SIMULATIONS OF CHARACTERISTICS OF TUNED LIQUID COLUMN DAMPER USING AN ELLIPTICAL FLOW PATH ESTIMATION METHOD

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ABSTRACT:

This paper reports the elliptical flow path estimation method. This newly proposed numerical method can be employed to simulate the vibration characteristics of the tuned liquid column dampers (TLCDs) and to validate their experimental results tested on the shake table. A TLCD is comprised of two vertical liquid columns connected by a horizontal cross-over duct of the same width. A variation of these with non-uniform vertical column and the horizontal cross-over duct is denoted as a liquid column vibration absorber (LCVA). In previous research, the simulation of the vibration characteristics of these dampers was based on the assumption that the liquid velocity within each leg of these dampers is constant and the transition between the vertical and horizontal flows occurs at a single point. This assumption is valid only for TLCD and LCVA in which the widths of both horizontal cross-over duct and vertical column are small compared to the overall dimension of the damper. For buildings with limited space, it is necessary to configure a TLCD whose column width is not small with respect to the overall dimension of the damper. This type of TLCD configuration has been hardly investigated in previous research. In this case, the variation of liquid velocity in the relatively large transition zone between the vertical columns and the cross-over duct cannot be ignored. A numerical potential-flow method, known as the numerical panel method, is then utilized to simulate the vibration characteristics of TLCDs and LCVA with the aforementioned configuration. The results obtained from the numerical panel method lead to the improved prediction of the vibration characteristics of TLCDs over a wide range of configurations. Unfortunately, the numerical panel method is too complicated to formulate and it cannot give the empirical form for TLCDs’ characteristics. As a result, the elliptical flow path estimation method is proposed to simulate the TLCDs’ vibration characteristics. The results obtained from the numerical model are compared with those obtained from the tests on the shake table. A significant improvement in the model accuracy can be obtained and this feature is essential in the control problem for the optimum performance of a building. Furthermore, the elliptical flow path estimation method can be utilized to investigate their efficiency as a vibration absorber.

KEYWORDS: LCVA, TLCD, Panel method, Elliptical flow path estimation method, Natural frequency
1. INTRODUCTION

Under dynamic loadings, several means can be used to improve the building performance in accordance with either serviceability or safety criteria. Generally, strengthening of a building or installing of the base-isolation system could be very costly or too difficult to perform. Incorporating a passive damper into the building is relatively less expensive and much simpler to perform. Therefore, many research works on the passive damper have been conducted in recent years. Among several kinds of passive dampers, a liquid damper is found to be the most attractive one due to its following advantageous features: a) lower cost; b) easy handling; c) low maintenance requirements; and d) practically no additional weight to the structure is required when the water in the TLCD (tank) is used for the fire fighting.

The tuned liquid column damper (TLCD) [1] is one kind of several passive liquid vibration absorbers used to control the flexible structures (e.g. tall buildings). This liquid column device in a U-shaped container with an orifice achieves the same vibration characteristic as a tuned mechanical damper consisting of a mass and spring system (TMD). It has been shown that TLCD is simpler than TMD since it requires no mechanical component. TLCD is well suited particularly for tall buildings since it usually contains water storage for potable or emergency uses. With the already available water utilized and proper modifications to the existing storage tanks, a TLCD can be formed without introducing an unnecessarily large additional mass to the system. Furthermore, its natural frequency and damping characteristics can be modified easily.

It is crucial that the fundamental frequency of the liquid motion be tuned to the natural frequency of structure, and the damping ratio of its motion be set to an optimal value to achieve the effective liquid dampers [2-5]. Unfortunately, the damping induced by the liquid motion is response-dependent. Therefore, the optimum damping ratio and tuning condition cannot be established a priori unless the excitation amplitude is known or can be estimated beforehand. However, previous researches have demonstrated the effectiveness of the liquid type damper under typical seismic excitations [6-11].

Watkins [12] proposed another variation of TLCD in which the column cross-section is not uniform. He denoted this damper as LCVA damper. The performance of LCVA damper was found to be as effective as or even more effective than TLCD damper [9]. As described subsequently, the natural frequency of the tank is determined by its “effective length” and this is related to the geometry of the tank and particularly to the ratio of the cross-section area of the vertical column and the cross-over duct [13].

The objective of this study is to develop the governing equation for the motion of the liquid inside the damper to simulate the vibration characteristic of a LCVA when the size of the transition zone between the vertical and horizontal portions (corner-to-corner width) is large compared to the horizontal width and the vertical height. Most researches done previously focus on LCVA with a small ratio of the transition zone between the vertical and horizontal portion (corner-to-corner width) to the horizontal length (in the range of 0.04 – 0.20). Various researchers have also suggested various definitions for the effective length of LCVA based on different idealization of the moving fluid, resulting in different values of the natural frequencies [3, 9 and 13]. For buildings with limited space, it will be necessary to configure a LCVA with a significantly larger ratio of the corner-to-corner width to the horizontal length, for which the relevant research work is still limited. This configuration of the LCVA does not allow an approximation of the effective average velocity within each portion of the moving liquid column, since there is a relatively larger transition zone in which the flow is highly non-uniform. As a more accurate prediction of the vibration characteristic of a damper is crucial in the control problems, a more refined analysis is deemed necessary.

A numerical potential-flow method known as the numerical panel method is employed to simulate the induced velocity distribution of the fluid inside the liquid damper. This method gives the accurate characteristic of TLCD and LCVA over a wide range of damper configurations [14]. However, the numerical panel method needs the careful implementation and it cannot give the empirical form to predict the vibration characteristic of TLCD and LCVA.
Based on the induced velocity distribution of the fluid inside the liquid damper obtained from the numerical panel method, it can be observed that the fluid in the bottom corner of the transition zone does not affect the effective mass or the effective length of the LCVA drastically. Therefore, the flow path of the liquid in the transition region can be assumed to be an elliptical shape. The elliptical flow path estimation method, the more simplified method, is then developed to simulate the vibration characteristic of LCVA with the large transition zone. The model accuracy is verified with those obtained from the existing experimental results.

2. FORMULATIONS

2.1 Mathematical model for liquid damper

Let us consider a LCVA with configuration shown in Fig. 1 excited by a base displacement, \( x(t) \). The vertical and the horizontal column cross-sectional areas are denoted by \( A_v \) and \( A_h \), respectively. As shown in Fig.1, during the motion, the liquid volume inside the LCVA can be divided into two vertical portions ( \( \forall_1 \) and \( \forall_3 \)), and one horizontal portion ( \( \forall_2 \)). In the vertical columns, the liquid is assumed to move vertically relative to the tube with an average velocity \( \dot{y} \). From continuity, the horizontal liquid velocity in the horizontal cross-over duct is approximately uniform with an average velocity, \( r\dot{y} \), where \( r \) is the area ratio of the vertical column to horizontal column of the LCVA and is defined as \( r = A_v / A_h \).

Based on the energy principles with the internal energy of the liquid assumed unchanged during the motion, the unsteady, non-uniform, and incompressible flow equation along the streamline for the LCVA can be derived following the Lagrange equation:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{y}} \right) - \frac{\partial T}{\partial y} + \frac{\partial V}{\partial y} = Q
\]  

(2.1)

where \( T \) and \( V \) are the total kinetic energy and the total potential energy of the system, respectively, and \( Q \) is the total non-conservative force in the direction of \( y \), which can be related to the head loss.

The governing equation for the motion of the liquid in the tube is (Gao and Kwok [3], Chang and Hsu [9])

\[
\rho A_v \ddot{y} + \frac{1}{2} \rho A_v r \delta \dot{y} + 2 \rho A_v g y = -\rho A_v B \ddot{x}
\]  

(2.2)

in which \( \rho \), \( \delta \), and \( g \) represent the density of liquid inside LCVA, the coefficient of head loss, and the acceleration due to gravity, respectively. The effective length of LCVA is defined in terms of the horizontal width.
B and the vertical height \( H \) of the liquid inside the LCV A as follows:

\[
L_c = rB + 2H
\]  
(2.3)

The natural frequency of the oscillation in the tube is \( \omega_f = \sqrt{2g/L_c} \).

### 2.2 Elliptical flow path estimation method

From the numerical results obtained from the panel method [14], the induced velocity distribution of the LCVAs with the large transition zone can be obtained. As observed from the induced velocity distribution of the liquid inside the damper (Fig. 2), the liquid at the bottom corner of transition boundaries between the vertical and horizontal portions has very low induced velocity especially in the LCV A with the larger transition zones (Fig. 2 (c)).

This implies that the vortices and separation in the flow occurs in the corner of the transition boundaries. Therefore, the liquid in these regions hardly moves compared to other portions of the LCV A. As a result it has a little effect on the effective mass or effective length of the LCV A. This type of phenomenon was also observed during the shake-table experiment of the damper model with large transition regions [14]. It is reasonable to neglect the flow at the bottom corner of the transition boundaries and assume the flow path in the transition boundaries to be of an elliptical shape.

As shown in Fig. 3, the flow path of the liquid inside LCV A at the transition boundaries is estimated to be an elliptical portion. In the vertical columns the liquid is assumed to move vertically relative to the tube with an average velocity, \( \dot{y} \). Based on the continuity condition, the horizontal liquid velocity in the horizontal cross-over duct is approximately uniform with an average velocity \( \dot{r} \). The liquid volume domain \( \forall \) inside the LCV A can be divided into five sub-domains, namely: two vertical portions (\( \forall _1 \) and \( \forall _5 \)), two elliptical portions (\( \forall _2 \) and \( \forall _4 \)), and one horizontal portion (\( \forall _3 \)). The total kinetic energy in the associated Lagrange equation (Eqn. (2.1)) can be expressed as:

\[
T = \int \frac{\dot{y}^2}{2} \rho dv + \int \frac{1}{2} \rho v_2^2 dv + \int \frac{(r\dot{y} + \dot{x})^2}{2} \rho dv + \int \frac{1}{2} \rho v_4^2 dv + \int \frac{\dot{y}^2}{2} \rho dv
\]

(2.4)

where \( v_2 \) and \( v_4 \) are the velocity of the liquid inside the elliptical portions no. 2 and 4, respectively. Representing the 2nd and 4th portions in the \( r-\theta \) coordinate system, the liquid velocities in those portions depend on the angle \( \theta \) as show in Fig.4 and is defined as: \( \frac{\dot{y}}{R} \) where \( a \) is the width of the vertical column of the LCV A and \( R \) is the distance from the center of the ellipse to the point on the perimeter coincided with the aforementioned
angle $\theta$, $R = \left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}\right)^{-1}$. Following this procedure, the liquid velocity inside the transition regions is varied from $\dot{y}$ at $\theta = 0^\circ$ to $r\dot{y}$ at $\theta = 90^\circ$. Thus, the more realistic velocity distribution of the liquid inside the large transition regions is obtained. The horizontal width $B'$ and the vertical height $H'$ are defined in Fig. 3. Using the $r-\theta$ coordinate system, the kinetic energy of the liquid inside the elliptical portions no. 2 ($T_2$) and 4 ($T_4$) can be expressed as:

$$T_2 = \int \frac{1}{2} \rho v^2 dv$$

$$= \int \frac{1}{2} \rho \left[ v \cos \alpha + \dot{x} \right]^2 dv + \int \frac{1}{2} \rho \left[ v \sin \alpha \right]^2 dv$$

$$= \frac{3}{2} \int \frac{1}{2} \rho \left[ \left(ay/R\right)^2 \cos^2 \alpha + 2\left(ay/R\right)\dot{x} \cos \alpha + \dot{x}^2 \right] \left(R^2/2-kd\theta\right) + \frac{3}{2} \int \frac{1}{2} \rho \left[ \left(ay/R\right)^2 \sin^2 \alpha \left(R^2-kd\theta\right) \right]$$

$$= \int \frac{1}{4} \rho a^2 \dot{y}^2 d\theta + \frac{3}{2} \rho \left[ 2aR\dot{y} \cos \alpha + R^2 \dot{\chi}^2 \right]$$

where $k = \text{tank width}$ ($A_v = ka$) and $\alpha = \text{angle of the tangent velocity of the liquid as shown in Fig. 4}$ ($\alpha = \tan^{-1}\left(\frac{b^2}{a^2} \cot \theta\right)$). Therefore, the total kinetic energy of the liquid can be expressed as:

$$T = \frac{1}{2} \rho A_v H' \dot{y}^2 + \frac{\rho A_v a}{4} \pi \cdot \dot{y}^2 + \frac{\rho k}{4} \left[ 2aR\dot{y} \cos \alpha + R^2 \dot{\chi}^2 \right]$$

$$= \rho A_v H' \dot{y}^2 + \frac{\rho A_v \pi a}{4} \dot{y}^2 + \frac{\rho A_v \pi a}{4} \dot{x}^2 + \rho A_v \dot{xy} \frac{3}{2} \pi R \cos \alpha d\theta$$

The potential energy of the liquid can be expressed as:

$$V = \rho g A \int_0^{H+y} \dot{z} dz + \rho g A \int_0^{H+y} \dot{z} dz$$

$$= \rho g A (H^2 + y^2)$$

The damping force associated with the liquid motion can be approximated as:
As a result, the governing equation for the motion of the liquid in the LCV As with large transition zone can be obtained by substituting Eqn. (2.6) – Eqn. (2.8) into Eqn. (2.1) as follows:

\[
\rho A_v \left(2H' + B'r + \frac{\pi a}{2} \right) \ddot{y} + \frac{1}{2} \rho A_v r \dot{y} \dot{y} + 2 \rho g A_v y = -\rho B' A_v \ddot{x} - \rho A_v \ddot{x} \int_0^{\pi/2} R \cos \alpha d\theta
\]  

The effective length \( L_e \) and an expression for the natural frequency \( \omega_f \) can be respectively expressed as:

\[
L_e = rB' + 2H' + \frac{\pi a}{2}, \quad \omega_f = \sqrt{\frac{2g}{L_e}} = \sqrt{\frac{2g}{rB' + 2H' + \frac{\pi a}{2}}}
\]  

3. Verifications of simulation results with the experimental results

The accuracy of the analytical prediction of the damper characteristic indicates the precision in estimating the natural frequency of the system. Three damper models tested in Chaiviriyawong’s tests [14] are shown in Fig.5. TLCD with insert type I and LCVA models with inserts types II and III are designed to have the same effective lengths of \( L_e = 2.07 \) m, based on the simplified effective length by Gao and Kwok [3] or Chang and Hsu [9], and are expected to have the natural frequencies, \( f_n \), of 0.49Hz. The corner-to-corner width to horizontal length ratio ranges from 0.35 to 0.75 in the models tested.
3.1 Verification of results from analytical method based on simplified effective length by [3,9]

By using the damping ratio determined from the free vibration tests [14], the frequency responses of the TLCDs can be analytically obtained by solving Eqn. (2.2) and are compared with the spectral test data [14] for three types of inserts as shown in Fig.6. The computation is based on the simplified effective length by Gao and Kwok [3] or Chang and Hsu [9].

It is found from Fig. 6 (a), that the analytical solutions based on the simplified effective length agree very well with the experimental results for the TLCD with insert Type I, but for the LCVAs with inserts types II and III, the simplified effective length method cannot give a satisfactory response. As depicted in Figs. 6 (b) and (c), both the analytical frequency response curves are significantly shifted to the left-hand side of those obtained from the experimental results. This clearly indicates that the theoretical natural frequency of the damper has not been estimated correctly, leading to mistuning of the system.

![Figure 6. Experimental and analytical results of spectral tests of TLCD models.](image)

3.2 Verifications of results from elliptical flow path estimation method with the experimental values in the case of LCVAs with large transition zones (corner-to-corner width to horizontal length ratio larger than 0.5)

The effectiveness of the elliptical flow path estimation method in simulating the vibration characteristics of LCVAs is first verified with the experimental results obtained in Chaiviriyawong’s tests [14] in case of LCVAs with large transition zones.

With the better estimates of the distribution of liquid velocity in the transition zones, the frequency response curves for LCVAs with inserts types II and III are determined by using Eqn. (2.9) obtained from the elliptical flow path estimation method. Fig. 7 shows the resulting frequency response curves together with those obtained earlier, numerical panel method [14] and test results.

![Figure 7. Frequency response curves for LCVAs with inserts types II and III.](image)

As evident from Fig. 7, the frequency response curves based on the elliptical flow path estimation method match the experimental results much better than the ones obtained from the existing analytical method based on the simplified effective length [3, 9]. The agreement is almost perfect for LCVA with insert type II and III. In the latter, at the resonance frequency, for instance, the analytical solution based on the elliptical flow path estimation method underestimates the displacement response by about 3.6% compared with an excessive error of about 52.6% for the existing method. For the numerical panel method, the agreement is almost perfect for LCVA with insert type II, and acceptable for LCVA with insert type III.

Table 1 lists the effective lengths and the natural frequencies computed from 3 analytical methods, based on simplified effective length (Gao and Kwok [3] or Chang and Hsu [9], Hitchcock [13]), the panel method and the elliptical flow path estimation. The experimental results are also included for comparison. It is found from Table 1, that the natural frequency based on the simplified effective length agrees very well with the experimental results for the TLCD with insert Type I, but for the LCVAs with inserts types II and III, the
simplified effective length method cannot give a satisfactory natural frequency. As presented in Table 1, both the natural frequencies are significantly less than the experimental results. The existing analytical model proposed by Gao and Kwok [3] or Chang and Hsu [9] shows the 3.9% and 7.5% of discrepancies of prediction of the natural frequencies compared with the experimental results for LCVAs with insert type II and III, respectively. While, for Hitchcock’s analytical model [13], the discrepancies of the prediction of the natural frequencies are 2.5% and 9.4%, respectively. This clearly indicates that the existing theoretical natural frequency of the damper has not been estimated correctly, leading to mistuning of the system.

The natural frequencies of the LCVAs with inserts types II and III are recalculated by Eqn. (2.10). It can be observed from Table 1 that the discrepancies of prediction of the natural frequencies obtained from the elliptical flow path estimation method are 0.6% and 2.1%, respectively which are close to the discrepancies of prediction of the natural frequencies obtained from the panel method. This reveals that the simplified effective length approximations in previous studies cannot represent the actual flow in the dampers with a relatively large size of the transition boundaries, while the elliptical flow path estimation method and the panel method yield the more realistic approximation.

3.3 Verifications of results from elliptical flow path estimation method with the experimental values in the case of LCVAs with small transition zones (corner-to-corner width to horizontal length ratio in the range of 0.04 – 0.20)

The bottom 5 LCVAs configurations shown in Table 1 are part of the first series of the Hitchcock’s experiments [13]. These dampers have the same horizontal fluid column length of 820 mm and vertical fluid column height of 180 mm, but different area ratios, which are 0.82, 1.39, 2.14, 3.14 and 4.11, respectively. The corner-to-corner width to horizontal length ratios range from 0.04 to 0.13. These dampers can represent the dampers with small transition zones and they are also simulated by the panel method for their characteristics [14] and their natural frequencies are also calculated by Eqn. (2.10) as shown in Table 1.

In Fig. 8, the test results of 20 LCVAs dampers obtained by Hitchcock [13] are compared with those computed based on the numerical panel method and the elliptical flow path estimation. It may be seen from the figure that the discrepancy between the measured and the calculated natural frequencies as proposed by Hitchcock [13] varies from about 1% to approximately 7%, while the discrepancy between the measured and the calculated natural frequencies based on the numerical panel method varies from less than 0.5% to 3%.

The numerical panel method clearly provides a better prediction of the natural frequencies of liquid motions for various LCVAs configurations. It should be noted, however, that the panel method is much more complicated than the other methods. For the elliptical flow path estimation, the discrepancy between the measured and the calculated natural frequencies varies from 1% to 7%. This means that when the transition zones are small, the elliptical flow path estimation yields the natural frequencies closer to those obtained from the existing analytical methods (Gao and Kwok [3] or Chang and Hsu [9], Hitchcock [13]).
Table 1: The damper’s effective lengths and natural frequencies from various procedures

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Figure 8 Variation of LCVA natural frequencies with area ratio and vertical column height.
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4. Conclusions

The elliptical flow path estimation method is proposed to simulate the vibration characteristics of the TLCDs and LCVAs with large transition zones. The governing equation for the motion of the liquid inside the LCVAs is developed to simulate their vibration characteristics. From the simulation results in case of the LCVAs with large transition zones, poor agreements on the prediction of the natural frequencies are observed between the analytical results based on the simplified effective length approach in existence and the experimental values. The numerical panel method is also a versatile and powerful tool for analysis and design of various configurations of liquid column dampers. Due to its complication in formulation process, however, the elliptical flow path estimation seems to be the best choice to simulate the vibration characteristics of the LCVAs with large transition zones.

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Reference