SEISMIC RESPONSE CONTROL
WITH INNOVATIVE NEGATIVE STIFFNESS DAMPERS

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ABSTRACT:

In this first part of this study, theoretical and numerical evaluation of negative stiffness appearing in the skyhook control is conducted. The skyhook control is widely known for the vibration control method in the mechanical engineering field. The skyhook control can also achieve absolute response reduction. In order to realize a negative stiffness, however, the control force that accelerates the deformation should be generated. At present, such a performance is achieved only by using active controlled actuators or semi-active devises with sophisticated controllers and sensors. In the second part of this research, a new damper realizing a negative stiffness and stable energy dissipation in a passive manner is proposed, and its dynamic performance is investigated through large-scale shaking table tests. It is confirmed that the innovative negative stiffness passive damper reduces both the absolute acceleration and the relative displacement of the bridge model.

KEYWORDS: Skyhook Control, Negative Stiffness Damper, Friction Damper, Shake Table Tests

1. INTRODUCTION

For seismic safety of urban flexible structures such as long-span and tall pier bridges, high rise buildings, tall towers and so on, effective vibration energy absorbing devices are necessary to reduce the acceleration and displacement response due to the severe earthquake ground motion. In this paper, innovative negative stiffness dampers are proposed and developed. In 2002, Iemura newly proposed the negative stiffness control based on the results of semi-active control of variable dampers. The negative stiffness control is one of the structural control methods of which characteristics are represented by a combination of a “negative” stiffness and a damping element. The stiffness of the total structural system is reduced with the negative stiffness element and displacement response is reduced with the damping element. The basic principle of the negative stiffness damper is illustrated in Figure 1.

![Figure 1 Basic principle of the negative stiffness damper](image)

In the first part of this study, theoretical and numerical evaluation of negative stiffness appearing in the skyhook control is conducted. The skyhook control is widely known for the vibration control method in the mechanical
In the skyhook control, the target load is proportional to the absolute velocity response, say,

\[ F_d = c_{sh}(\dot{x} + \dot{z}) \]  

(2.1)

where \( c_{sh} \) is the viscosity coefficient of the skyhook dashpot, \( x \) is the deformation of the damper and \( z \) is the ground motion. Eqn. 2.1 indicates that absolute velocity, which is difficult to directly measure, is needed in calculating the target load. This is one of the disadvantages of the skyhook control.

A typical numerical analysis of the SDOF system with the skyhook control is shown in this section. Figure 3 shows the SDOF system with the skyhook dashpot. The parameters used in Figure 3 are shown in Table 2.1. The sinusoidal wave with the amplitude of 1.0[m/s²] and the angular frequency of \( \omega = (k_0 / m_0)^{1/2} \) is used as the ground motion. The result of the hysteretic loop is shown in Figure 4. As seen in Figure 4, the hysteretic loop of the skyhook control is similar to that of the negative stiffness control. In other words, negative stiffness appears in the skyhook control. Figure 4 indicates that the skyhook control can be represented by negative stiffness and viscosity.
Table 2.1 Parameters of SDOF System with Skyhook Dashpot

<table>
<thead>
<tr>
<th>$m_0$</th>
<th>$c_0$</th>
<th>$k_0$</th>
<th>$c_{sh}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0[kg]</td>
<td>1.0[N.s/m]</td>
<td>100[N/m]</td>
<td>3.0[N.s/m]</td>
</tr>
</tbody>
</table>

The theoretical stationary solution of the SDOF system with the skyhook dashpot shown in Figure 3 is calculated in this section. The equation of motion of the SDOF system shown in Figure 3 is

\[ m_0 \dddot{x} + c_0 \ddot{x} + k_0 x + c_{sh} (\dot{x} + \dot{z}) = -m_0 \ddot{z} \]  
\[ \dddot{x} + (2h_0 \omega_0 + 2h_{sh} \omega_0) \ddot{x} + \omega_0^2 x = -\ddot{z} - 2h_{sh} \omega_0 \dot{z} \]

where

\[ \omega_0 = \sqrt{\frac{k_0}{m_0}}, \quad h_0 = \frac{c_0}{2m_0 \omega_0}, \quad h_{sh} = \frac{c_{sh}}{2m_0 \omega_0} \]

The frequency response function is obtained as follows.

\[ \frac{X_{sh}}{Z} = \frac{r_{\omega}^2 - 2i h_{sh} r_{\omega}}{1 - r_{\omega}^2 + 2i(h_0 + h_{sh}) r_{\omega}} \]
\[ = \frac{r_{\omega}^2 (1 - r_{\omega}^2) - 4h_{sh} (h_0 + h_{sh}) r_{\omega}^2}{(1 - r_{\omega}^2)^2 + 4(h_0 + h_{sh})^2 r_{\omega}^2} + i \frac{-2h_0 r_{\omega}^3 - 2h_{sh} r_{\omega}}{(1 - r_{\omega}^2)^2 + 4(h_0 + h_{sh})^2 r_{\omega}^2} \]

where

\[ r_{\omega} = \frac{\omega}{\omega_0} \]

In order to simplify the equations, following symbols are defined.

\[ a_{sh}(\omega) = \text{Re} \left( \frac{X_{sh}}{Z} \right) = \frac{r_{\omega}^2 (1 - r_{\omega}^2) - 4h_{sh} (h_0 + h_{sh}) r_{\omega}^2}{(1 - r_{\omega}^2)^2 + 4(h_0 + h_{sh})^2 r_{\omega}^2} \]
\[ b_{sh}(\omega) = \text{Im} \left( \frac{X_{sh}}{Z} \right) = \frac{-2h_0 r_{\omega}^3 - 2h_{sh} r_{\omega}}{(1 - r_{\omega}^2)^2 + 4(h_0 + h_{sh})^2 r_{\omega}^2} \]

It should be noted that $b_{sh} \leq 0$. Therefore, the frequency response function of the load of the skyhook dashpot is...
The relative displacement response and the damper load can be obtained as follows:

\[
\frac{F_{sh}}{Z} = c_{sh} \frac{\ddot{Y}_{sh}}{Z} = c_{sh} \frac{i\omega Y_{sh}}{Z} = c_{sh} \frac{i\omega(X_{sh} + Z)}{Z} = -\omega c_{sh} b_{sh}(\omega) + i\omega c_{sh} a_{sh}(\omega) + 1
\]  

The time history of the relative displacement response and the damper load can be obtained as follows.

\[
x(t) = \text{Re}(X_{sh} e^{i\omega t}) = |X_{sh}| \text{Re}(e^{i\omega t + \varphi_x})
\]

\[
f(t) = \text{Re}(F_{sh} e^{i\omega t}) = \omega c_{sh} |X_{sh}| \text{Re}\left(\frac{b_{sh}(\omega)}{a_{sh}(\omega) + b_{sh}(\omega)} + i \left(1 + \frac{a_{sh}(\omega)}{a_{sh}(\omega) + b_{sh}(\omega)}\right) e^{i\omega t + \varphi_x}\right)
\]

where \(\varphi_x\) is the phase angle of \(X_{sh}\). When \(\omega t + \varphi_x = 0\), the relative displacement response is maximum and the damper load is

\[
f(t) = \omega c_{sh} |X_{sh}| \frac{b_{sh}(\omega)}{a_{sh}(\omega) + b_{sh}(\omega)} < 0
\]

Eqn. 2.11 indicates that the value of the damper load of the skyhook damper is negative when the relative displacement response reaches the maximum value. Hence, the hypothesis that the skyhook control has the negative stiffness is proved. This fact suggests that the skyhook control can be represented by the negative stiffness control.

### 3. DEVELOPMENT OF A NEGATIVE STIFFNESS FRICTION DAMPER

Figure 5 shows the overview of the proposed negative stiffness damper, or simply called “NSD”. The components of the device are quite similar to the ordinal friction pendulum support or FPS, except that the inverted curve is introduced to the stainless-steel slide plate. Since the vertical weight induced on the unstable convex slide plate accelerates the horizontal deformation due to the gravitational effect, the device’s force is negatively proportional to the deformation. It should be noted that the proposed device generates a negative stiffness in a passive manner. The convex plate is supported by a PTFE portion to introduce friction energy dissipation. In order to attain a stable friction behavior while the slide plate moves horizontally, the portion is attached to the pivot support that rotates smoothly.
The mechanism of the damper is shown in Figure 6. The equation of motion with regard to the girder subjected to an earthquake motion is written as follows:

\[
\frac{\ddot{W}}{g} + F(x, \dot{x}) = -\frac{\ddot{z}}{g}
\]

\[
F(x, \dot{x}) = -N_h + \mu \cdot W \cdot \text{sgn}(\dot{x}) \cos \theta
\]

\[
x = R \sin \theta
\]

where \( W \) is the vertical weight, \( x \) is the horizontal deformation, \( \text{sgn}(\dot{x}) \) is the signum function of the velocity, \( N_h \) is the horizontal force of the damper, \( \mu \) is the friction coefficient, \( \theta \) is the rotating angle and \( R \) is the curvature radius of the stainless-steel slide plate. In Eqn. 3.3, the size of a pivot shown in Figure 5 is assumed to be considerably small compared to \( R \).

The horizontal force of the damper can be written by decomposing the vertical weight, namely,

\[
N_h = W \sin \theta \cos \theta
\]

It is obvious that the amount of the negative stiffness is controlled by the vertical load and the curvature radius of the convex slide plate.

4. VERIFICATION TESTS USING SHAKING TABLE

4.1. Test Setup

The dynamic behavior of the proposed negative stiffness damper was assessed by using the large-scale shaking table facility at Disaster Prevention Research Institute of Kyoto University. The shake table has a capability to drive the table (5.0 m × 3.0 m) up to 1.0 G in acceleration and 150 kine in velocity with the maximum specimen weight of 150 kN. The maximum strokes are 300 mm in longitudinal, 250 mm in transverse, and 200 mm in up-down directions.

As shown in Photo 1, a large-scale model of an isolated girder was assembled on the shake table. The model of the girder was a steel-made slab (W4150 mm × D2650 mm × H400 mm), weight of which is approximately 100 kN. The square steel plate (300 mm × 300 mm) was welded at each corner of the slab in order to attach the developed negative stiffness damper. Moreover, two cylindrical natural rubber bearings were installed to adjust the natural frequency of the total test system without dampers to approximately 1.25 sec. The total stiffness of these rubber bearings was 254 kN/m, corresponding to the structure’s positive stiffness mentioned in Figure 1(i).

For measuring the behavior of the test system, accelerometers and laser displacement sensors were utilized. The
force generated by the developed damper was directly measured by a tri-directional load transducer attached on the bottom of each damper. Total test setup is shown in Figure 7 and Photo 1.

![Figure 7 Schematic view of the test setup](image)

4.2. Input Motions for Earth Quake Excitation Test

In the series of earthquake loading tests, the shake table was driven by uni-directional accelerations. The east-west component of the strong motion observed at JMA Kobe observatory during the 1995 Hyogoken-Nanbu earthquake was selected as acceleration input. This motion has been commonly used in Japan for seismic design of civil engineering structures constructed on good soil conditions. The modified waveform instead of the original records was used for the test, which was fitted to the design spectrum designated in the seismic code for highway bridges in Japan. Moreover, maximum acceleration of the wave was scaled down by the factor of 0.4 of the original motion due to the stroke limitations of the girder model and the shaking table. The absolute acceleration response spectrum of the motion for 5% damping is shown in Figure 8.

![Figure 8 Absolute acceleration response spectra](image)

4.3. Test Results and Discussion

The earthquake loading tests were carried out to clarify the effect of the proposed damper under strong motions. Figure 9 shows the hysteretic responses of the inertia force of the girder versus relative displacement between the girder and the shake table subjected to the Kobe EW motion. This figure is depicted as a comparison of the cases with and without the proposed dampers. Figure 10 shows the relationship between the total force of the four NSDs, versus the displacement. It is clearly observed in Figure 10 that the proposed damper successively generated the force that was negatively proportional to the deformation. It consequently follows that the negative stiffness introduced by the damper reduced the test system’s total stiffness, which contributed to the significant reduction in the maximum inertia force as shown in Figure 9.
The maximum inertia force, relative and absolute displacements in the positive and negative directions as a comparison of the cases with and without the proposed damper are summarized in Table 4.1. In this table, maximum responses in the negative are shown in the bracket. The time histories with regard to the absolute displacement and absolute acceleration for the cases with and without the NSD are shown in Figures 11 and 12, respectively. It is found that maximum inertia force of the test system with the NSD was reduced from that with flat friction damper by the factor of approximately 40%. This reduction is clearly observed in Figure 12 as the NSD suppressed absolute acceleration to the small extents. In addition, the measured maximum response showed good agreement with that obtained by preliminary simulations, since the desired negative stiffness was realized in a passive manner by controlling the vertical weight as well as the curvature radius of the damper.

<table>
<thead>
<tr>
<th>Device</th>
<th>Max. Inertia Force (kN)</th>
<th>Max. Relative Displ. (mm)</th>
<th>Max. Absolute Displ. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat friction damper</td>
<td>26.79 (-19.01)</td>
<td>58.46 (-33.81)</td>
<td>45.79 (-50.49)</td>
</tr>
<tr>
<td>NSD</td>
<td>16.07 (-14.05)</td>
<td>43.51 (-21.73)</td>
<td>50.39 (-36.41)</td>
</tr>
</tbody>
</table>

Table 4.1 Maximum responses of the girder with and without NSD

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Figure 9 Comparisons of inertial force vs. relative displacement

Figure 10 Hysteretic loops of the NSD

Figure 11 Comparisons of absolute displacements with and without NSD

Figure 12 Comparisons of absolute accelerations with and without NSD
It should also be emphasized that the relative and absolute displacements of the NSD were not significantly magnified compared to those with the flat damper. It indicates that the proposed damper is particularly applicable to structures such that both absolute acceleration and displacement induce damage, such as railway bridges with rail tracks.

5. CONCLUSION

In the first part of this study, the negative stiffness is newly found in the hysteretic restoring force of a skyhook damper plotted against relative velocity of a structure. It is also shown that the hysteretic restoring of a skyhook damper can be approximated by the combination of the negative stiffness spring and the general viscous damper.

In the second part of this research, the new device realizing a negative stiffness in a passive manner was developed, and its performance was investigated by the shaking table test using a girder model with rubber bearings. It is clarified that the proposed device successfully generates the stable negative stiffness as well as energy dissipation under both sinusoidal and earthquake motions. It consequently follows that the proposed device significantly reduced absolute acceleration of the girder without considerably increasing the absolute and relative displacements, compared to the ordinary friction bearings.

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