Delta t}$ on the line cd /which is the border of $\mathcal{F}$ / the data are lacking in the vicinity of the nodes on these lines.

We can, however, use in both cases the relation / 1 /. As the values $U_{i\atop t+\Delta t}$, $V_{i\atop t+\Delta t}$ are already determined /by formulae / 2 / /, we find for the nodes i on the footing line cd :

$$\Delta u_{i\atop t+\Delta t} = U_{i\atop t+\Delta t} - u_{i\atop t+\Delta t} , \qquad \Delta v_{i\atop t+\Delta t} = V_{i\atop t+\Delta t} - v_{i\atop t+\Delta t} \tag{8}$$

where $u_{i\atop t+\Delta t}$, $v_{i\atop t+\Delta t}$ are given. For the nodes i on the line ef where $\Delta u_{i\atop t+\Delta t}$, $\Delta v_{i\atop t+\Delta t}$ are already determined /also by / 2 / / we got

$$U_{i\atop t+\Delta t} = u_{i\atop t+\Delta t} + \Delta u_{i\atop t+\Delta t} , \qquad V_{i\atop t+\Delta t} = v_{i\atop t+\Delta t} + \Delta v_{i\atop t+\Delta t} \tag{9}$$

Here $u_{i\atop t+\Delta t}$, $v_{i\atop t+\Delta t}$ are known by / 6 /.

We proceed analogously, using / 9 / for the nodes outside[I] the footing on the segments ac and $d\ell$, where u, v are given[I];

Thus, for the moment $t+\Delta t$ all the displacements U_i, V_i in the $\mathcal{D}$ -domain, as well as the quantities $\Delta u, \Delta v$ in the $\mathcal{F}$ -domain, are fully determined, and the cycle of operations is repeated after that.

As the initial moment for the computational operations we take t_0 /see above/, when all the necessary quantities in both $\mathcal{D}$ - and $\mathcal{F}$ -domains are known.

Let us add that because of the limitations of computor memory we may need cutting the network domain by a "conventional boundary" /see fig. 3 , line $k\ell nm$ / which allows the elastic waves to pass into the half-plane. On the conventional boundary the following approximate relations hold:
for the segment ℓn $\quad \Delta u_{i-\Delta y\atop t+\Delta t} = \Delta u_{i\atop t}$, $\quad \Delta v_{i-\Delta y\atop t+\Delta t} = \Delta v_{i\atop t}$
and analogously
for the segments mn, $k\ell$

$$\Delta u_{i\pm\Delta x\atop t+\Delta t} = \Delta u_{i\atop t} , \qquad \Delta v_{i\pm\Delta x\atop t+\Delta t} = \Delta v_{i\atop t} \tag{10}$$

6. Under the unteraction of the elastic domain with the

[I] The length of each of the segments ac and $d\ell$ must be not less than $2\Delta x$.

ideal compressible liquid /see fig. 4 / the behavior of the latter is described by the wave equation $\nabla^2\varphi = \frac{1}{C_w}\frac{\partial^2\varphi}{\partial t^2}$, where φ is the velocity potential, C_w - the sound velocity in the liquid.

For inner nodes i of the network approximating the liquid this equation assumes in finite differences the following form [3] :

$$\varphi_{\substack{i\\t+\Delta t}} = -\frac{2}{n^2}\varphi_{\substack{i\\t}} + \varphi_{\substack{i+\Delta\xi\\t}} + \varphi_{\substack{i-\Delta\xi\\t}} + \frac{1}{n^2}\left(\varphi_{i+\Delta\eta} + \varphi_{i-\Delta\eta}\right)_t - \varphi_{\substack{i\\t-\Delta t}} = M(\varphi)_{t,\,t-\Delta t}\,, \tag{11}$$

where $\Delta\xi, \Delta\eta$ are the network steps in the liquid,

$$n = \frac{\Delta\eta}{\Delta\xi} \quad (n > 1)\,, \qquad \Delta t = \frac{\Delta\xi}{C_w} = \frac{\Delta x}{C_1}$$

The stability of difference scheme / 11 / is ensured, as above, by the operation / 4 /.

Velocities and pressures in the liquid are expressed by the relations $\alpha_x = \frac{\partial\varphi}{\partial x}$, $\alpha_y = \frac{\partial\varphi}{\partial y}$, $p = \rho_w\frac{\partial\varphi}{\partial t}$ / ρ_w is the density of the liquid/.

On the contact lines at the common nodes of both networks a condition must be sutisfied of the velocities of both elastic and liquid media: $\frac{\partial V}{\partial t} = \frac{\partial\varphi}{\partial y}$ on the contact line parallel to the x -axis and $\frac{\partial U}{\partial t} = \frac{\partial\varphi}{\partial x}$ on that parallel to the y -axis. In addition, the hydrodynamic pressure $\rho_w\frac{\partial\varphi}{\partial t}$ is introduced as a normal load on the contact surface of the elastic domain.

Allowing for all these the equations are set up for the contact lines. At the nodes i on the contact line parallel to the x -axis the quantity $\varphi_{i-\Delta\eta}$ in equation / 11 / must be replaced by the quantity[I]

$$\bar{\varphi}_{\substack{i-\Delta\eta\\t}} = \frac{\rho C_1 mn - \rho_w C_w}{\rho C_1 mn + \rho_w C_w}\,\varphi_{\substack{i+\Delta\eta\\t}} - \frac{\rho_w C_w n^2}{\rho C_1 mn + \rho_w C_w}\left(\frac{-2}{n^2}\varphi_{\substack{i\\t}} - 2\varphi_{\substack{i\\t-\Delta t}} + \varphi_{\substack{i+\Delta\xi\\t}} + \varphi_{\substack{i-\Delta\xi\\t}}\right) -$$
$$-\frac{\rho_w C_w C_1 mn^2}{\rho C_1 mn + \rho_w C_w}\left[2\left(1+\mu-\frac{1}{m^2}\right)V_{\substack{i\\t}} - 2V_{\substack{i\\t-\Delta t}} + \frac{2}{m}V_{\substack{i-\Delta y\\t}} - \frac{\mu\Delta y}{m\Delta x}\left(U_{i+\Delta x} - \right.\right. \tag{12}$$
$$\left.\left. - U_{i-\Delta x}\right)_t - \mu\left(V_{i+\Delta x} + V_{i-\Delta x}\right)_t\right].$$

The motion equations at the same contact nodes i are as follows

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[I] At the nodes φ on the contact line which do not coincide with the nodes of the elastic domain network /node j in fig. 4 / the quantities φ_j are determined by interpolation.

$$\underset{t+\Delta t}{U_i} = 2\left[1-\frac{1-\mu}{2}\left(2+\mu+\frac{1}{m^2}\right)\right]\underset{t}{U_i} + \frac{(1-\mu)(2+\mu)}{2}\left(U_{i+\Delta x}+U_{i-\Delta x}\right)_t + \frac{1-\mu}{2m^2}\left(\bar{U}_{i+\Delta y}+U_{i-\Delta y}\right)_t +$$

$$+\frac{(1+\mu)\Delta x \rho_W}{8C_1\Delta\xi\rho}\left[\left(\varphi_{i+\Delta\xi}-\varphi_{i-\Delta\xi}\right)_{t+\Delta t}-\left(\varphi_{i+\Delta\xi}-\varphi_{i-\Delta\xi}\right)_{t-\Delta t}\right]-\underset{t-\Delta t}{U_i} = \mathcal{K}(U,V,\varphi)_{t,t-\Delta t}\,; \tag{13}$$

$$\underset{t+\Delta t}{V_i} = 2\left(1+\mu-\frac{1}{m^2}\right)\underset{t}{V_i} - \mu\left(V_{i+\Delta x}+V_{i-\Delta x}\right)_t + \frac{1}{m^2}\left(\bar{V}_{i+\Delta y}+V_{i-\Delta y}\right)_t - \underset{t-\Delta t}{V_i} = \mathcal{L}(U,V)_{t,t-\Delta t}\,,$$

where the quantities $\underset{t}{\bar{U}_{i+\Delta y}}$, $\underset{t}{\bar{V}_{i+\Delta y}}$ are

$$\underset{t}{\bar{U}_{i+\Delta y}} = \underset{t}{U_{i-\Delta y}} - \frac{\Delta y}{\Delta x}\left(V_{i+\Delta x}-V_{i-\Delta x}\right)_t\,,$$

$$\underset{t}{\bar{V}_{i+\Delta y}} = \frac{\rho C_1 mn - \rho_W C_W}{\rho C_1 mn + \rho_W C_W}\left[2\left(1+\mu-\frac{1}{m^2}\right)V_i - \mu\left(V_{i+\Delta x}+V_{i-\Delta x}\right)\right]_t + \tag{14}$$

$$+\frac{\rho_W m^2 n}{\rho C_1 mn+\rho_W C_W}\left[\frac{2}{n^2}\left(\varphi_{i+\Delta\eta}-\varphi_i\right)_t + \underset{t}{\varphi_{i+\Delta\xi}} + \underset{t}{\varphi_{i-\Delta\xi}} - 2\underset{t-\Delta t}{\varphi_i}\right].$$

Analogous equations are set up for the contact line parallel to the y -axis, for angular points on the contact line, etc.

The hydrodynamic pressure p on the border of the half-plane plays the same role as the reactions along the dam footing, and generates in the half-plane displacements Δu and Δv as well which are superimposed on the displacements u and v . The course of the solution remains the same as above with the sole difference that together with the $\mathcal{D}$ - and $\mathcal{G}$ -domains the domain W appears as well in which the quantities φ_i are determined by / 11 /. The domain W is also bounded by a conventional contour /the line in fig. 4 / where approximate relations $\underset{t+\Delta t}{\varphi_{i-\Delta\xi}} = \underset{t}{\varphi_i}$ hold analogous to /10 /.

For the nodes i on the contact line of the elastic foundation with the liquid /line C_1C in fig. 4 / and for those on the line $e_1 f$ situated below the formulae / 8 / resp. / 9 / remain valid, only that the quantities $\underset{t+\Delta t}{U_i}$, $\underset{t+\Delta t}{V_i}$ entering / 8 / are computed in the contact segment C , as distinct from the footing segment cd , by / 13 /.

As at the moment of the arrival of the displacement wave front at the contact line, all the necessary values are known to start the computation for determining the U, V

7. Thus an algorithm is established to determine the strain-stress state of the dam if the displacements on the foundation surface in the region of the future dam footing are known caused by some seismic action.

A test example was computed for the strain-stress state of a rectangular dam on the rock foundation /see fig. 5 /. The elastic properties of the dam and foundation material are equal. The underlying data are as follows: $\mu = 0{,}167$, $\frac{\Delta y}{\Delta x} = m = 1{,}25$, $E = 2 \cdot 10^6$ T/м²

Dimensions are shown in the figure.

A segment of the foundation surface at a certain distance from the dam was exposed to an impuls action of a load q of duration $3\Delta t$ *which simulated the seismic source.*

The displacements in the segment ab of the half-plane boundary caused by the "seismic source" in the absence of the dam. These displacements where used as only initial data for computing the dam in accordance with the above method.

As the seismic source is known in this case, the problem of the wave motion in the dam-foundation domain can be solved in a straightforward way by the methods developed earlier [1, 2], and that was done. The results of both computations coincided exactly. The stress-to-time curves are shown in fig. 5 for one of the nodes of the dam /point i /.

References

I. Л.И.Дятловицкий, К решению плоской динамической задачи теории упругости методом конечных разностей. Прикладная механика т.II, в.IO, К., 1966.

2. Л.И.Дятловицкий, С.И.Ключникова, Решение плоской задачи о распространении упругих волн методом конечных разностей при прямоугольной (не квадратной) сетке, тр.2-й Вс.конф. по числ. мет. решения задач теории упругости и пластичности, Новосибирск, I97I.

3. Л.И.Дятловицкий , Э.Д.Лемберг, Плоская задача гидроупругости, Тр. 5-ой международной конф. по нелинейным колебаниям, Т.3, I970.

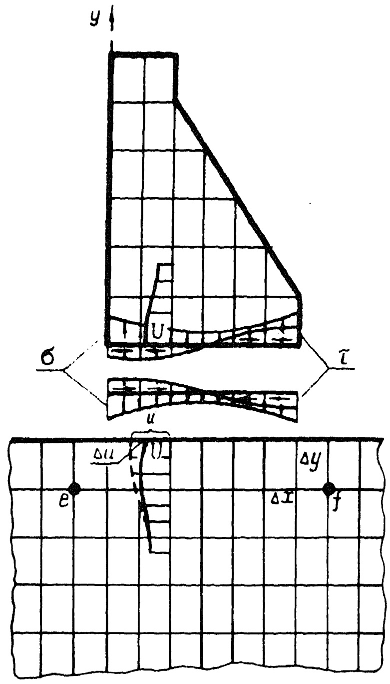

Fig 1

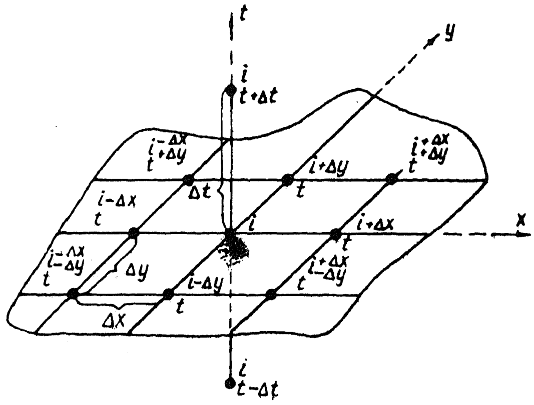

Fig 2

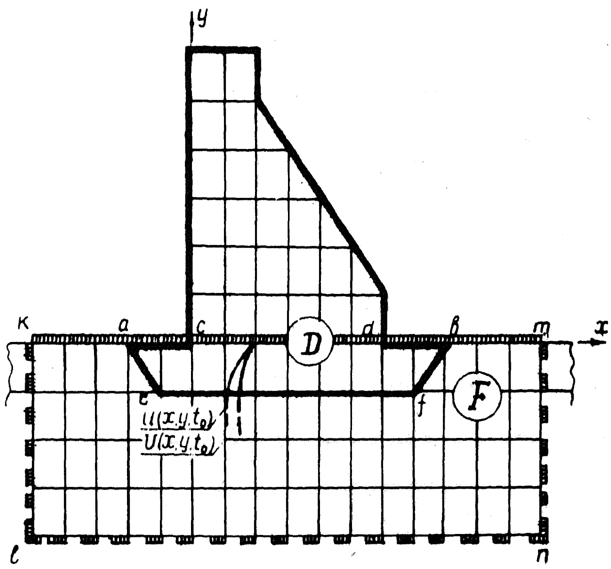

Fig 3

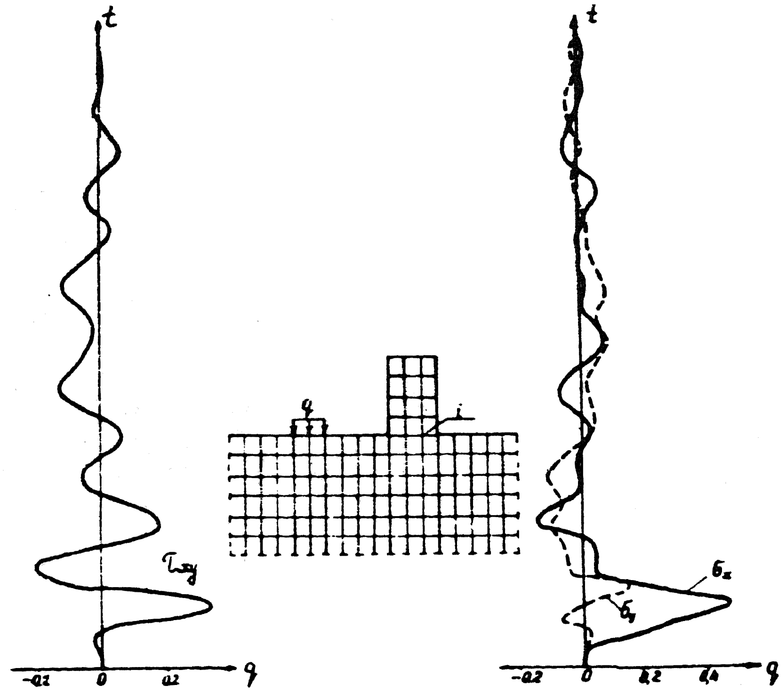

Fig 5

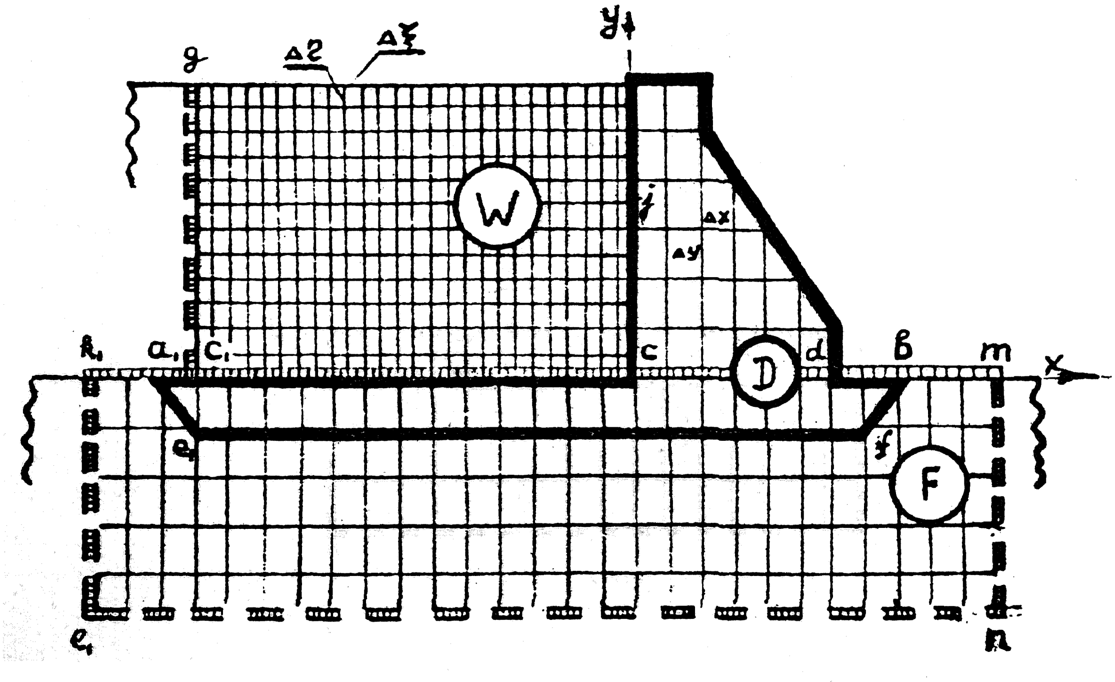

Fig. 4.