

IDENTIFICATION OF FIXED-BASE AND RIGID BODY FREQUENCIES OF VIBRATION OF SOIL-STRUCTURE SYSTEMS FROM RECORDED RESPONSE WITH MINIMUM INSTRUMENTATION

M.I. Todorovska¹

¹ Research Professor, Dept. of Civil Eng., Univ. Southern California, Los Angeles, California 9089-2531, U.S.A.
Email: mtodorov@usc.edu, URL: http://www.usc.edu/dept/civil_eng/Earthquake_eng/

ABSTRACT :

Civil engineering structures are founded on soils, which are not infinitely rigid, and alter the structural dynamic response by modifying its frequencies of vibration and introducing additional degrees of freedom and dissipation. The energy of the vibrational response of the coupled soil-structure system is concentrated around the frequencies of vibration of the *system*, which depend on the properties of the soil, structure and foundation. Consequently, observed changes in the frequencies of vibration, as identified via Fourier type of analyses, can be due to changes in any of these quantities. For structural health monitoring, it is essential to be able to isolate the changes in the fixed-base frequencies - task thought to be impossible unless the structural instrumentation had been specially designed, which is not the case for most instrumented structures. This paper shows how the fundamental fixed-base frequency of a structure deforming primarily in shear can be measured, using impulse response functions, and then the rigid body rocking frequency can be computed using a relationship between fixed-base, rigid-body and system frequencies. This requires data from only two sensors recording horizontal motion - at ground level and at the roof. Results are shown for the NS response of Millikan Library in Pasadena, California, during several earthquakes between 1970 and 2003, which reveal to which degree the observed “wonder” of this building NS system frequency has been due to structural deterioration vs. nonlinear elastic behavior of the building and nonlinear behavior of the foundation soil.

KEYWORDS: Structural health monitoring; structural identification; fixed-base frequency of vibration; impulse response analysis; soil-structure interaction; rigid body frequency of vibration; Millikan library.

1. INTRODUCTION

Long term seismic monitoring of structures has demonstrated that their resonant frequencies of vibration (as determined by Fourier analysis, which are those of the soil-structure system, and depend on the properties of the structure, soil and foundation) can vary significantly from one earthquake to another and with time. Udwardia and Trifunac (1974) showed that these frequencies drop during strong shaking, but recover partially or totally. Recoverable changes, not related to damage, appear to reach and exceed 20% (Trifunac et al., 2001a,b; Todorovska et al., 2006). Unfortunately, for the majority of significant recordings, it has been difficult to tell to what degree the observed changes have been due to change of the properties of the soil and foundation as opposed to the structure, because of inadequate instrumentation to separate the effects of the soil-structure interaction. For the same reason, it has been also difficult to estimate, directly from earthquake observations, the fixed-base frequencies of the structure. Distinguishing between fixed-base and system frequency is important because the observed frequencies are often erroneously used to calibrate the stiffness of structural models (leading to underestimation of the structural stiffness), and to infer the change in the health of a structure.

Recently, Todorovska (2008a) showed how both the building fixed-base frequency f_1 and rigid-body rocking frequency f_R can be estimated using earthquake response data from only two horizontal sensors, one at the roof and the other one at the base, extending the usability of structural response data from past earthquakes. This paper presents an application to the NS earthquake response of Millikan library (Fig. 1) during four earthquakes, for which data was available for both basement and roof response (more detailed analysis can be found in Todorovska 2008b). Millikan library is a 9-story reinforced concrete building in Pasadena, California, instrumented over a

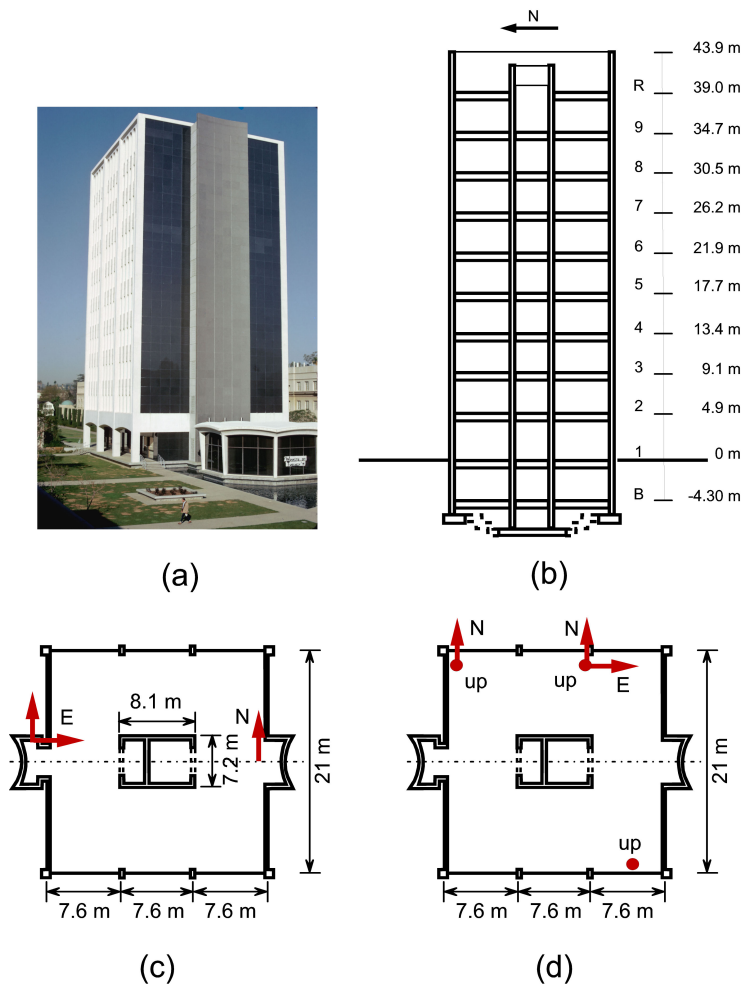


Fig. 1 Millikan library: a) photo (taken by M. Trifunac); b) vertical cross-section and c) typical floor layout (redrawn from Snieder and Şafak, 2006); d) sensor locations at basement.

entirely on earthquake records. Clinton et al. (2006) also documented the variability of the resonant (i.e. system) frequencies of the building, for ambient noise excitation, due to environmental effects (strong winds, heavy rainfall, and temperature), and changes in mass. Finally, wave propagation through the building has also been studied, for small earthquake excitation (Loma Linda, 2002, earthquake), and wave travel times through the structure have been measured using impulse response functions (Snieder and Şafak, 2006).

2. METHODOLOGY

The method for estimation of the fixed-base and rigid-body rocking frequencies is based on measuring the wave travel time from the base to the top of a building using impulse response functions (Snieder and Şafak 2006), and on a relationship between f_1 , f_R and $f_{1,appar}$, where $f_{1,appar}$ is the system resonant frequency obtained from transfer-function between the roof and base horizontal responses, which assumes rigid foundation (Luco et al. 1987). The method is described in detail and proven on numerically simulated data using a 2D soil-structure interaction model with coupled horizontal and rocking motion in Todorovska (2008a). The fixed-base frequency f_1 is estimated using the relation

$$f_1 = 1/(4\tau) \quad (1)$$

long period of time (for 40 years), and tested extensively. Structural deformation patterns obtained from detailed ambient vibration tests showed that, for its NS response, rigid foundation model may be appropriate, and that as much as 30% of its roof response can be accounted for by the rigid body rocking, suggesting significant soil-structure interaction effects (Foutch et al., 1975; Luco et al., 1987). Its vibrational properties have been identified from forced vibration test data, considering the effects of soil-structure interaction (Luco et al., 1986, 1987, 1988; Wong et al., 1988), which provides an independent reference point to compare the results of this study. The San Fernando, 1971, earthquake, produced a significant drop of $f_{1,appar}$ (Udwadia and Trifunac, 1974), which did not recover completely to its pre-earthquake value. The cause of the permanent change has been attributed to degradation of the structural stiffness based on ambient and forced vibration tests before and after this earthquake (Luco et al., 1987). Since the 1971 earthquake, smaller drops of its system frequencies have been observed over time in data from many forced vibration tests, documented most recently in Clinton et al. (2006). These results represent other independent reference to check the consistency of the results in this paper, which are based

where τ is the wave travel time from the base to the top, measured from impulse response functions. This relation is based on the assumption that the building deformation is primarily in shear, and that its stiffness and mass are distributed uniformly along the height. Such estimation of f_1 differs from Snieder and Şafak (2006). The rigid-body rocking frequency f_R is estimated based on the relation (Luco et al., 1987)

$$\frac{1}{f_{1,\text{sys}}^2} \approx \frac{1}{f_H^2} + \frac{1}{f_R^2} + \frac{1}{f_1^2} \quad (2)$$

where f_H and f_R are the *horizontal* and *rocking* rigid-body frequencies, and $f_{1,\text{sys}}$ is the fundamental *system* frequency. If $f_H \rightarrow \infty$ (infinite foundation horizontal stiffness), then $f_{1,\text{sys}} \rightarrow f_{1,\text{app}}$, the apparent building frequency. This special case of Eqn (2) is

$$\frac{1}{f_{1,\text{app}}^2} \approx \frac{1}{f_R^2} + \frac{1}{f_1^2} \quad (3)$$

Eqn (2) is based on the assumption that the foundation does not deform. Frequency $f_{1,\text{app}}$ is estimated from the peaks of the transfer function between the roof response and the response of the building at ground level, which corresponds to the condition $f_H \rightarrow \infty$. Given f_1 and $f_{1,\text{app}}$, f_R is computed from Eqn (3).

The wave travel time τ is measured from impulse response functions obtained by deconvolution of the roof motion with the motion of the ground (Snieder and Şafak 2006)

$$h(t; \xi = H) = FT^{-1} \left\{ \hat{u}(\omega, \xi = H) / \hat{u}_{\text{ref}}(\omega) \right\} (t) \quad (4)$$

with $\hat{u}_{\text{ref}}(\omega)$ being the Fourier transform response at ground level, $\hat{u}(\omega, \xi)$ being the Fourier transform of the response at some level ξ measured from ground level, and FT^{-1} indicates inverse Fourier transform. Then $h(t; \xi = H)$ corresponds to the roof response to excitation that results in horizontal impulse at ground level (ideally a Dirac delta-function $\delta(t)$). Because $\delta(t)$ is zero at all t except at $t=0$, this corresponds to zero horizontal motion of the foundation, i.e. $f_H \rightarrow \infty$ but $f_R < \infty$ (Todorovska 2008a). For the earthquake records, a regularized version of eqn (4) was used (Snieder and Şafak 2006).

3. RESULTS AND ANALYSIS

3.1 The Building and Data

Millikan library is a 9-story RC building in Pasadena, California, instrumented since 1967. It has plan dimensions 21×23 m, and vertically extends 43.9 m above grade and 48.2 m above basement level. Resistance to lateral forces in the NS direction is provided by RC shear walls on the east and west sides of the building. The RC central core houses the elevators and provides resistance to lateral forces in the EW direction. The foundation system is composed of a central pad 9.75 m wide by 1.2 m deep, which extends across the building to the shear walls on the east and west ends. The local soil can be characterized as alluvium, with average shear

wave velocity in the top 30 meters of about 300 m/s, and depth to “bedrock” of about 275 m. The alluvium consists of medium to dense sands mixed with gravels, and the water table appears to be at about 11 m depth (Luco et al., 1986;

Table 1. Earthquakes recorded in Millikan Library analyzed in this study

No.	Name	Data Source	Date	M	R km	PGA cm/s ²	PGV cm/s	θ_{max} 10 ⁻³ rad
1	Lytle Creek	Caltech	12 Sep 1970	5.3	57	54	4.1	0.065
2	San Fernando	Caltech	9 Feb. 1971	6.6	31	301	25	0.516
3	Whittier-Narrows	CDMG	1 Oct. 1987	6.1	19	543	64	1.866
4	Yorba Linda	USGS	3 Sep. 2002	4.8	40	.8	0.58	0.012
5	San Simeon	USGS	22 Dec 2003	6.4	323	14.2	1.92	0.035

Clinton et al., 2006). Fig. 1 shows (a) photo of the building, (b) NS cross-section, (c) typical floor plan, and (d) basement showing the location of the sensors and orientation.

The first earthquake recorded in the building was the Lytle Creek of 1970, which produced small amplitude response. It was followed by the San Fernando of 1971, which caused a significant drop of its resonant frequencies (Udwadia and Trifunac, 1974). Many other earthquakes were recorded in the building over the past 40 years, most notably the Whittier-Narrows earthquake of 1987 ($M_L=5.9$, $R=19$ km), the Northridge earthquake of 1994 ($M_L=6.4$, $R=34$ km) and their aftershocks.

At the time of this study, only data from five of the earthquakes were available both at basement and roof level. These earthquakes are listed in Table 1. The last three columns show roof peak acceleration, roof peak velocity and the rocking angle $\theta(t)$ (sum of rigid body rocking and rocking due to deformation of the structure). Fig. 2 shows $\theta(t)$ vs. time, which can be used as an estimate of the average drift of the

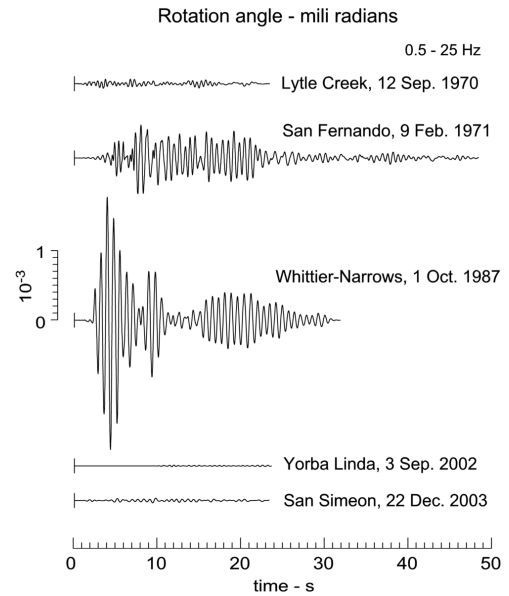


Fig. 2 Roof NS rocking angle.

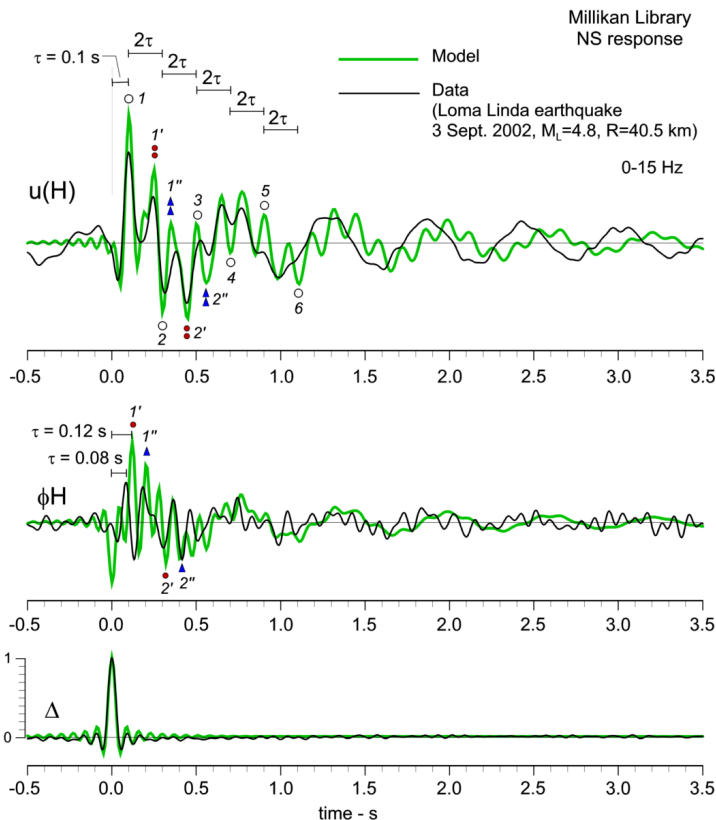


Fig. 4 Comparison of impulse response functions for model and small earthquake.

beam 44 m high, and with fundamental *fixed-base* frequency $f_1 = 2.5$ Hz (chosen by trial and error to match the observed system frequency).

building (including drift due to rigid body rocking) (e.g. $\theta(t)=10^{-3}$ rad corresponds to drift of 0.1%). The peak drift during the San Fernando earthquake was 0.052% and during Whittier-Narrows earthquake it was 0.187%, which approaches but is less than the drift considered to cause damage of moment resistant frames (0.2%; Ghobarah, 2004).

3.2 Comparison of System Functions

Figs. 3 and 4 show comparisons of system functions, respectively computed in the frequency and time domains, for the soil-structure interaction model in Todorovska (2008a) and for recorded small earthquake data (Loma Linda earthquake of 2002). The model parameters are same as those in Todorovska and Al Rjoub (2006, 2008) and Todorovska (2008a,b), chosen to correspond *approximately* to the NS response of Millikan library. In that model, the foundation is semi-circular with radius $a = 12$ m, and the soil is homogeneous elastic half-space with Poisson ratio 0.3 and shear wave velocity 300 m/s. The building is a uniform shear

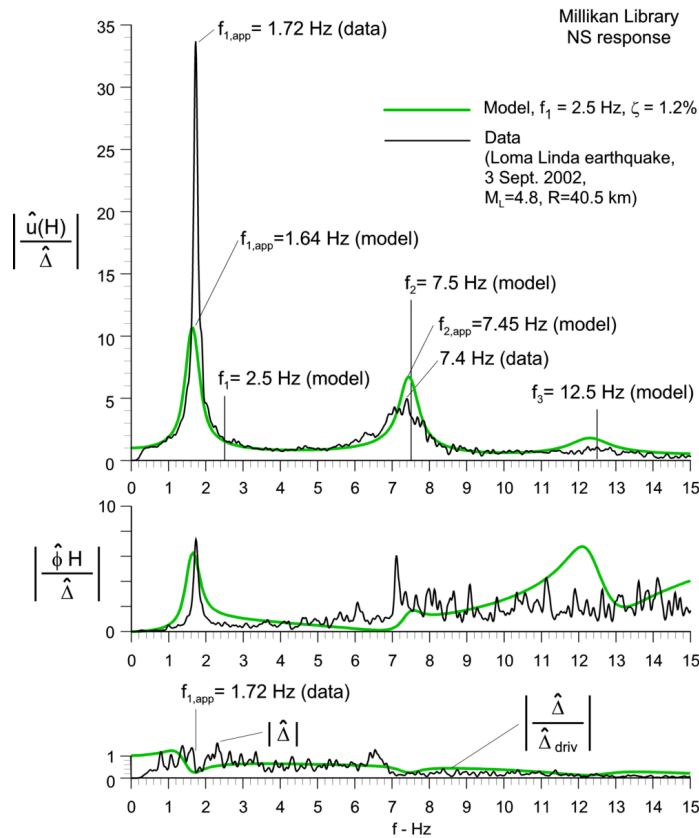


Fig. 3 Comparison of transfer-functions for model and small earthquake.

Fig. 3 shows comparison of transfer-functions, where Δ_{driv} is the foundation horizontal driving motion, Δ is the resultant horizontal displacement of the foundation at ground level (sum of the driving motion and motion due to feedback forces), φ is the foundation rocking, and H is the height to the top sensor (hence φH represents the horizontal roof displacement due to foundation rocking). The model gave $f_R \approx 2.055$ Hz, and $f_{1,\text{app}} \approx 1.64$ Hz. The transfer functions for the earthquake data gave $f_{1,\text{app}} \approx 1.72$ Hz. There is good qualitative agreement between the approximate theoretical and observed transfer-functions.

Fig. 4 compares theoretical and experimental impulse response functions for “input” impulse at ground level, obtained from transfer-functions windowed between 0 and 15 Hz, for a meaningful comparison, because the earthquake did not excite the higher modes. The impulse first arrivals in the *horizontal* roof responses agree well, and give wave travel time between *ground level* and *roof* $\tau \approx 0.1$ s. The different pulses marked by symbols are explained in Todorovska (2008a,b).

3.3 Fixed-base, Rigid-body and Apparent Frequencies during Earthquakes

For each earthquake, $f_{1,\text{app}}$ was measured from the first peak of the transfer-function between *roof* and *basement* responses, f_1 was estimated from the wave travel time τ between *basement* and *roof* using eqn (1), f_R was then computed using eqn (3). For the San Fernando earthquake, these frequencies were computed for six segments (SF1: 0–3.5 s; SF2: 3–6.5 s; SF3: 6.5–10 s; SF4: 10–15 s; SF5: 15–22 s; and SF6: 22–50 s), and for the Whittier-Narrows earthquake for three segments (WN1: 2.2–7.2 s; WN2: 7.2–15 s and WN3: 15–30 s). Percentage changes in f_1 , $f_{1,\text{app}}$ and f_R relative to their values during the Lytle Creek earthquake.

The results are presented graphically in Fig 5a,b,c,d. Parts a, b and c show f_1 , f_R and $f_{1,\text{app}}$ versus θ_{max} , while part d shows a correlation plot of the percentage changes in f_R and f_1 . Each data point is represented by the corresponding symbol for the earthquake/segment. For this set of earthquakes, $0.07 \text{ mrad} < \theta_{\text{max}} < 1.87 \text{ mrad}$, $2.12 \text{ Hz} < f_1 < 3.05 \text{ Hz}$ (30% variation), $1.44 \text{ Hz} < f_R < 2.47 \text{ Hz}$ (42% variation), and $1.19 \text{ Hz} < f_{1,\text{app}} < 1.92 \text{ Hz}$ (38% variation).

Fig. 5a shows that f_1 dropped by 24% during the first 10 s of shaking by the San Fernando earthquake (segments SF2 and SF3), and increased slightly during the subsequent smaller amplitude response. It further dropped during the first 7 s of the Whittier-Narrows earthquake, but much less (by 8.5%) considering the large increase in amplitudes of response (θ_{max} reaching 1.87 mrad), and recovered with decreasing response during the small amplitude Yorba Linda earthquake following practically the same path. The two trends of variation of f_1 vs. θ_{max} are shown by thick fuzzy lines, drawn by hand (the first through points LC, SF1, SF2 and SF3, and the

second through points SF4, SF6, WN1 and YL), which indicate permanent change in f_1 due to structural degradation caused by the San Fernando earthquake, and amplitude dependent and mostly recoverable (within the accuracy of the estimates) change of f_1 for the cracked structure during the subsequent earthquakes. Therefore, no significant additional degradation of stiffness occurred during the Whittier-Narrows earthquake.

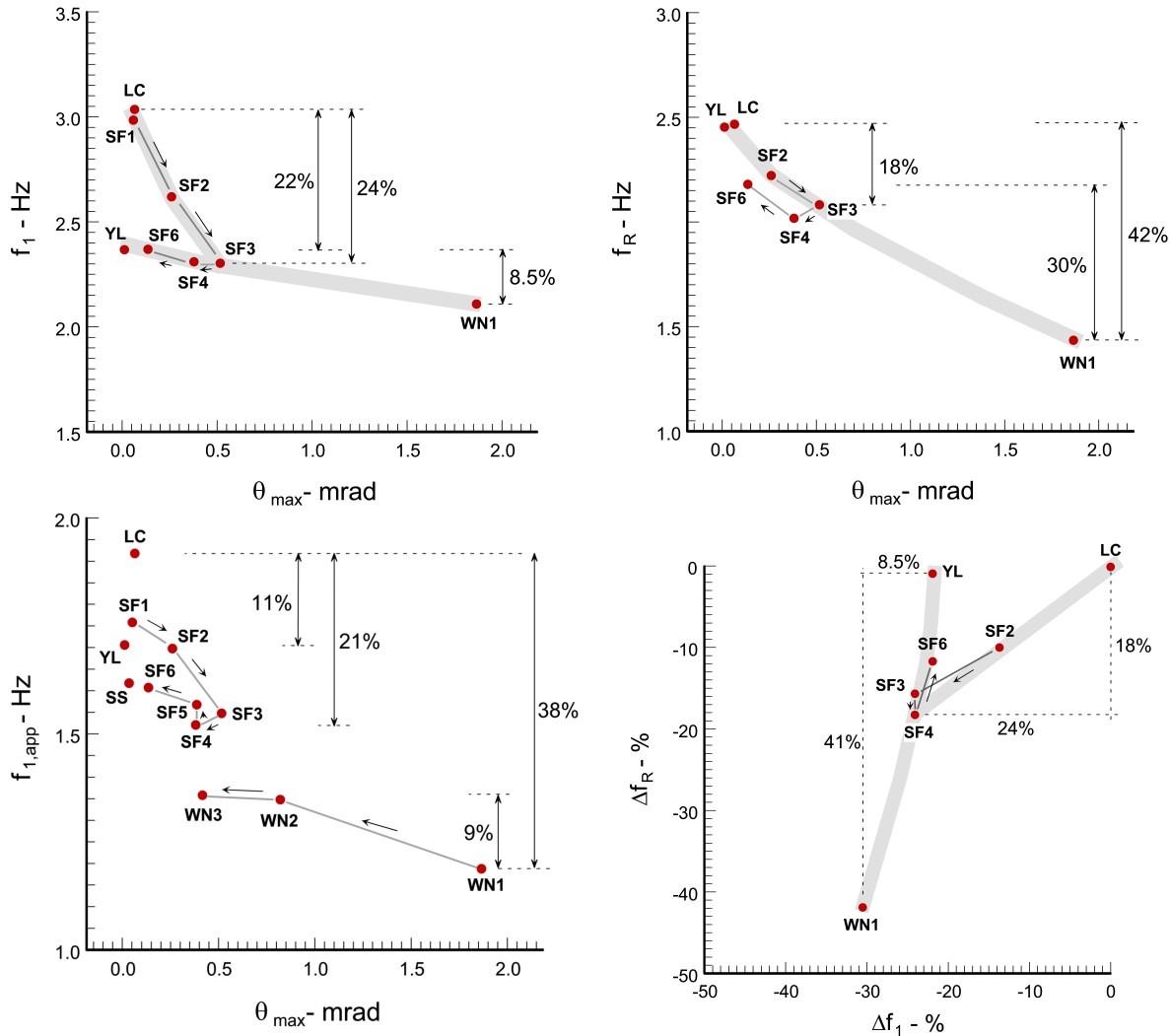


Fig. 5 (a) Fixed-base frequency (NS) vs. level of response. (b) Rigid body rocking frequency (NS) vs. level of response. (c) Apparent frequency (NS) vs. level of response. (d) Percentage change of rocking frequency (NS) vs. percentage change of fixed-base frequency (NS) for four earthquakes.

Fig. 5b shows that f_R decreased by 18% with increasing θ_{max} during the first 10 s of shaking by the San Fernando earthquake (segments SF2 and SF3), continued to decrease during segment SF4 although θ_{max} decreased, and recovered partially during segments SF6, as θ_{max} continued to decrease. During the Whittier-Narrows earthquake, it dropped markedly, by 30%, and recovered during the Yorba Linda earthquake approximately to its value during the Lytle Creek earthquake. This pattern suggests that, in the long term, f_R recovered completely, but the recovery was not instantaneous (as for f_1 , see Fig. 5a) following the strong shaking during the initial 10 s of the response to the San Fernando earthquake. The total change of f_R was 42%.

Fig. 5c shows that $f_{1,app}$ dropped during the San Fernando earthquake and recovered partially towards the end of shaking. Then it dropped further during the initial 7 s of the response to the Whittier-Narrows earthquake, recovered partially towards the end of the shaking, and further recovered during the smaller Yorba Linda and San Simeon earthquakes. The total change of $f_{1,app}$ was 38%.

The data from the Lytle Creek and Yorba Linda earthquakes, which caused similar amplitude responses, give an opportunity to examine permanent changes in f_1 , f_R and $f_{1,app}$ over the period 1970 to 2002. The data shows no change of f_R , change in f_1 of -22%, and change in $f_{1,app}$ of -11%. The much smaller change in $f_{1,app}$, which represents the combined effect of change in f_1 and f_R , compared to the change in f_1 alone, is due to the nature of their combination rule (see eqn (3)). These changes in f_1 and $f_{1,app}$ correspond to about -39% change in the overall structural stiffness and about -21% change in the equivalent stiffness of the structure and the “rocking soil spring”. Hence, observing change in $f_{1,app}$ instead of f_1 will underestimate the changes in structural stiffness.

An interesting question is to what degree the observed changes in $f_{1,app}$ of this building have been due to changes in f_1 as opposed to f_R , during the San Fernando earthquake and after. Fig. 5d shows that, between the Lytle Creek earthquake and San Fernando earthquake (segment SF4), f_1 dropped by 24% and f_R dropped by 18%, while $f_{1,app}$ dropped by 21% (see Fig. 5c). Between the Whittier-Narrows earthquake (segment WN1) and the Yorba Linda earthquake, f_1 changed (recovered) by 8.6%, f_R by 41%, while $f_{1,app}$ by 27%. This suggests that f_1 and f_R changed comparably during the San Fernando earthquake, when degradation of the structural stiffness occurred (with the change in f_1 being slightly larger), while after the San Fernando earthquake, the observed changes in $f_{1,app}$ have been to a much larger degree (by a factor of almost 5) due to changes in f_R .

4. CONCLUSIONS

The trends in the variations of the NS fixed-base (f_1) and rigid body rocking (f_R) frequencies of Millikan library during four earthquakes (1970 - 2002) suggest the following. All variations are relative to the values during the Lytle Creek earthquake of 1970. (1) Both f_1 and f_R are amplitude dependent, (2) significant permanent reduction of frequency occurred over the years, ~22% for f_1 and 11% for $f_{1,app}$, mostly caused by the San Fernando earthquake of 1971, while (3) the changes of f_R have been amplitude dependent and recoverable. (4) During the San Fernando earthquake, both f_1 and f_R dropped, respectively by ~24% and ~18%, resulting in 21% drop of $f_{1,app}$. (5) After this earthquake, the changes in the observed resonant frequencies (which are those of the system) have been to a much larger degree (4-5 times) due to changes of f_R than of f_1 . (6) The small permanent changes in f_1 that appear to have occurred after the San Fernando earthquake cannot be deciphered with certainty because of the small number of earthquakes recorded since 1971, and because strong motion records from the period 1988 to 2002 have not been released.

The analysis and conclusions of this study are preliminary and based on data from only four earthquakes. Other earthquakes have also been recorded, e.g. Sierra Madre, 1988, M=5.8, R=18 km; Northridge, 1994, M=6.7, R=34 km and its aftershocks; Beverly Hills, 2001, M=4.2, R=26 km, listed in Clinton et al. (2006). The Landers (M=7.5) and Big Bear (M=6.5) earthquakes of 1992 should have also been recorded. Unfortunately, no data recorded by the CR-1 array after 1987 has been digitized and released. Once these and other future earthquake records become available, it will be possible to refine and to verify the trends discussed in this paper.

ACKNOWLEDGEMENTS

The digitized records of the Lytle Creek and San Fernando earthquakes were made available for this study by M.D. Trifunac, of the Whittier-Narrows earthquake – by the California Geological Survey, and of the Loma Linda and San Simeon earthquakes by the U.S. Geological Survey.

REFERENCES

1. Clinton JF, Bradford, SK, Heaton TH, and Favela J. (2006). The observed wander of the natural frequencies in a structure. *Bull. Seism. Soc. Am.*; **96**(1):237-257.
2. Foutch DA, Luco JE, Trifunac MD, Udwardia FE. (1975). Full scale three-dimensional tests of structural deformations during forced excitation of a nine-story reinforced concrete building, *Proc. U.S. National Conf. on Earthquake Engineering*, Ann Arbor, Michigan, 206-215.
3. Ghobarah A (2004). On drift limits associated with different damage levels, *Proc. International Workshop on Performance-Based Seismic Design Concepts and Implementation*, Bled, Slovenia, 28 June – 1 July 2004, P. Fajfar and H. Krawinkler, Eds., PEER Report 2004/05, Pacific Earthquake Engineering Research Center, University of California, Berkeley, California.
4. Kawakami H, Oyunchimeg M. (2004). Wave propagation modeling analysis of earthquake records for buildings. *J. of Asian Architecture and Building Engineering*; **3**(1):33-40.
5. Luco JE, HL Wong, MD Trifunac (1986). Soil-structure interaction effects on forced vibration tests, Technical Report 86-05, University of Southern California, Department of Civil Engineering, Los Angeles, California.
6. Luco JE, Trifunac MD, Wong HL (1987). On the apparent change in the dynamic behavior of a nine-story reinforced concrete building, *Bull. Seism. Soc. Am.*, **77**(6): 1961-1983.
7. Luco JE, Trifunac MD, Wong HL (1988). Isolation of soil-structure interaction effects by full-scale forced vibration tests, *Earthqu. Eng. Struct. Dyn.*, **16**: 1-21.
8. Snieder R, Şafak E. (2006). Extracting the building response using interferometry: theory and applications to the Millikan Library in Pasadena, California, *Bull. Seism. Soc. Am.*, **96**(2):586-598.
9. Todorovska MI (2008a). Seismic interferometry of a soil-structure interaction model with coupled horizontal and rocking response, *Bull. Seism. Soc. Am.*, accepted for publication.
10. Todorovska MI (2008b). Fixed-base and rigid body rocking frequencies of Millikan library NS response during four earthquakes (1970-2002), *Bull. Seism. Soc. Am.*, submitted for publication.
11. Todorovska MI, Al Rjoub Y (2006). Effects of rainfall on soil-structure system frequency: examples based on poroelasticity and a comparison with full-scale measurements, *Soil Dyn. Earthq. Eng.*, **26**(6-7): 708-717.
12. Todorovska M.I., Y. Al Rjoub (2008). Environmental effects on measured structural frequencies – model prediction of short term shift during heavy rainfall and comparison with full-scale observations, *Structural Control and Health Monitoring*, in press, DOI: 10.1002/stc.260.
13. Todorovska, MI, M.D. Trifunac (2008a). Earthquake damage detection in the Imperial County Services Building III: analysis of wave travel times via impulse response functions, *Soil Dyn. Earthq. Eng.*, **28**(5), 387–404.
14. Todorovska MI, Trifunac MD. (2008b). Impulse response analysis of the Van Nuys 7-storey hotel during 11 earthquakes and earthquake damage detection, *Struct. Control Health Monit.* **15**(1), 90-116.
15. Todorovska MI, Trifunac MD, Hao TY (2006). Variations of apparent building frequencies-lessons from full-scale earthquake observations, *Proc. First European Conference on Earthquake Engineering and Seismology* (a joint event of the 13th ECEE & 30th General Assembly of the ESC), Geneva, Switzerland, 3-8 September 2006, Paper No.: 1547, pp. 9.
16. Trifunac MD, Ivanović SS, Todorovska MI (2001a). Apparent periods of a building I: Fourier analysis, *J. of Struct. Engrg.*, ASCE, **127**(5), 517-526).
17. Trifunac MD, Ivanović SS, Todorovska MI (2001b). Apparent periods of a building II: time-frequency analysis, *J. of Struct. Engrg.*, ASCE, **127**(5), 527-537.
- [1] Trifunac MD, Todorovska MI, Manić MI, Bulajić BD (2008). Variability of the fixed-base and soil-structure system frequencies of a building – the case of Borik-2 building, *Structural Control and Health Monitoring*, DOI: 10.1002/stc.277, in press.
18. Udwardia FE, Trifunac MD (1974). Time and amplitude dependent response of structures, *Earthqu. Eng. Struct. Dyn.*, **2**, 359-378.
19. Wong HL, Trifunac MD, Luco JE (1988). A comparison of soil-structure interaction calculations with results of full-scale forced vibration tests, *Soil Dyn. Earthq. Eng.*, **7**(1), 22-31.