PROBABILITY-BASED DESIGN EARTHQUAKE LOAD CONSIDERING ACTIVE FAULT

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SUMMARY

The probability-based structural design can provide a specific safety performance demand for the earthquake resistant design as a next generation code. The purpose of this study is to show examples of determining a design load on the basis of minimum total cost principle considering damage cost models with multi-limit states, a hazard model with active faults and soil amplification characteristics. Parametric studies are conducted for sites in Tokyo, Osaka and Sendai, which are major cities in Japan with different earthquake activity characteristics. Numerical examinations clarify the influences of mean occurrence rate of active faults and data period for earthquake catalogue. Moreover, we discuss the usefulness of GIS application for determining various parameters such as active fault data and soil characteristics by examining their information on the design load.

INTRODUCTION

Structural engineers are required to make their own decision for insufficient information such as seismic activities. More and more information on active faults is available and readily applied to hazard analysis. Such up-to-date information should be considered in the structural design or the design earthquake load. Clients are now paying attention to the safety performance of structure as well as the life cycle cost. Then the optimum reliability based load determination in a useful tool for considering these situations. The procedure for the optimum reliability which minimizes the total cost including the expected failure cost is developed in order to determine an optimum load adopting multi-failure cost model based on the statistics for repair costs due to the Hyogo-ken Nanbu Earthquake in 1995 (Kanda and Hirakawa 1997). With respect to the soil amplification uncertainty, the coefficient of variation of PGA and response spectra are quantitatively estimated due to differences of analytical methods, phase contents and soil characteristics (Ahmed and Kanda 1996). The purpose of this paper is that by fully utilizing existing results, we apply probability models of proposed seismic hazard analysis with active faults information to cost benefit analysis and examine effects of various seismic uncertainties on the optimum design load in order to determine the design earthquake load.

2. ANALYTICAL METHOD

A general analysis flow is shown in Figure 1 for a procedure to obtain the optimum reliability-based earthquake load. The analytical method consists of three stages, i.e. seismic hazard analysis, soil amplification analysis and cost benefit analysis.
2.1 Seismic Hazard Analysis

An empirical extreme value distribution with both upper and lower bounds was used for a basic seismic hazard model based on historical earthquake data (Kanda 1994). The cumulative probability distribution of the 50-year maximum peak ground acceleration (PGA) at bedrock, $A_{50}$, can be obtained from the 50-th power of that of the annual maximum model. Historical earthquake data for 100 to 400 years were utilized to calculate the annual maximum bedrock velocity (Ahmed and Kanda 1995) together with Kanai's attenuation formula, which is most commonly used in Japan for point source earthquake models. Then the bedrock velocity was converted to the bedrock PGA in a simple manner by multiplying 15 (sec$^{-1}$) (A.I.J. 1996). The empirical extreme value distribution for the 50-year maximum PGA at the bedrock is expressed as:

$$F_{A_{50}}(y) = \left\{ \exp \left[ - \left( \frac{w - y}{u y} \right)^{k} \right] \right\}^{50}$$

(1)

where $u$, $k$ and $w$ are the scale parameter, the shape parameter estimated for the annual maxima and the upper bound PGA, respectively.

Since we obtained the 50-year maximum bedrock velocity based on an attenuation formula by Kanai, it is necessary to consider dispersion due to the attenuation formula. An error coefficient $\varepsilon$ due to dispersion of the attenuation is produced. Then the 50 year probability of exceedance of $\tilde{y}$ is obtained by:

$$P_{\varepsilon}(\tilde{y}) = \text{Prob}\{ y > \tilde{y} \} = \int_{0}^{\varepsilon} \int_{\tilde{y}}^{\infty} f_{A_{50}}(y) f_{\varepsilon}(\varepsilon) \, d\varepsilon \, dy$$

(2)

where $f_{A_{50}}(y)$ and $f_{\varepsilon}(\varepsilon)$ are the probability density function of $y$ and $\varepsilon$ respectively and $\varepsilon$ is assumed to be log-normal with the median being 1.0 and the logarithmic standard deviation is assumed to be 0.3 is introduced.

A probabilistic model based on active fault data is then constructed. Assuming that earthquakes occur according to the Poisson arrivals, the exceeding probability of an intensity $y$ in 50 years is given by:

$$P_{f}(y) = 1 - \exp\left\{ 50v_{f}(y) \right\}, \quad v_{f}(y) = \sum_{i=1}^{l} v_{i} q_{i}(Y > y)$$

(3)

where $l$ is the total number of active faults under consideration, $v_{f}(y)$ is the mean annual occurrence rate at a site due to all earthquakes caused by the active faults, $v_{i}$ is the mean annual occurrence rate of each earthquake caused by the $i$-th active fault, and $q_{i}(Y > y)$ is the probability of exceeding an intensity $y$ when the earthquake caused by the $i$-th active fault occurs expressed as:

$$q_{i}(Y > y) = \iint_{Y > y} f_{M}(m) f_{R}(r) f_{\varepsilon}(\varepsilon) \, dm \, dr \, d\varepsilon$$

(4)

where $M$, $R$ and $\varepsilon$ are the earthquake magnitude, the distance from an active fault and the error coefficient of the attenuation, and $f_{M}(m)$, $f_{R}(r)$ and $f_{\varepsilon}(\varepsilon)$ are the probability density function of $M$, $R$ and $\varepsilon$ respectively. The following formula (Fukushima and Tanaka 1991) is used for an attenuation formula for earthquake fault model:

$$\log A_{\text{max}} = 0.51M - \log(R + 0.006 \cdot 10^{0.51M}) - 0.0034R - 1.41$$

(5)
where $A_{max}$ is the peak ground motion in m/sec$^2$, $R$ is the minimum distance in km from the fault plane and $\varepsilon$ is assumed to be log-normal with the median being 1.0 and the logarithmic standard deviation is assumed as 0.5 (Fukushima and Tanaka 1991). In this study we conduct seismic hazard analysis in terms of the bedrock acceleration, $A_{50}$, which is defined as a simple conversion from the peak ground acceleration on the surface $A_{max}$, into $A_{50}$ as $A_{50} = 0.6 \cdot A_{max}$ (Fukushima and Tanaka 1991).

Only active faults which did not cause earthquakes listed in historical earthquake catalogue are considered in order to avoid double count. Finally, a combined hazard curve defined in terms of the 50-year probability exceeding $A_{50}$ is obtained by:

$$H(A_{50}) = 1 - (1 - P_h(A_{50})) (1 - P_f(A_{50}))$$

(6)

### 2.2 Soil Amplification Analysis

The uncertainty of soil amplification is a major concern in determining design loads. In general, the more information can be utilized, the less the estimation error becomes for soil amplification (Ahmed, Kanda and Iwasaki 1996). When the soil layer data above the bedrock are available in terms of the density and the shear wave velocity, the equivalent linear analysis wave-propagation can be carried out for simulated earthquake ground motions (Ohsaki 1982). A conventional design spectrum for the bedrock motion was used to specify a general spectral characteristics (B.R.I. 1991). Seven simulated ground motions for each PGA level were generated and the 5% damping acceleration response spectra of motions at the ground surface were calculated. The acceleration response spectra were then obtained by minimizing error term between the periods 0.0 and 2.0 sec and then load effect $Q$ can be estimated as the value for acceleration constant range of fitted AIJ design spectra. The relationship between $Q$ and $A_{50}$ can then be modeled by the following formula through a regression analysis:

$$Q = a \cdot A_{50}^b$$

(7)

where $a$ and $b$ are empirical constants obtained from the regression analysis.

It is convenient to use a log-normal variable for optimum reliability study (Kanda and Ellingwood 1991). The logarithmic mean and logarithmic standard deviation for $A_{50}$ is calculated from a log-normal distribution which has an equivalent value at 50% and 1% fractile point of probability of exceedance obtained from the seismic hazard model. When eq.(7) is established as a deterministic relationship, the mean and standard deviation of $\ln Q$ can be written as:

$$\lambda_Q = \ln a + \lambda_{A_{50}} , \quad \zeta_Q = b \cdot \zeta_{A_{50}}$$

(8)

where $\lambda$, and $\zeta$, are the mean and the standard deviation of $\ln(\cdot)$ respectively.

When the uncertainty of soil amplification is considered, the mean and the coefficient of variation (cov) of $Q$ may be written as:

$$\mu_Q = \exp \left( \lambda_Q + 0.5 \cdot \zeta_Q^2 \right)$$

$$\zeta_Q = \sqrt{b \cdot \zeta_{A_{50}}} + \zeta_{amp}$$

$$V_Q = \sqrt{e^{\zeta_Q^2} - 1}$$

(9)

where $\zeta_{amp}$ is the logarithmic standard deviation due to soil amplification, which is later referred to as 0.3 (Ahmed, Kanda and Iwasaki 1996).

### 2.3 Cost Benefit Analysis

The optimum reliability can be defined as the reliability at which the total expected life-time cost is minimum. Damages are expected to occur even at lower load effect levels which have higher probabilities. Then, multi-damage criteria can be considered in the total cost formation by expanding eq.(15) as (Kanda and Hirakawa 1997):
\[ C_T = C_f + \sum_{i=1}^{n} P_{fi} \Delta C_{fi} \]  

(10)

where \( C_f \) is the initial cost of a structure, \( P_f \) is the probability of failure and \( C_T \) is the cost due to the failure.

When the load effect, \( Q \), and the resistance, \( R \), are assumed to be log-normal, the probability of failure, \( P_f \), and the corresponding reliability index, \( \beta \), can be obtained from:

\[ P_f = \Phi(-\beta) \]  

(11)

where \( \Phi(\cdot) \) is the standard normal distribution function.

The design load effect, \( r_0 \), can be written in a load factor format as:

\[ r_0 = \gamma \mu_Q \]  

(12)

and \( \gamma \) is the load factor expressed as

\[ \gamma = \frac{1}{\sqrt{1+V^2}} \exp(\alpha_Q \beta \zeta_Q) \]  

(13)

where \( \mu \) and \( V \) are the mean and the coefficient of variation, respectively, and \( \alpha_Q \) is the separation factor: \( \alpha_Q = 0.85 \) was used for simplification.

where \( P_{fi} \) is the probability of exceedance of the \( i \)-th limit state and \( \Delta C_{fi} \) is the failure cost increase at the \( i \)-th limit state.

The failure cost model may be written in the following form (Kanda and Hirakawa 1997):

\[ g(x) = g_2 + \left( \frac{x-x_0}{1-x_0} \right)^\eta \cdot (g_1-g_2) \]  

(14)

where \( g \) is the normalized failure cost; i.e. \( C_f = g C_{io} \), \( C_{io} \) is a reference initial cost, \( x \) is the load effect variable normalized by that corresponding to the ultimate limit state, \( x_0 \) is the normalized load effect corresponding to minor damage or the slightest damage, \( g_i \) is the normalized failure cost at \( x=1 \), \( g_2 \) is the normalized failure cost at \( x=x_{io} \), and \( \eta \) is a constant which may be estimated from a failure cost study.

The initial cost \( C_i \) in eq.(10) can be written using a cost-up constant \( k \) as:

\[ C_T = C_{io} \left\{ 1 + k (\gamma - 1) \right\} \]  

(15)

where

\[ k = \frac{\mu_Q}{100} \]  

(16)

and the mean of acceleration response \( \mu_Q \) is expressed in m/sec\(^2\) (Kanda and Ahmed 1997).

For the ultimate limit state, it is sufficient to consider the probability of only one exceedance since the probability is fairly low. However, when the probability of failure \( P_f \) for minor or moderate damages in eq.(10) is considered, more than one occurrences are probable. Corresponding to the failure cost model by eq.(14), \( R \) can be the resistance variable at a damage limit state \((x_0 \leq x \leq 1)\). Then the reliability index for the limit state designated by \( x \) can be obtained from (Kanda and Hirakawa 1997):

\[ \beta(x) = \frac{\lambda_R - \lambda_Q}{\sqrt{\frac{1}{\mu} + \zeta_Q}} = \frac{\ln x + \lambda_R - \lambda_Q}{\sqrt{\frac{1}{\mu} + \zeta_Q}} \]  

(17)

By substituting \( \beta \) for \( x=1 \) corresponding to the ultimate limit state, we obtain:
An approximate annual probability of exceedance for the limit state of $x$ is:

$$P = 1 - \left\{1 - \Phi(-\beta(x))\right\}^{1/50}$$

Then the probability of failure $P_{fi}$, considering more than one exceedance of the limit state, can be obtained as:

$$P_{fi} = \sum_{n=0}^{\infty} P_{n} (1-P)^{50-n}$$

$$= \sum_{n=0}^{\infty} n \cdot (50P)^n \cdot e^{-50P} = 50P$$

When $\beta > 1$, eq.(20) is practically the same as the probability of one failure during 50 years. The optimum design load and corresponding optimum reliability $\beta_{opt}$ can be obtained by minimizing eq.(10) by substitution of eqs.(12), (13),(14),(15),(18),(19) and (20).

3. APPLICATION OF GIS TO HAZARD ANALYSIS

Application of GIS to seismic hazard analysis enables structural engineers to determine design load easily with their own decisions. Active fault information can be viewed on GIS screen as in Figure 2. Locations and fault lengths are recognized and estimated earthquake occurrence rates are sometimes available. In the following case studies only limited combination of parameters are examined, but alternative choices of parameter values can be demonstrated fairly easily once the procedure is established. Soil characteristics are also suitable information for GIS but further efforts may be required for practical uses. The relation between parameter values for active faults and the optimum design load can be shown to the client who has to approve the final decision on the safety performance.

4. CASE STUDIES

4.1 Seismic Hazard Analysis

Case studies in three sites, i.e. Sendai, Tokyo and Osaka, are conducted in order to consider different hazard potential among various sites. These are major cities Japan with different earthquake activity characteristics.
The parameters of the extreme value distribution function eq.(1) with historical earthquake data obtained from a previous study (Ahmed and Kanda 1995) are shown in Table 1.

In modeling active faults, Arakawa fault in Tokyo are considered. Regarding Arakawa fault, the largest PGA could be expected at the site when the fault caused an earthquake. Nagamachi-rifusen fault in Sendai and Uemachi fault in Osaka are selected since according to recent researches these faults seem to have high hazard potentials. Parameters of active fault models are shown in Table 2. The mean occurrence rate per year, $\nu$, is assumed to be 0.002, 0.001, 0.0005, 0.0002 since $\nu$ can be a high or low value depending on adopted fault model type and accuracy of these data. In Poisson process, $\nu$ is equivalent to the inverse of mean recurrence time, $T_R$.

The hazard curves are shown in Figures 3. Hazard curves by the solid line, broken line and dotted line represent the probability of exceedance in 50 years of bedrock acceleration based on the combination of historical and fault data, that approximated by the Log-normal distribution and the response spectrum explained in 4.2 respectively.

### 4.2 Soil Amplification

Soil profile models are shown in Table 3. The coefficient of variation of amplification was assumed to be 0.3, referring to the variability study of soil amplification for typical soil configuration in Tokyo (Ahmed and Kanda 1996). An example of acceleration response spectra is shown in Figure 4 for each site. Figure 5 shows the relationship between $A_{50}$ and $Q$. $a$ and $b$ in eq.(7) are obtained by the regression analysis. According to Figure 4, the property of soil in Osaka might be that of relatively soft soil. Therefore the definition of $Q$ in this study may provide a rather un-conservative estimation in particular for a structure with the period around 1 sec.

### 4.3 Numerical Results for Optimum Design Load

Two failure cost models are adopted to examine their effects on the optimum load. Model Type 0 is a model where only the ultimate failure cost is considered with $g_1=2.0$. Parameters for model Type 1 in eq.(14) for these models are : $x_0=0.3$, $g_2=2.0$, $g_0=0.1$ and $\eta=3.0$. Model Type 1 is a basic model and constructed based on damage data statistics due to the Hyogo-ken Nanbu Earthquake (Kanda and Satoh 1998).
The Japanese conventional ultimate limit state criteria called the capacity limit can be regarded as corresponding to the severe damage by considering the damage statistics survey for the Hyogo-ken Nanbu Earthquake (Kanda 1997). Then results are shown for cases where the collapse criteria is directly used for the design and where the severe damage criteria is used in Figure 6. For the severe criterion ratio to the ultimate limit state is assumed to be 0.85.

Results shown here are obtained based on various assumptions for seismic hazard information, nevertheless the conventional design load of 9.8m/sec², i.e. the base shear coefficient of 1.0 in Japan may be regarded as approximately optimal for a typical failure cost model such as Type 0 at Tokyo and Type 1 at Osaka. When the severe damage criterion represents the conventional design, a slightly higher load may recommended for Tokyo site and the conventional load is recommended for Osaka site.

5. CONCLUSION

Active fault information is utilized to form a hazard probability model of seismic load effect together with equivalent linear soil amplification with a help from GIS information. Numerical examples for three sites, i.e. Sendai, Tokyo and Osaka show significant effects of the initial failure cost on the optimum reliability. Failure cost models are also considered to examine the effects of initial and intermediate damage on the optimum reli-

<table>
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<th>Table 3 Soil profile in three sites</th>
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<tr>
<td>(a) Sendai</td>
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<tr>
<td>Thickness of Layer (m)</td>
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<td>1.5</td>
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| (b) Tokyo                         |
|Thickness of Layer (m) | S Wave Velocity (m/sec) | Density (ton/m³) |
| 5 | 120 | 1.4 |
| 1 | 180 | 1.8 |
| 6 | 260 | 2.0 |
| 10 | 395 | 2.0 |
| 10 | 405 | 2.0 |
| 9 | 270 | 1.9 |
| 6 | 800 | 1.8 |
| 14 | 345 | 2.0 |
| 7 | 290 | 2.0 |
| 14 | 420 | 2.0 |
| 1300 | 680 | 2.0 |
| 1000 | 1500 | 2.3 |
| ∞ | 3000 | 2.6 |

| (c) Osaka                         |
|Thickness of Layer (m) | S Wave Velocity (m/sec) | Density (ton/m³) |
| 8 | 130 | 1.7 |
| 13 | 150 | 1.7 |
| 85 | 380 | 1.8 |
| 185 | 500 | 1.9 |
| 280 | 650 | 2.0 |
| 110 | 800 | 2.1 |
| 1000 | 1000 | 2.3 |
| ∞ | 2400 | 2.3 |

Figure 4 Acceleration response spectra with AIJ design spectra at three sites
ability-based design load. Specific design load values obtained in the proposed procedure are discussed by
comparing the conventional one. The usefulness of optimum reliability-based earthquake load utilizing the fail-
ure cost model is demonstrated.

0.00000.00050.00100.00150.00200
2 4 6 8 10 12 14 16 18 20

Figure 6 Optimum design load considering equivalent lognormal distribution

[Ahmed and Kanda 1995] "Characteristics of Maximum Bedrock Velocity Estimated From Extreme Value Dis-

[Ahmed, Kanda and Iwasaki 1996] "Estimation of Uncertainties in the Dynamic Response of Urban Soils in Ja-

