PREDICTION OF SHEAR CONTRIBUTION OF WALLS IN RC WALL-FRAME BUILDINGS UNDER EARTHQUAKE MOTIONS

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ABSTRACT:

This paper shows two earthquake response evaluation methods for multi-story wall-frame buildings. One is a method of redistributing the representative shear in an equivalent SDOF system reduced from the wall-frame building to the first mode components of the story shears contributed by the shear walls and frames, and the other is a method of evaluating the higher mode components of the story shears. Time history earthquake response analysis and pushover analysis for three types of 12 story RC wall-frame building with different wall layouts are performed to examine the prediction accuracy of earthquake responses by the proposed methods. Good agreements between the results of the proposed methods and the earthquake response analysis are obtained in terms of the story shears contributed by the shear walls and frames.

KEYWORDS: RC wall-frame buildings, Earthquake response evaluation, Equivalent SDOF system, Higher mode response, Story shear contribution ratio.

1. INTRODUCTION

In order to evaluate the peak response of each story or each member in a multi-story building under an earthquake motion in the capacity spectrum method, the technique of using the peak response of the equivalent SDOF system which represents the structural characteristics of the building is adopted (Freeman, 1978). However, since the effects of higher mode components cannot be reflected through the response values evaluated from the equivalent SDOF system alone, a key issue for improving the capacity spectrum method is on how to take these effects into account within the peak earthquake response evaluation. In recent years, some researchers in the US have tried to solve this issue (e.g., Paret, et al., 1996; Chopra and Goel, 2002; Kalkan and Kunnath, 2006). The authors have also addressed this issue in the past few years through the proposal of a method for evaluating higher mode response components on multi-story frame buildings (Kuramoto, 2006 and 2007) which have been shown to be able to evaluate the inter-story drift and shear with good accuracy.

For reinforced concrete (RC) wall-frame buildings consisting of pure frames and multi-story shear walls, on the other hand, the formation of an overall yielding mechanism due to flexural yielding at the bottom of the wall or to overturning of the foundation is generally desirable. In order to achieve this, prevention of shear failure at the earthquake-resisting shear walls is of crucial importance, and a more precise evaluation of the shear force response contributed by the shear wall during an earthquake becomes necessary. However, the wall-frame buildings differ from pure frame buildings in that a complicated interactive transmission of shear force occurs through the boundary beam between the shear wall and the frame (i.e. boundary effect), making it more difficult to evaluate the shear force response contributed by each shear wall and frame.

Two earthquake response evaluation methods for multi-story wall-frame buildings are proposed to evaluate the shear force response including higher mode component contributed by the shear walls and the frames in this paper. One is a method of redistributing the representative shear in an equivalent SDOF system reduced from the wall-frame building to the first mode component of story shears contributed by the shear walls and the frames, and the other is a method of evaluating the higher mode component of the story shears. The prediction accuracy of earthquake responses by the proposed methods is discussed based on the results of time history earthquake response analysis and pushover analysis for three types of 12 story RC wall-frame building with different shear wall layouts.
2. REDUCTION TO EQUIVALENT SDOF SYSTEM AND DISTRIBUTION OF THE RESPONSE VALUE

Extending methods proposed for multi-story pure frame buildings (Kuramoto, 2006 and 2007), a method based on nonlinear pushover analysis (hereafter referred to as the static reduction method) and a method based on modal decomposition of the time history earthquake response analysis results (hereafter referred to as the dynamic reduction method) are described for the equivalent SDOF system reduction of multi-story wall-frame buildings in this chapter. A method to distribute the shear response of the equivalent SDOF system to each story and to evaluate the shear force contributed by the shear walls and the frames is also shown. Note that Modal Pushover Analysis (Kuramoto and Matsumoto, 2004; hereafter referred to as MAP analysis), in which the horizontal load distribution is proportional to the elasto-plastic first mode of vibration, is employed for the static reduction method.

2.1. Static Reduction

The static representative shear versus representative displacement relationship ($S_a - S_d$ relationship) of the equivalent SDOF system of a multi-story building can be obtained from the MAP analysis results using equations below (Kuramoto and Matsumoto, 2004; Kuramoto, 2006 and 2007).

\[
\frac{\sum_{i=1}^{N} m_i \cdot \delta_i}{\sum_{i=1}^{N} \delta_i} = \frac{\sum_{i=1}^{N} m_i \cdot \delta_i^2}{\sum_{i=1}^{N} \delta_i^2} \quad (1a, 1b)
\]

where $m_i$ is the $i$th story mass, $\delta_i$ is the relative displacement of the $i$th story with respect to the first story floor position and $P_i$ is the horizontal load acting on the $i$th story. If the horizontal loads acting on the shear walls and the frames at the $i$th story in a multi-story wall-frame building are $w_i P_i$ and $f_i P_i$ respectively, the response acceleration components of the shear walls and the frames in the equivalent SDOF system, $w_i S_a$ and $f_i S_a$, are given by the following equations.

\[
\frac{\sum_{i=1}^{N} w_i m_i \cdot \delta_i}{\sum_{i=1}^{N} \delta_i} = \frac{\sum_{i=1}^{N} f_i m_i \cdot \delta_i}{\sum_{i=1}^{N} \delta_i} \quad (2a, 2b)
\]

Furthermore, similar to the case for the entire building, the representative displacements of the shear walls and the frames can be obtained by Eqn. (1b). Note that as a matter of course, the following relationship exists between the horizontal load acting on the $i$th story, $P_i$, $w_i P_i$ and $f_i P_i$.

\[
P_i = w_i P_i + f_i P_i \quad (3)
\]

2.1. Dynamic Reduction

The representative shear versus representative displacement relationship ($S_a(t) - S_d(t)$ relationship) of the entire building during the seismic response of the equivalent SDOF system which represents the multi-story wall-frame building, and the acceleration response components of the shear walls and the frames, $w_i S_a(t)$ and $f_i S_a(t)$, are respectively provided by equations below (Kuramoto, 2006 and 2007).

\[
\frac{\sum_{i=1}^{N} P_i(t) \cdot \delta_i(t)}{\sum_{i=1}^{N} \delta_i(t)} = \frac{\sum_{i=1}^{N} m_i \cdot \beta_i \cdot u_i \cdot \delta_i(t)}{\sum_{i=1}^{N} \delta_i(t)} \quad (4a, 4b)
\]

\[
\frac{\sum_{i=1}^{N} w_i P_i(t) \cdot \delta_i(t)}{\sum_{i=1}^{N} \delta_i(t)} = \frac{\sum_{i=1}^{N} f_i P_i(t) \cdot \delta_i(t)}{\sum_{i=1}^{N} \delta_i(t)} \quad (5a, 5b)
\]

where $P_i(t)$ is the horizontal load acting on the $i$th story at time $t$, $w_i P_i(t)$ is the horizontal load acting on
the \(i\)th story frame at time \(t\), \(P_i(t)\) is the horizontal load acting on the \(i\)th story shear wall at time \(t\), \(\delta_i(t)\) is the relative displacement of the \(i\)th story at time \(t\) with respect to the first story floor position, and \(\beta_i\) is the first mode component of \(\delta_i(t)\) (i.e., \(\beta_i(t)=\delta_i\)).

The first mode participation function \(\beta_i\) is given by Eqn. (6), obtained by getting the load step in the MAP analysis results corresponding to the peak response displacement of the equivalent SDOF system reduced through Eqns. (4a) and (4b), and using the relative displacement with respect to the first story floor position \(\delta_1\) of each story at this step.

\[
\beta_i = \left( \frac{\sum_{j=i}^{N_i} m_j \cdot \delta_j}{\sum_{j=i}^{N_i} m_j \cdot \delta_j^2} \right) \delta_i
\]  

(6)

Note that in obtaining \(\beta_i\) by the above equation, reduction is conducted assuming \(\beta_i\) at the load step in the MAP analysis results corresponding to the peak response displacement of a certain story in the MDOF system, since the equivalent SDOF system is not given at the outset. Thereafter, a \(\beta_i\) satisfying a predefined convergence criterion can be obtained by repeating the same steps once or twice (Kuramoto 2006).

### 2.3. Redistribution of the Equivalent SDOF System Response Value to Each Story

The first mode component of story shear force and interstory drift at time \(t\) in the \(i\)th story of a multi-story building, \(Q_i(t)\) and \(\delta_i(t)\), can be obtained by the next equations, using the equivalent SDOF system response values, \(S_{a1}(t)\) and \(S_{u1}(t)\), and the first mode participation function \(\beta_i\) (Eqn. (6)).

\[
\delta_i(t) = \beta_i u_i \cdot S_{a1}(t)
\]  

(7)

\[
Q_i(t) = \sum_{j=1}^{N_i} P_j(t)
\]  

(8)

where \(P_j(t)=\beta_{ij} u_j \cdot m_{ij} \cdot S_{a}(t)\)

(9)

Even for wall-frame buildings, response values at each story of a building can be evaluated by Eqns. (7) and (8) using the first mode participation function \(\beta_i\). However, the equivalent SDOF system response values cannot be distributed and evaluated for each shear wall and frame through \(\beta_i\), since the proportion of the horizontal loads applied to the shear walls and the frames differs for each story. Thus, the equivalent SDOF system response values are distributed at each story by setting the apparent first mode participation function for the shear walls and the frames (hereafter referred to as the distribution coefficient), \(\beta_{iw} u_i\) and \(\beta_{if} u_i\).

Applying the relationship in Eqn. (9), the distribution coefficients of the shear walls and the frames at \(i\)th story, \(\beta_{iw} u_i\) and \(\beta_{if} u_i\), are assumed to be given by the following equations.

\[
\beta_{iw} \cdot u_i = \frac{P_{i_{\max}}}{m_{iw} \cdot S_{a_{\max}}}
\]

\[
\beta_{if} \cdot u_i = \frac{P_{i_{\max}}}{m_{if} \cdot S_{a_{\max}}}
\]

(10a, 10b)

where \(P_{i_{\max}}\), \(S_{a_{\max}}\) and \(P_{i_{\max}}\), \(S_{a_{\max}}\) are the acting horizontal load and acceleration on the \(i\)th story of the shear walls and the frames respectively, at the MAP analysis load step corresponding to the time of peak deformation response of the equivalent SDOF system reduced by the dynamic reduction. The shear responses of the shear walls and the frames at each story \(i\)th story \(\beta_{iw} u_i\) and \(\beta_{if} u_i\) can be obtained by the following equations, using the distribution coefficients \(\beta_{iw} u_i\) and \(\beta_{if} u_i\) from Eqns. (10a) and (10b).

\[
Q_i(t) = \sum_{j=1}^{N_i} P_j(t)
\]

(11a, 11b)

where \(P_j(t)=\beta_{ij} u_j \cdot m_{ij} \cdot S_{a}(t)\) and \(P_j(t)=\beta_{if} u_j \cdot m_{ij} \cdot S_{a}(t)\)

(12a, 12b)

Similar to the case of the entire building, note that the displacement responses of the shear walls and the frames at each story can be obtained by Eqn. (7).
3. OVERVIEW OF ANALYSIS

A total of 3 types of 12 story RC wall-frame building, from Model-1 to Model-3, are to be analyzed. The ground plan and elevation for each model are shown in Fig. 1. All buildings consist of a 6.0m x 3 spans for both the lateral and longitudinal directions in the ground plan, with a building height of 42.5m (1st story: 4.0m, 2nd story and above: 3.5m). Model-1 has its continuous shear walls placed at the center span of the inner frames. Model-2 is the same as Model-1 with shear walls removed from stories 4, 8 and 12, while Model-3 is the same as Model-2 with shear walls removed from stories 2, 6 and 10 as well. The shear wall thickness for all models is equal to 300mm, while the vertical and horizontal bars are both D13@200 double reinforcements (\( p_v = 0.4\% \)). Using Model-1 as the standard model, Model-2 and Model-3 are the case models with lower wall quantity as well as discontinuously distributed shear walls. For the time history earthquake response analysis, 3 records, namely El Centro NS (1940), JMA-Kobe NS (1995) and Taft EW (1952), were selected and standardized to a maximum velocity of 75cm/sec. These records were used as the input ground shaking for each of the model, giving 9 analysis cases. The analysis was conducted in one direction on 2 frames, as shown by dotted lines in Fig. 1, with the beams modeled as springs at both ends using the Takeda model for its hysteretic characteristics, while columns and shear walls were idealized as multi-spring models (MS model; Gu, Inoue and Shibata, 1998).

4. ANALYSIS RESULTS

4.1. Validity of the Equivalent SDOF System Reduction

Figure 2 is a comparison of analytical results of the dynamic reduction and the static reduction for Model-1 and Model-3 using the El Centro record (75cm/sec) as input. The first row of figures shows the results for the entire building, the second row shows those for the shear wall, while the third row shows those for the frames. As shown in Eqns. (2a) and (5a), the horizontal loads acting on the shear walls at every story are necessary in order to obtain the acceleration response at the shear walls in the equivalent SDOF system, \( S_\alpha \) and \( S_\delta \). Since the shear walls are discontinuous for Model-3 and from the sense that the horizontals loads are supposed to be acting on the frame with the shear wall, the horizontal load acting on the column (i.e., the load portion acting on the two center columns of the inner frame) was used for stories without walls.

It can be seen from Fig. 2 that for both models, the response values at the time of maximum deformation response of the equivalent SDOF system from the dynamic reduction shown as filled circles in the figures, as well as the maximum experienced deformation responses at some point in the response history before the time of maximum deformation shown as hollow circles in the figures, generally appear in the \( S_\alpha \) and \( S_\delta \) curve of the static reduction, not only for the entire building but also for the shear wall and the frame. In other words, as shown in the previous study for pure frame buildings (Kuramoto, 2006) which in a statement says that “In the response history before the time of maximum response of the equivalent SDOF system, the maximum deformation response point experienced up to a point in time is nearly the same as the MAP analysis result for the first mode.”, this tendency can also be seen in wall-frame buildings regardless of the layout of shear walls. Moreover, this also shows that the same tendency can be seen even for shear walls and frames. Thus, the static and dynamic reduction methods shown in Sections 2.1 and 2.2 can be taken as valid.
4.2. Validity of Distribution Method to Each Story from Equivalent SDOF System Response

Figure 3 is a comparison of MAP analysis results and first mode component of the time history response results at the 1st, 7th and 11th story of Model-3, which is the building model with discontinuously distributed shear walls, subjected to the El Centro record input. Similar to the equivalent SDOF system in Fig. 2, the maximum values of the first mode component of time history response at each story may be considered to be generally on the story shear force versus interstory drift curves of the MAP analysis, for the entire building as well for the shear walls and frames. Similar results are also obtained for the continuous shear wall type of Model-1. Generalizing from these results, it may be considered that the distribution method to each story from the equivalent SDOF system response value described in Section 2.3 is valid regardless of the shear walls layouts.
5. EVALUATION OF HIGHER MODE COMPONENTS IN STORY SHEAR

Considering that the equivalent mass for the first mode of vibration of the entire building, $\overline{M}$, and the horizontal load, $P_{max}$, are given by Eqs. (13) and (14) respectively, the relationship between $\overline{M}$ and the shear wall and frame components, $\overline{M}_w$ and $\overline{M}_f$, can be obtained by Eqs. (15), (16a) and (16b), using the distribution coefficients $i_w \beta_i u_i$ and $i_f \beta_i u_i$ from Eqs. (10a) and (10b).

\[
\overline{M} = \sum_{i=1}^{N} m_i \bar{\beta_i} u_i \quad \text{(13)}
\]

\[
P_{max} = m_1 \bar{\beta_i} u_i \cdot S_{max} \quad \text{(14)}
\]

\[
\overline{M} = i_w \overline{M} + i_f \overline{M} \quad \text{(15)}
\]

\[
i_w \overline{M} = \sum_{i=1}^{N} m_i \bar{\beta_i} u_i \quad i_f \overline{M} = \sum_{i=1}^{N} m_i \bar{\beta_i} u_i \quad \text{(16a, 16b)}
\]

Where, $S_{max}$ designates acceleration at a loading step in MAP analysis at which the displacement is corresponding to the maximum displacement of the equivalent SDOF system reduced by the dynamic reduction method. The following relationship is also made by defining $Q_B(t)$, $Q_w(t)$ and $Q_f(t)$ as the higher mode components of base shear for the entire building, shear walls and frames, respectively.

\[
hQ_B(t) = hQ_w(t) + hQ_f(t) \quad \text{(17)}
\]

On the other hand, $hQ_B(t)$ is approximated by Eqn. (18), considering that the higher mode component of base shear of the entire building is roughly proportional to the acceleration of input ground motion (Kuramoto, 2006).

\[
hQ_B(t) \approx -\left(M - \overline{M}\right) \cdot \ddot{x}(t) \quad \text{(18)}
\]

From Eqs. (15) and (18), the following relationship can be obtained.

\[
hQ_B(t) \approx -\left(M - \overline{M}\right) \cdot \left(\frac{i_w \overline{M}}{\overline{M}} + \frac{i_f \overline{M}}{\overline{M}}\right) \cdot \ddot{x}(t) = \frac{i_w \overline{M}}{\overline{M}} hQ_w(t) + \frac{i_f \overline{M}}{\overline{M}} hQ_f(t) \quad \text{(19)}
\]

Therefore, the higher mode component of base shears contributed by the shear walls and the frames, $hQ_w(t)$ and $hQ_f(t)$, can be given by Eqs. (20a) and (20b), considering the relationships of Eqs. (15) and (21).

\[
hQ_w(t) = \frac{i_w \overline{M}}{\overline{M}} hQ_B(t) \quad hQ_f(t) = \frac{i_f \overline{M}}{\overline{M}} hQ_B(t) \quad \text{(20a, 20b)}
\]

The relationship among the higher mode component of story shear in the $i$th story for the entire building, $hQ_i(t)$, and the shear wall and frame components, $hQ_w(t)$ and $hQ_f(t)$ is given by

\[
hQ_i(t) = hQ_w(t) + hQ_f(t) \quad \text{(21)}
\]

Similar to the case of the base shear above, $hQ_w(t)$ and $hQ_f(t)$ are given by Eqs. (22a) and (22b), considering the relationships of Eqs. (15) and (21).

\[
hQ_w(t) = \frac{i_w \overline{M}}{\overline{M}} hQ_i(t) \quad hQ_f(t) = \frac{i_f \overline{M}}{\overline{M}} hQ_i(t) \quad \text{(22a, 22b)}
\]

where $hQ_i(t)$ can be approximated by Eqn. (23), using the second mode component of story shear, $sQ_i(t)$, and the participation function for $s$ th mode, $\beta_s u_j$ (Kuramoto, 2006).

\[
hQ_i(t) \approx sQ_i(t) - \sum_{j=1}^{N} \left(m_j \left(1 - \sum_{s=1}^{N} \beta_s u_j \right) \right) \cdot \ddot{x}(t) \quad \text{(23)}
\]
Figure 4 shows time histories of the higher mode component of story shear contributed by the shear walls and the frames, \( Q_{b,w}(t) \) and \( Q_{b,f}(t) \), at the 1st, 7th and 11th story for all building models in which the input ground shaking are El Centro NS (1940) for Model-1, JMA-Kobe NS (1995) for Model-2 and Taft EW (1952) for Model-3, which are standardized to a maximum velocity of 75cm/sec. In the figures, gray lines show the analytical results while black lines designate the predicted by Eqns. (22a) and (22b).

For Model-1 with multi-story shear walls, the predicted results of both shear walls and the frames show good agreement with the analytical results. For Model-2 and Model-3 with discontinuous shear wall layouts, almost good agreements between the predicted and the analytical results are observed, while it is found a tendency that the prediction accuracy by Eqn. (22b), which is for the frame contribution, somewhat reduces due to less peak shear response of the analytical results. Thus, these results indicate that the time history of higher mode components of story shear contributed by the shear walls and frames can be evaluated by Eqns. (22a) and (22b) without regard to the shear wall layouts or input ground motions.

Figure 5 shows the ratio of mass shared by the shear walls to equivalent mass for the first mode of the entire building, \( \frac{M_{w}}{M} \), in each model every input ground motion. A contribution ratio of higher mode shear response in the shear walls increases with the increase of \( \frac{M_{w}}{M} \), as been clear from Eqns. (20a) and

![Figure 4](image4.png)  
**Figure 4** Prediction of higher mode component in story shear contributed by shear wall and frame

![Figure 5](image5.png)  
**Figure 5** Ratios of contributed mass by shear wall to equivalent mass for the 1st mode
The values of $\mu M_1 \omega_1$ for Model-2 and Model-3 is larger than that for Model-1, although the shear walls in Model-2 and Model-3 are discontinuously arranged and the amount of shear walls is less than that in Model-1. Moreover, the difference of the values tends to be significant so that the degree of discontinuity becomes large. Thus, it is found that the contribution ratio of higher mode shear response in shear walls is influenced by shear wall layouts.

6. CONCLUSIONS

This paper presented two methods for the equivalent SDOF system reduction of multi-story wall-frame buildings, the static reduction method and the dynamic reduction method, modifying methods proposed for multi-story pure frame buildings (Kuramoto, 2006), to evaluate the shear force response including higher mode component contributed by the shear walls and frames. The prediction accuracy of earthquake responses by the proposed methods was discussed by comparing with the results of time history earthquake response analysis and pushover analysis for three types of 12 story RC wall-frame building with different shear wall layouts. The findings obtained in this study may be summarized as follows.

(1) The proposed static and dynamic methods for the equivalent SDOF system reduction and the method to redistribute the equivalent SDOF system response to each story response are applicable to multi-story wall-frame buildings without eccentric planning regardless of the quantity and layouts of shear walls.

(2) The time history of higher mode components of story shear contributed by the shear walls and frames can be evaluated by Eqns. (22a) and (22b).

(3) The equivalent mass ratios of the shear wall and frame components to the entire building for the first mode are almost equal to those for the remaining higher modes.

(4) The higher mode shears contributed by shear walls in buildings with discontinuous shear wall layouts is larger than that in buildings with multi-story shear walls.

REFERENCES


