A GENERATION METHOD OF SIMULATED EARTHQUAKE GROUND MOTION CONSIDERING PHASE DIFFERENCE CHARACTERISTICS

T. Yamane\(^1\) and S. Nagahashi\(^2\)

\(^1\) Senior Structural Engineer, Structural Engineering Group, Nikken Sekkei Ltd., Tokyo, Japan
\(^2\) Professor, Dept. of Architecture and Civil Engineering, Chiba Institute of Technology, Chiba, Japan
Email: yamanet@nikken.co.jp, nagahashi.sumio@it-chiba.ac.jp

ABSTRACT:
A method of simulating earthquake ground motions for testing the structural design of a building is proposed. In this method, a seismic wave is synthesized by combining the Fourier amplitude prescribed by the model with the Fourier phase from observation records selected on the basis of the standard deviation of the phase differences, which reflects the source and propagation characteristics of an earthquake. The proposed method can simulate seismic waves for different types of earthquakes. The dynamic behavior of a high-rise building subjected to earthquake ground motions having a long-period component was found to differ from that of a building subjected to a crustal earthquake in the vicinity of a construction site. This finding suggests the importance of considering the earthquake environment around construction sites and using simulated earthquake ground motions to test the structural designs of buildings.

KEYWORDS: Waveform generation method, Simulated earthquake ground motion, Phase difference

1. INTRODUCTION
Performance-based design has been employed recently for the structural design of buildings in Japan. The basic concepts of performance-based design can be summarized as (i) examine the performance of each building, and (ii) clearly explain the results to the public. To verify resistance to earthquakes, the earthquake environment, soil conditions at the construction site, etc., must be considered and the dynamic behavior of each building evaluated. Since step-by-step integration is performed when evaluating the dynamic behavior of high-rise or base-isolated buildings, in order to satisfy the basic concepts of performance-based design, the dynamic analyses must include the time histories of the acceleration of seismic waves, which reflect the magnitude of the earthquake, the hypocentral distance, and the soil conditions. Thus, performance-based design requires a practical method of simulating earthquake ground motions that can be easily applied to a variety of projects.

The purpose of this paper is to propose a method of simulating earthquake ground motions for testing the structural design of buildings. In this method, the Fourier amplitude is prescribed by the \(\omega^2\) model, and the Fourier phase is selected from observation records. The phase characteristics are quantitatively evaluated by adopting the standard deviation of the phase differences as an index of the properties of the hypocentral region, including the rupture process, and the dispersion during wave propagation. Relational expressions of the standard deviation of the phase differences against the hypocentral distance, which are obtained by analyzing the observation records, are employed as criteria for selecting the Fourier phase.

The seismic waves of inter-plate and crustal earthquakes synthesized by the proposed method differ in the shape of their wave envelopes and their frequency characteristics. Furthermore, numerical analyses of high-rise buildings using these waves show that the dynamic behavior of a high-rise building subjected to an inter-plate earthquake with a long-period component differs from that of a building subjected to a crustal earthquake in the vicinity of the construction site. This finding suggests the importance of using simulated earthquake ground motions to evaluate the dynamic behavior of buildings in performance-based design. Although this paper focuses on a method for generating a body wave, a method for generating a surface wave (Love wave) in a sedimentary basin also has been developed using the same framework. These synthesized Love waves have been applied to the design of a tower about 600 m in height that currently is being constructed in Tokyo.
2. PHASE DIFFERENCE CHARACTERISTICS OF EARTHQUAKE GROUND MOTIONS

2.1. Definition of Phase Difference Characteristics

As is well known, a discrete time history, \( f(t) \), is expressed by a finite Fourier series:

\[
f(t) = \sum_{k=0}^{N/2} a_k \cos(\omega_k t + \phi_k)
\]  

(2.1)

where \( a_k, \phi_k, \omega_k \), and \( N \) denote the Fourier amplitude, the Fourier phase, an angular frequency of the \( k \)th degree, and the number of discrete data points, respectively, and providing \( \omega_0 = \phi_0 = \phi_{N/2} = 0 \) based on Fourier expansion. The difference in the Fourier phase for any adjacent frequencies, which are arranged so as to have the value of an interval \([0, -2\pi]\), is defined as the phase difference (Ohsaki, 1979):

\[
\Delta \phi_k = \phi_{k+1} - \phi_k \quad (k = 1, 2, \cdots, N/2 - 1)
\]  

(2.2)

Examples of a distribution of the phase differences that is represented as a histogram by dividing the interval \([0, -2\pi]\) into 32 are shown in Figure 2.1. These examples suggest that the convergence of the phase differences corresponds to the envelope of the acceleration waveform. It also suggests that this feature can be used when considering the characteristics of a time history, i.e., the phase characteristics. This study proposes the adoption of the standard deviation of the phase differences as an index, which will permit quantitative evaluations of the phase characteristics. For instance, the standard deviation of the phase differences normalized by \( \pi \) is 0.18 in the case of the example in Figure 2.1(a) and 0.05 in the case of the example in Figure 2.1(b), respectively. Analysis of a number of records confirmed that this index \( \sigma / \pi \) can be used to continuously evaluate the differences in the phase characteristics of earthquake ground motions.

![Figure 2.1 Examples of accelerograms and distributions of the phase differences](image)

2.2. Standard Deviation of the Phase Differences for Crustal Earthquakes

Quantitative evaluations of the phase characteristics apply the standard deviation of the phase differences, \( \sigma / \pi \). For crustal earthquakes, plots of \( \sigma / \pi \) against the hypocentral distance, \( r_0 \), for the records from KiK-net (a system of sending strong-motion data on the Internet in Japan) seismic stations with respect to the 2000 Western Tottori Prefecture Earthquake are shown in Figure 2.2.

![Figure 2.2 Examples of accelerograms and distributions of the phase differences](image)
The plot in Figure 2.2 shows that the longer the hypocentral distance, the greater the standard deviation of the phase differences. In other words, the effect of the dispersion in the wave propagation is reflected in the index $\sigma / \pi$. A regression analysis of the data shown in Figure 2.2 gives the following relational expression.

$$\sigma / \pi = 0.06 + 0.0003 r_0$$  \hspace{1cm} (2.3)$$

In the proposed method, Eqn. 2.3 is used as the criteria for selecting the Fourier phase from a number of records of earthquake ground motions in the case of crustal earthquakes. The value of $\sigma / \pi$ is based on the hypocentral distance under the given conditions. Thus, the appropriate Fourier phases having the specified value of $\sigma / \pi$ are selected from records of earthquake ground motions.

### 2.3. Standard Deviation of the Phase Differences for Inter-plate Earthquakes

For inter-plate earthquakes, the relational expressions are determined by analyzing the observation records for the 2003 Tokachi-oki Earthquake. Figure 2.3(a) shows $\sigma / \pi$ with respect to the NS component of the records from K-NET (another system similar to KiK-net) seismic stations in Hokkaido.

![Figure 2.3 Standard deviation of the phase differences at the K-NET observation sites in Hokkaido with respect to the 2003 Tokachi-oki Earthquake](image)

Assuming the strike slip, i.e., the direction of rupture toward the northeast on the fault plane of this earthquake, the data is analyzed in each area divided by the solid line in Figure 2.3(a). $\sigma / \pi$ of the observed data in the eastern and western areas are plotted against $r_0$ in Figures 2.3(b) and (c), respectively. An analysis of these data produces the following relational expressions.

- In the direction of rupture propagation: $\sigma / \pi = 0.05 + 0.0003 r_0$  \hspace{1cm} (2.4)$$
- In the orthogonal direction of rupture propagation: $\sigma / \pi = 0.08 + 0.0003 r_0$  \hspace{1cm} (2.5)$$
It should be noted that Eqns. 2.4 and 2.5 are applicable in the case of inter-plate earthquakes with a fault length of about 100 km because the fault plane for the 2003 Tokachi-oki Earthquake was about 100 km in length. The equality of the gradients among Eqns. 2.3, 2.4, and 2.5 shows that the effects of dispersion in wave propagation are generally similar and not dependent on the type of earthquake or the direction of propagation. On the contrary, the differences in the intercepts in those equations correspond to the source characteristics of earthquakes, including the rupture process.

3. METHOD OF SIMULATING EARTHQUAKE GROUND MOTIONS

3.1. Amplitude Characteristics

The Fourier Amplitude with respect to the source characteristics is given by the \( \omega^2 \) model (Aki, 1967) as

\[
\Omega_\lambda(f) = M_0 (2\pi f)^2 (f \leq f_c) \\
\Omega_\lambda(f) = M_0 (2\pi f_c)^2 (f > f_c)
\]

where \( \Omega_\lambda(f) \), \( M_0 \), and \( f_c \) denote the source spectrum of acceleration, seismic moment, and corner frequency. The propagation characteristics are written as

\[
\left| \hat{U}(r_0, f) \right| = \frac{R_{0\theta}}{4\pi \rho V_S^2} \frac{1}{r_0} \Omega_\lambda(f)
\]

where \( \left| \hat{U}(r_0, f) \right| \), \( \rho \), \( V_S \), and \( R_{0\theta} \) denote the Fourier Amplitude of acceleration on the bedrock, density and velocity of a shear wave in bedrock, and the radiation pattern coefficient (Kanamori and Anderson, 1975). In this study, \( R_{0\theta} = 0.63 \) (Aki and Richards, 1980). Although the published values in papers or from public institutions are basically utilized in the estimation of \( M_0 \) and \( f_c \), \( f_c = 1/(\pi \tau) \) is also used, where the rise time, \( \tau \), is given by

\[
\tau = 4.3\times10^{-7} M_0^{1/5}
\]

Eqn. 3.3 was formulated from the relationships of \( \tau = 0.5, 1.6, 5.0 \) sec. as \( M_0 = 1.6\times10^{18}, 5.0\times10^{19}, \) and \( 1.6\times10^{21} \) Nm, respectively (Sato, 1979).

3.2. Phase Characteristics

The selection criteria for the phase characteristics are discussed here because the application of the Fourier phase from the observation records is presupposed in the proposed method. For crustal earthquakes, Eqn. 2.3 is used for selecting the phase characteristics from a number of records. In other words, the Fourier phases, which have the value of \( \sigma/\pi \) as specified by Eqn. 2.3 based on the hypocentral distance, are applied when simulating earthquake ground motions. On the other hand, the selection criteria for the Fourier phase for inter-plate earthquakes that have faults 100 km in length are given by Eqn. 2.4 in the direction of rupture propagation and by Eqn. 2.5 in the orthogonal direction. Based on Eqns. 2.4 and 2.5, the relationship of \( \sigma/\pi \) at the observation sites to a fault plane typically can be represented as shown in Figure 3.1, including the interpolated evaluation in the oblique directions.
As for the site characteristics of the phase differences, the shape of the distribution of the phase differences at the surface of the ground has been confirmed to be similar to that in seismic bedrock, as shown in Figure 3.2. In other words, the values of $\sigma/\pi$ for those waves are almost the same. Thus, the property of the phase differences is fairly independent of the site characteristics. Consequently, the Fourier phase of the time history recorded at the surface of the ground can be used to synthesize a wave in seismic bedrock.

3.3. Flow Chart of the Proposed Method

A seismic wave in the bedrock below a construction site is synthesized by combining the Fourier amplitude and Fourier phase, as described above. The simulated earthquake ground motion at the surface of the ground can be obtained by applying a multiple reflection theory of the horizontally layered soil model to the seismic wave in the bedrock. A flow chart of this method is shown in Figure 3.3.

[Diagram of Flow Chart]
4. VERIFICATION OF THE PROPOSED METHOD

The proposed method was validated by comparing the synthesized waves with observed waves. Figure 4.1 shows the observed waves at Yubara, Okayama Prefecture, during the 2000 Western Tottori Prefecture Earthquake, the synthesized waves generated by this method, and their response spectra. In this simulation, \(1.3 \times 10^{19}\) Nm was adopted as \(M_0\) (Sagiya et al., 2002) and 2.1 sec. as \(\tau\) (Satoh, 2002). For the phase characteristics, a Fourier phase with \(\sigma / \pi\) of 0.07, which is evaluated by Eqn. 2.3 based on \(r_0 = 31\) km, was selected from records of the same earthquake taken at a different observation site. Figure 4.1 shows that the synthesized waves can appropriately simulate the observed waves in terms of the maximum values, the shapes of the wave envelopes, and the frequency characteristics.

The other verification is shown in Figure 4.2, which shows a comparison of the waves at Sapporo, Hokkaido, during the 2003 Tokachi-oki Earthquake. In this simulation, \(1.0 \times 10^{19}\) Nm was adopted as \(M_0\) (Earthquake Research Institute, University of Tokyo, 2003) and 4.3 sec. as \(\tau\) based on Eqn. 3.3. The Fourier phase with \(\sigma / \pi\) of 0.17, which is evaluated by Eqn. 2.5 based on \(r_0 = 286\) km, was selected from observation records for the 2000 Western Tottori Prefecture Earthquake. In other words, a record for a different type of earthquake is applied. As shown in Figure 4.2, the proposed method can simulate not only the duration of a wave but also the long-period component, which appears in the velocity waveform (Figures 4.2(c) and (d)). Though an \(f_{\text{max}}\) (Hanks, 1982) of 10Hz is used in this method, the \(f_{\text{max}}\) in the case of Sapporo is assumed to be 2Hz by considering the reduction in the high-frequency components due to the reflection of waves in the thick deposits of the sedimentary basin. This is not a significant problem because dynamic analysis of a high-rise or base-isolated building does not require an accurate estimate for frequencies higher than 2 Hz.
5. EXAMPLES OF SIMULATED EARTHQUAKE GROUND MOTIONS

Examples of simulated earthquake ground motions for inter-plate and crustal earthquakes are described in this section. The ground motion of an inter-plate earthquake at Chiyoda-ku, Tokyo, is synthesized after the Great Kanto Earthquake in 1923, which had a moment magnitude, seismic moment, and fault length of $M_w = 7.9$, $M_o = 8 \times 10^{38} \text{Nm}$, $L = 130 \text{km}$, respectively (Wald and Somerville, 1995). $\sigma / \pi = 0.09$ in consideration of the positional relationship of the site to the fault plane shown in Figure 3.1. On the other hand, for the example of a crustal earthquake with a fault length of 17 km, $M_w = 6.5$ ($M_o = 6.2 \times 10^{38} \text{Nm}$). Assuming that this earthquake would occur near the site, i.e., the hypocentral distance is 0 km in Eqn. 2.3, $\sigma / \pi$ is prescribed for 0.06. Furthermore, the proposed method provides that the lower bound of $r_0$ in Eqn. 3.2 is the fault length, namely the upper bound of Fourier amplitude in this equation is given thus. Table 5.1 is a list of the soil conditions at Chiyoda-ku, Tokyo.

Table 5.1 Soil conditions at Chiyoda-ku, Tokyo

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Thickness (m)</th>
<th>Velocity of shear wave (m/s)</th>
<th>Density (t/m$^3$)</th>
<th>Damping factor (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>130</td>
<td>520</td>
<td>1.88</td>
<td>2.0</td>
</tr>
<tr>
<td>130</td>
<td>260</td>
<td>690</td>
<td>1.93</td>
<td>2.0</td>
</tr>
<tr>
<td>390</td>
<td>1170</td>
<td>940</td>
<td>2.01</td>
<td>0.5</td>
</tr>
<tr>
<td>1560</td>
<td>1010</td>
<td>1500</td>
<td>2.19</td>
<td>0.5</td>
</tr>
<tr>
<td>2570</td>
<td>-</td>
<td>3110</td>
<td>2.71</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Figure 5.1 shows the synthesized waves of the scenario earthquakes mentioned above, the response spectra of those waves, and the distribution of the phase differences of observed waves selected in these simulations. The data in Figure 5.1 confirms that these waves differ considerably in the shape of the wave envelope and frequency characteristics.

Numerical analyses of high-rise buildings were performed by these synthesized waves. Time histories of the displacement at the top of a 60-story building having a height of 240 m, a fundamental period of 6.0 sec., and damping factor of 2% are shown in Figure 5.2. Reflecting the difference in the shapes of the wave envelopes and the frequency characteristics shown in Figure 5.1, the high-rise building was vigorously shaken for a long time by the inter-plate earthquake (Figure 5.2(a)). This was because the synthesized wave had the long-period component exited by the thick deposits of the sedimentary basin in the Kanto Plain (Figures 5.1(a) and (c)). On the other hand, the vibration of the building subjected to a crustal earthquake in the vicinity of the construction site gradually attenuated in a manner similar to free oscillation (Figure 5.2(b)). This finding suggests that it is important for designers to consider the differences in the properties of various types of earthquake ground motions.
6. CONCLUSIONS

Relational expressions of the standard deviation of the phase differences with respect to earthquake ground motions against the hypocentral distance exhibit the source and propagation characteristics of seismic waves. On the basis of those expressions, a method of simulating earthquake ground motions for testing the structural designs of buildings is proposed. In this method, a seismic wave is synthesized by combining the Fourier phase selected from observation records by means of a quantitative evaluation using those expressions and the Fourier amplitude prescribed by the $\omega^2$ model. The proposed method was validated by comparing the synthesized wave with observed earthquake ground motions. Examples of the simulated earthquake ground motions for inter-plate and crustal earthquakes indicate differences in those waves in terms of the shape of their wave envelopes and their frequency characteristics. Furthermore, time history analyses using these synthesized waves show that a high-rise building subjected to an inter-plate earthquake with a long-period component excited in a sedimentary basin will shake vigorously for a long time. On the other hand, a high-rise building subjected to a crustal earthquake in the vicinity of a construction site will experience vibrations that gradually attenuate in a manner similar to free oscillation. In performance-based design, it is important to consider the earthquake environment around the construction site and appropriately apply simulated earthquake ground motions to the dynamic analyses of the buildings.

REFERENCES

Earthquake Research Institute, University of Tokyo (2003). http://www.eri.u-tokyo.ac.jp/sanchu/Seismo_Note/EIC_News/030926.html