FREQUENCY DEPENDENT EQUIVALENT-LINEARIZED TECHNIQUE FOR FEM RESPONSE ANALYSIS OF GROUND

Yoshinori FURUMOTO¹, Masata SUGITO² And Atsushi YASHIMA³

SUMMARY
A frequency-dependent equivalent linearized technique for FEM response analysis of ground is developed. This technique improves the conventional FEM for seismic response analysis of ground in terms of the definition of the equivalent stain, which controls dynamic properties of soil such as shear modulus and damping factor. The frequency-dependent equivalent stain in proportion to the Fourier amplitude of strain time history is defined in frequency domain. The new technique is compared with the conventional technique for both analyses of horizontal layered ground and embankment. In the former case when the very strong motion is input to the very soft ground, it is demonstrated that the proposed technique can estimate the amplification correctly even in high frequency region, in contrast, the conventional technique underestimates the amplification considerably. In the latter case, the large level input motion presents the differences of the calculations between these techniques.

INTRODUCTION
An equivalent linearization method, in which non-linear characteristics of shear modulus and damping factor of soils are modeled as equivalent linear relations of the shear strain, has been applied commonly for seismic response analysis of ground, especially in practical fields. Computer programs based on this method, such as SHAKE (Schnabel et.al, 1972) and FLUSH (Lysmer et.al, 1975) have been widely used for long time. SHAKE is on the basis of multi-reflection theory for the analysis of horizontally layered ground, and FLUSH is used as a Finite Element Method for 2-Dimentional analysis of ground. However, it has been pointed out that the calculated responses of sediments contradict observations in cases of strong ground accelerations. In particular, the results of high frequency components are underestimated comparing with the observed records in soft layers.

In conventional equivalent linearized technique, if strain level is large, the equivalent shear modulus and damping factor will be regulated at the large strain level. In the case of soft soil, as a result, damping factor of high frequency component is estimated exceedingly large. Since the time history of a strain, in general, is strongly characterized by the frequency, the frequency-dependent equivalent stain is defined for evaluation of equivalent shear modulus and damping factor of soil in frequency domain (Sugito et.al, 1994). This technique, which is named FDEL, successfully improved SHAKE on the analysis of horizontally layered soft ground.

In this paper, we define the frequency-dependent equivalent stain for the FEM-based seismic response analysis and attempt to improve FLUSH. The new method based on FEM is examined with strong motion array records in both cases of a very soft horizontally layered ground and of an embankment. Several parametric case studies are carried out to examine the effect of in-homogeneity on the ground amplification characteristic. The responses for the cases of strong input motions are calculated and compared with the conventional technique.

¹ Department of Civil Engineering, Faculty of Engineering, Gifu University, Gifu, JAPAN Email: furumoto@cive.gifu-u.ac.jp
² Department of Civil Engineering, Faculty of Engineering, Gifu University, Gifu, JAPAN Email: sugito@cive.gifu-u.ac.jp
³ Department of Civil Engineering, Faculty of Engineering, Gifu University, Gifu, JAPAN Email: yashima@cc.gifu-u.ac.jp
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2.1 Definition of Frequency Dependent Equivalent Strain

The shear modulus and damping factor of soils have been examined in a large amount of laboratory tests, regarding the shear strain magnitude and the confining pressure. They have been modeled as a function of shear strain. In the conventional equivalent linearized technique, which is called CVT in this paper, some percentage of peak shear strain is used to determine the shear modulus and damping factor according to these modeled functions on G-γ, h-γ relations. In the program SHAKE, one of the CVT, the equivalent mean shear strain, γₑ, is fixed as 65% of the maximum strain, γₘₐₓ; namely, γₑ = 0.65 γₘₐₓ. The numerical calculations are carried out until the deviation of γₑ from that given in the former calculation converges to the expected level.

Generally the ground motion include some spectral characteristics, and the contribution of the frequency contents to strain time history strongly depend on frequency. Since the strong spectral characteristic of shear strain amplitude are included in the seismic ground response, it may be derived that appropriate shear modulus and damping depending on the frequency characteristic could be used for equivalent linearization analysis. According to this assumption the frequency-dependent equivalent strain is proposed in the following equation:

\[ \gamma_f(\omega) = C\gamma_{\text{max}} \frac{F(\omega)}{F_{\gamma_{\text{max}}}} \]

where \( C = \text{constant} \), \( \gamma_{\text{max}} = \text{maximum shear strain} \), \( F(\omega) = \text{Fourier spectrum of shear strain} \), and \( F_{\gamma_{\text{max}}} \) represents the maximum of \( F(\omega) \). The definition of \( \gamma_f(\omega) \) in the left side of equation (1) is described that the equivalent strain, which controls equivalent shear modulus and damping factor, is given in proportional to the spectral amplitude of shear strain in frequency domain. The constant \( C \) controls the level of equivalent strain uniformly along the frequency axis. The condition \( F(\omega)/F_{\gamma_{\text{max}}} = 1.0 \) and \( C = 0.65 \) gives the same condition as CVT. The technique proposed here is called FDEL (Frequency-Dependent Equivalent Linearized technique).

2.2 Convergence Criterion

The numerical calculations are carried out by using the equivalent shear modulus and damping factor for each layer, which are given by equation (1). The equivalent strain for each frequency given by the equation is compared with that given in the previous calculation. The iterative calculations are carried out until the error in the equivalent strain defined by the equation compared with the previous value, is converged into some given level. In FDEL the convergence judgement is performed individually in three frequency regions such as (a) low frequency region (1 Hz or lower), (b) middle frequency region (1 to 5 Hz), and (3) high frequency region (5 Hz or higher). The average of deviation in each frequency region is calculated, and the iterative procedures are continued until the deviation in each frequency region is converged into the given level. These divisions of the convergence judgement on the frequency axis are incorporated to consider the following ground motion characteristic.

(1) low frequency region (lower than 1 Hz): low strain amplitude and long wave length [linear response region]

(2) middle frequency region (from 1 to 5 Hz): large strain amplitude and large non-linear effect [non-linear response region]

(3) high frequency region (higher than 5 Hz): low strain amplitude and short wave length [large effect of damping, and linear or non-linear response region]

In this study the reference deviation in the iterative calculation is fixed as 5% for each frequency region. The continuity of equivalent strain \( \gamma_f(\omega) \) along the frequency axis is still kept even the convergence judgement is separately performed in each frequency region. In case of response analysis in the following, the number of iterations in FDEL is in the range of 5 to 10 times which is similar to that of CVT.

Figure 1 shows flow-chart of the numerical calculation of FDEL. The three parts represented by thick solid line (a), (b), and (c) in the figure represent the characteristic parts of FDEL, which differ from CVT. The equivalent strain is given according to the Fourier amplitude of strain time history in each soil layer [(a), (c)], and the convergence of iterative calculation is evaluated in each three frequency ranges [(b)].
2.3 Verification of Techniques in a Case of Horizontally Layered Ground

The numerical analysis on the basis of FDEL is verified by strong motion records obtained at borehole array station in Kobe Portisland during the 1995 Hyogoken nambu earthquake. Soil properties of this ground are very soft, and massive liquefaction occurred during the earthquake. Figure 2 shows the soil profile of the site and locations of accelerographs. There are 4 accelerographs at GL.0m, -16m, -32m, and -83m depth.

The accelerograms of N-S component at GL-83m is used for response analyses. Closed circles in figure 3 show the maximum accelerations of the array records. Lines in figure 3 represent the calculated acceleration responses. Comparing FDEL with SHAKE, it is noted that the result of FDEL is consistent with the observed records except at GL.0m point where liquefaction occurred. The results of SHAKE and FLUSH in figure 3 are in agreement. The result of FDEL also agrees with the result of FDEL-FEM as to be mentioned later.

Figure 4 shows acceleration transfer functions from GL-83m to GL-16m. The transfer function of FDEL is consistent with that from observed records, although the transfer function of SHAKE is much lower than the others in high-frequency region.
Fig. 2 Soil Profile of Kobe Port Island Site

Fig. 3 Amplification ratio of Amax

Fig. 4 Observed and calculated ground motion amplification
3. FEM RESPONSE ANALYSIS OF EMBANKMENT USING FREQUENCY-DEPENDENT EQUIVALENT LINEARIZED TECHNIQUE

3.1 Application of FDEL to FEM-based Analysis of Embankment ground

Generally response analysis of ground should be considered two types of non-linearities. One is associated with stress-strain relation of soil material, and another is associated with geometric deformation characteristic. FDEL (based on multi-reflection theory) is regarded as a solution of the material non-linearity; however, it can not apply to the analysis of spatially in-homogeneous ground such as embankment. A FEM-based technique, here called FDEL-FEM, is applied considering the frequency-dependent equivalent strain as well as FDEL.

3.2 Array Records and Configuration of Array Observation System in Embankment

Figure 5 shows configuration of array observation system installed in an embankment (Takewaki et.al, 1978). The embankment is getting thick along its slope, and the thickness is about 40m in the flat part. Seismographs (accelerographs) are installed at 13 points. No.3 (GL+102m), No.4 (GL+87m), and No.5 (GL+67m) points constitute a vertical array observation system. The average of shear wave velocities on the ground is about 400 m/sec in the embankment body. Array records obtained during the earthquake occurred in Jul.15, 1993 are examined after correction of the orientation-error for borehole accelerographs. The acceleration record obtained at No.5 seismograph is de-convoluted to the analytical basement by using multi-reflection theory and applied as the input motion for FEM analysis. Fig.6 shows waveforms and Fourier spectrum of input ground motion for the analyses.

3.3 Verification of FEM Analysis by Observed Data in a Case of Embankment

The calculated response of embankment is verified by the observed array records. Figure 7 shows the peak acceleration response for each element in each cross section: section A; the vertical plain of the left side of the mesh model, section B; the vertical plain of the center, and section C; the horizontal plain on the top of the embankment body. The observed records are located in the section B, and plotted in the graphs by closed circles. The calculated responses of FLUSH and FDEL-FEM are plotted in the graphs. The both outside of FEM mesh is assumed horizontally layered ground. The peak acceleration of input ground motion for both FEM analyses are 4.2 (cm/sec²).

Since the input motion level is very small and soils are a little hard, those two analyses are almost in the same result. In such the case, the strain level of soil is so small that the shear strain remains in linear domain. The results of two FEM-based analyses are consistent with the observed data.

3.4 Parameter Study

Figure 8 shows the result in the case of large strain level. The maximum value of input ground motion is 300 (cm/sec²). As the equivalent shear modulus and damping factor are regulated at large strain level, they are strongly affected in frequency-dependent non-linearity of the equivalent strain. The equivalent strain based on a conventional technique has tendency to overestimate damping factor of soil and underestimate the responses of elements, comparing with the FDEL-based equivalent strain. In the section A, which is regarded as a part of horizontal layers, the acceleration response by FLUSH is lower than that by FDEL-FEM. In the section C, the responses of these analyses are getting large against the top slope of the embankment.
Fig. 5. Configuration of Array Observation System (Takewaki et al., 1978)

Fig. 6. Input Ground Motion for Analysis (de-convoluted record)
Fig. 7 case 1 (input acc.=4.2 (cm/sec/sec))

Fig. 8 case 2 (input acc.=300(cm/sec/sec))
4. CONCLUSIONS

In this study, frequency-dependent equivalent linearized technique for FEM response analysis of ground and its applicability are examined on the basis of the strong motion records obtained at borehole array observation station. Major conclusions derived from this study may be summarized as follows.

(1) The frequency-dependent equivalent strain, which is proportional to the Fourier amplitude of strain time history is defined and applied for FEM response analysis in frequency domain. The idea is based on the concept that the equivalent strain used for the regulation of equivalent shear modulus and damping factor should be given according to their contribution of each frequency contents to strain time history, since the spectral intensity of shear strain generally depend strongly on the frequency.

(2) The applicability of the technique (FDEL) is examined for the strong motion array records observed at the very soft horizontally layered ground. In case of the strong acceleration level, it is demonstrated that the frequency-dependent equivalent linearized technique can estimate the amplification correctly even in high frequency region where the conventional technique underestimates the amplification considerably. The proposed technique is consistent with the observed data. Both analyses based on multi-reflection theory and based on FEM agree in results.

(3) The applicability of the technique is also examined in a case of response analysis of embankment ground. It is demonstrated that small level input motion gives almost no difference between the FDEL-based FEM and a conventional FEM such as FLUSH. However, in case of the large level input motion, it is presented the difference in the response analyses between these analyzers.

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