ESTIMATION OF LATERAL SPREAD OF A CAISSON TYPE QUAY CAUSED BY BACK FILL LIQUEFACTION

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SUMMARY

It is said on the lateral spread induced by liquefaction that firstly, a quay structure such as a caisson displaces toward the sea, and then the back fill ground displaces laterally following the quay movement. Lateral displacement of a caisson is to be calculated if the “forces” acting on the caisson is known, whatever properties the back fill subsoil has. From such viewpoint, a method to calculate the lateral displacement of a quay structure is proposed. The continuity equation and the momentum equation within a time interval from t=0 to t are obtained and numerically integrated. The result is represented as a relation between re-sedimentation time of the once liquefied ground and caisson’s displacement. The elapsed time necessary for the pore water pressure to dissipate in the back fill influences largely the caisson displacement. The water depth in front has large restricting effect on the motion of the caisson. The observed lateral displacements at the 1995 Hyogo-ken Nanbu Earthquake are classified into three types of diminishing characteristics with distance from the quay and well simulated by proposed calculation, where the dominant factor is not the rigidity of the liquefied layer, but the tensile strength of the non liquefied layer.

INTRODUCTION

When the back fill of a quay wall liquefies and spreads laterally, bridge foundations in the vicinity of the quay wall are subjected to excessive lateral force. The intensity of the force depends on the subsoil’s properties and the degree of ground movement. Several methods have been proposed to estimate the lateral displacement of a quay wall and/or back fill ground. It is said that firstly, a quay structure such as a caisson displaces toward the sea, and then the back fill ground displaces laterally following the quay movement. Although such consideration of the lateral spread seems to be interesting and may be acceptable, it is not essential here whether the above explanation for the mechanism of lateral spread is true or not.

Lateral displacement of a caisson is to be calculated if the “forces” acting on the caisson is known, whatever properties the back fill subsoil has. From such viewpoint, a method to calculate the lateral displacement of a quay structure will be shown, as well as a case study showing its application to a model structure in Rokko island (Kobe) with a brief discussion of the calculated results. Because lateral displacement in the proposed method does not account for the inertia force by the earthquake, the time origin t = 0 is taken to be just after the end of vibrating ground motion. At t = 0, the back fill is assumed to be in a state of perfect liquefaction and the caisson begins to move with an initial lateral velocity v₀=0.

FUNDAMENTAL EQUATIONS

The lateral resultant force acting on a caisson at any time t is denoted by u(t) per unit width considering plain-strain condition. The continuity equation and the momentum equation within a time interval from t=0 to t are deduced with reference to Fig.1.
<continuity equation> Sea water displaced by the movement of the caisson must be moved to the surface of the sea and is assumed to proceed forward as if like a surge (hydraulic bore). The continuity condition of the displaced water is then given as follows.

\[ h_0 \int_0^{\ell(t)} v(t) dt = \int_0^{\ell(t)} \left( h(\xi) - h_0 \right) d\xi \]  \hspace{1cm} (1)

Differentiating (1) by \( t \), the next equation is obtained.

\[ v(t) = \frac{h(\ell(t))}{h_0 + h(\ell(t))} \xi(t) \]  \hspace{1cm} (2)

<momentum equation> A force product of \( u(t) \) acting on the back face of the caisson from \( t = 0 \) to \( t \) is equated to the increment in momentum of the quay wall together with the sea water in front.

\[ \int u(t) dt = \left\{ M + \rho_0 h_0 \right\} c(t) + \int_0^{\ell(t)} \left( h(\xi) - h_0 \right) v(t) \]  \hspace{1cm} (3)

Differentiating (3) by \( t \),

\[ u(t) = \rho \omega \frac{h_0 \xi(\ell(t))}{h_0 + h(\ell(t))} c(t) \] \hspace{1cm} (4)

Eq. (2) and eq. (4) involve three unknown functions, i.e., \( v(h), c(t) \) and \( h(\ell(t)) \). If the following expression is used to approximate the surge velocity \( c(t) \)\[1\],

\[ c(t) = \sqrt{gH} \]  \hspace{1cm} (5)

where \( H \) is the water depth in front of the caisson, the set of three equations (2), (4) and (5) containing the three unknown functions can be solved.

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**Fig.1 Model for fundamental equations**

HORIZONTAL FORCES ACTING ON A CAISSON

Because a state of liquefaction can be sustained after the end of vibrating effect of an earthquake, the relative velocity of soil grains with respect to the surrounding water should be the only cause of generation of excess pore pressure and its loss of effective weight of soil grains. Resettlement or dropping of the grains maintains the excess pore pressure in the liquefied soil. The back fill ground which has once liquefied due to an earthquake solidifies, as the once mutually separated soil grains drop in the surrounding water and form ‘re-sediment’. The dissipating process of the excess pore pressure which makes the soil grains recover their contact forces determines, therefore, the rising velocity of the newly formed surface of re-sedimentation. The rising velocity is presently assumed to have a constant value \( V \). The total water pressure on the back of the caisson at any time \( t \) is simplified as shown in Fig. 2. Such simplification would be consistent with some experimental results [2] even quantitatively.
In addition to the above total water pressure, horizontal forces exist, consisting of earth pressure at rest \( (k_0=0.5) \) in the re-sedimentary back fill, hydrostatic pressure in front of the caisson and frictional force at its base equal to \( (Mg - B)\tan 30^\circ \). \( B \) is total water pressure on the base and is calculated as the total water pressure on the back face of the caisson at the depth of the base, multiplied by the caisson’s width.

**CALCULATED RESULTS AND DISCUSSION**

Calculated results on the caisson model illustrated in Fig. 3 are shown in Fig. 4. The abscissa is the elapsed time for re-sedimentation, during which a re-sedimentary surface, starting from the base at \( t = 0 \), ascends to reach the ground water table. The upper and lower lines of the figure represent the cases where the water depth in front is a constant value of 6 m and 10 m, respectively, while the water depth in the back fill is the same as the ground water table.

The follows are observed from Fig. 4.

1. The elapsed time necessary for the pore water pressure to dissipate in the back fill influences largely the caisson displacement. This might become significant if the elapsed time is long as in the case of a soil layer with low permeability near the ground surface. For example, in the case where the depth of the liquefied layer is 6 m, the settlement is 3 % and the dissipation time is 400 sec, the re-sedimentation velocity corresponds to 0.045 cm/s.

![Fig.2 Model for dissipation of pore water pressure (at any time t)](image)
(2) The water depth in front has large restricting effect on the motion of the caisson. The deeper the water depth is, the larger is the inertia mass of water in front of the caisson.

**OBSERVED LATERAL DISPLACEMENTS**

After proposing the method described above to estimate the caisson's movement, the features of lateral displacement of the back fill ground will be presented next, which must diminish with distance from the caisson. Lateral displacements of quay walls and their backfill ground in the Kobe district observed during the 1995 Hyogo-ken Nanbu Earthquake are shown in fig. 5[3].

They are in variety of shapes and can be classified into three types as follows.
- Type(I): rapid decrease in displacement with distance from the quay wall
- Type(II): gentle decrease similar to a straight line
- Type(III): a certain part adjacent to the quay wall moves together with the quay wall, while the remaining part moves little.

The reasons why there exists such difference in these types are discussed below and their validity will be numerically examined using FEM analysis.

![Fig.3 Caisson model (unit:m)](image)

![Fig.4 Elapsed time and caisson’s displacement](image)

![Fig.5 Observed lateral displacement of back fill ground](image)
MECHANISM OF LATERAL SPREAD

When back fill ground spreads due to liquefaction, the ground starts moving and after a certain time has elapsed motion will stop. In the same way as the previous calculation of caisson’s displacement, it is most important to identify the "forces" acting on the ground. Without the acting forces the ground could neither start moving nor stop moving.

Typical geographical conditions for lateral spread induced by liquefaction are an inclined ground surface and one with lateral opening, as shown in Fig. 6(a) and (b), respectively. Horizontal forces acting on a soil block ABCD at the instant of perfect liquefaction are also indicated in the figure. Horizontal forces on AB and DC are "hydrostatic pressure + excess pore water pressure = total hydraulic pressure during liquefaction". In both cases (a) and (b), the force on DC is larger than that on AB. The force acting on BC is the shear resisting force and is null at the instance of perfect liquefaction, at which time the lateral component of the resultant force acting on the soil block ABCD could therefore not be zero and the block would start moving leftward: i.e., the ground spreads laterally.

We now consider the force required to stop the movement. During the dissipation of the excess pore water pressure, the forces on AB and DC decreases due to the recovery of the effective weight of soil grains accompanied by the dissipation of the excess pore water pressure. As a result, the total lateral force decreases and at the same time, the shear resisting force on BC increases due to the recovery of effective weight of the soil block ABCD.

From the above considerations, the mechanism of lateral spread from macroscopic view point consists of the appearance of excess pore water pressure, the resulting difference of two forces on two vertical planes, the loss of the shear resisting force on a horizontal plane, and also the dissipation of the excess pore water pressure, resulting in the decrease of the unbalanced lateral water pressure and the recovery of the shear resisting force. The same mechanism would be equally applicable to the movement of each isolated soil grain in motion.

DOMINANT FACTOR IN LATERAL DISPLACEMENT

What mechanism governs the three types of lateral spread of back fill ground as described in Sec. 5? A possible mechanism is explained here while a numerical verification for this mechanism is shown in the next section.

Spreading ground may be simplified as in Fig. 7. A overlying non liquefied layer is generally subjected to shear stress $\tau$ on its lower face by a liquefied layer. If the dropping velocity of grains in the liquefied layer is greater than the settling speed of the upper layer and the boundary of the two soil layers forms a horizontal plane, a thin water film might develop at the uppermost part of the liquefied layer and there could ideally exist no shear stress in the water film. In most cases nevertheless, it would be reasonable to imagine that there exists somewhat a shear stress $\tau$ on the boundary surface of the two layers.

As a result, a stress state is induced in the non liquefied layer and the horizontal stress $\sigma_x$ becomes tensile. The authors postulate that the difference of the three types depends on this tensile stress and consequently, on tensile strength of the non liquefied layer, i.e., whether the non liquefied layer can move following lateral movement of the liquefied layer. In other words, the dominant factor to determine the displacement type of back fill is not the rigidity (numerically, modulus of deformation) of the liquefied layer, but the tensile strength of the non liquefied layer.

The horizontal shear stress on the lower plane of a non liquefied layer is maximum at the back face of a caisson and this diminishes with distance, while tensile stress increases with the distance if the non liquefied layer is considered to be a continuous body. A non liquefied layer which consists of sandy soil with little cohesion can not resist the tensile stress and its displacement follows that of a liquefied layer, which diminishes rapidly with the distance; this case corresponds to type( ). A non liquefied layer having some cohesion behaves as a resistant solid body and its displacement does not follow that of a liquefied layer, which diminishes gradually with the distance tracing a small curvature; the case corresponds to type( ). The magnitude of modulus of deformation of the non liquefied layer is the same as that before liquefaction, or its strain dependency may be taken into account.

A non liquefied layer fully compacted or having a sufficient depth and strength near a caisson behaves as one body with the caisson, and a tension crack is then expected to appear in a certain position where the depth is
relatively thin or strength is relatively small. When a crack appears in the non liquefied layer of type( ), sand and/or water must spout up on the ground surface from the internal parts of the liquefied layer and lateral displacement may show some discontinuity in the horizontal direction. The resistant non liquefied layer does not follow the movement of the liquefied layer near a caisson, while far from the caisson, the less resistant non liquefied layer follows the liquefied layer similar to type( ); this case corresponds to type( ).

**VERIFICATION BY NUMERICAL ANALYSIS**

The lateral displacement of a caisson can be calculated considering the time history of forces acting on the caisson in the process of generation and dissipation of excess pore water pressure as already described. On the other hand, soil grains in the liquefied layer of back fill ground move in the horizontal direction following the lateral displacement of the caisson as well as in the vertical direction according to sinking or re-sedimentation, and then become stable by recovering their mutual contact forces. The soil grains in the re-sedimentary layer are in a static state relative to surrounding pore water and then regain their effective weight. The re-sedimentary layer shows now frictional resistance depending on its depth, while the upper part of the liquefied layer has frictional resistance yet because of the loss of effective weight of sinking soil grains. This upper part continues to spread until the grains stop sinking.

![Diagram of Geographical conditions of lateral spread](image1)

**Fig. 6 Geographical conditions of lateral spread**

![Diagram of Spreading Ground](image2)

**Fig. 7 Model of spreading ground**
An overlying non liquefied layer seems to have a variety of displacement types, i.e. whether it can follow that of a liquefied layer or not, depending on its tensile strength and its depth.

Now numerical calculations based on the explanation for the mechanism described in the previous section to obtain the variety of types (Ⅰ) (Ⅱ) and (Ⅲ) of a non liquefied layer will be presented. In general, the displacement of a non liquefied layer may be calculated dynamically if the time history of forces and their distribution are known, together with its strength and rigidity. Here, the mechanical properties of the non liquefied layer are nevertheless assumed and expressed in terms of modulus of deformation \( E_N \), while that of the liquefied layer is expressed in terms of modulus of deformation \( E_L \) and Poisson’s ratio \( \nu_L \). Although it is almost zero, the value of \( E_L \) is now to be determined such that the value of caisson’s displacement coincides with the value calculated from the condition that each solid element of the FEM model is subjected to a body force equal to the unbalanced lateral pressure at the instant of perfect liquefaction. \( E_L \) is thus determined to be 2.8 MPa for a caisson’s displacement of 1.5 m (see Fig. 5). The \( E_N \) written in Fig. 8 are used to verify the explanation given in Sec. 7. The body force mentioned is obtained from the distribution of effective maximum principal stress \( \sigma_1' \) at preliquefaction, i.e. excess pore water pressure at post-liquefaction.

Fig. 8 Models for FEM analysis (\( E_N \): modulus of deformation of non liquefied layer)

Fig. 9 Results of FEM analysis
Fig. 9 represents the calculated results. The abscissa is the distance from the back face of the caisson while the ordinate represents the diminishing characteristics of lateral displacement of the ground surface as a ratio with respect to that of the caisson. The figure seems to well simulate the types of lateral displacement with reference to Fig. 5.

From Fig. 9, the spread of back fill ground can be characterized as follows depending on the properties of the non liquefied layer as in Fig. 8.

- **In case of loose sandy soil or with small $E_s$ ($\varphi$-material), the upper layer moves following the lower layer and the displacement decreases rapidly with distance.**
- **In case of large modulus of deformation such as dense sandy soil, thick layer or high tensile strength ($c$, $\varphi$-material), the upper layer does not move following the lower layer and displacement reaches farther from the caisson with gradual decrease, such as a straight line.**
- **In case of combination of the above two cases, the nearer part of the upper layer moves as if acting as one body with the caisson, while the other part moves little. Cracks and sand spouts might appear in the boundary of the two parts and the displacement shows some discontinuity.**

**CONCLUSIONS**

(1) Lateral displacement of a caisson was calculated by integrating the momentum equation of the caisson without reference to the properties of the back fill ground.

(2) Independent from this calculation, the characteristics of lateral spread of ground was estimated based on the properties of the non liquefied layer, without reference to the properties of the liquefied layer.

(3) The results presented herein well simulated the displacements observed during the 1995 Hyogo-ken Nanbu Earthquake.

For more appropriate estimation of displacement induced by lateral spread of back fill ground, it would be necessary to identify exactly the settling speed of a non liquefied layer, and to study the influence of the constitution of soil layers and water depth in front of a caisson; especially, to determine experimentally the sinking velocity (with acceleration if possible) of soil grains within a liquefied layer and to study in detail the effect of deformation/strength characteristic of the non liquefied layer.

Two mechanism, a mechanism of development of pore water pressure during vibration, which has been considered largely in relation to dilatancy, and a mechanism of pore water pressure after the end of vibration, should be consistently explained in fundamental studies dealing with the process of liquefaction.

**REFERENCES**

1) HOMMA, M. & AKI, K. (1962): “Mononobe Hydraulics”, Iwanami Shoten, Publisher