AMD BASED ACTIVE STRUCTURAL VIBRATION CONTROL USING $H^\infty$ ROBUST DESIGN

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ABSTRACT:

A mathematical model of any real system is always just an approximation of the true, physical reality of the system dynamics. There are always uncertainties in the system modeling. This paper outlines a general approach to the design of an $H^\infty$ control of an Active Mass Damper (AMD) for vibration reduction of a building with mass and stiffness uncertainties. Linear fractional transformation (LFT) is introduced in this paper for uncertainties modeling. To facilitate computation of the $H^\infty$ controller, an efficient solution procedure based on Linear Matrix Inequalities (LMI), or the so-called LMI problem, is employed. The controller uses the acceleration signal for feedback. A two-story building model with an AMD is used to test the designed $H^\infty$ controller. Earthquake ground motion is introduced by a shaking table. A pair of diagonal shape memory alloy (SMA) wire braces are installed in the first floor to introduce stiffness uncertainty to the structure by controlling the temperature of the SMA wire brace. Masses are added to the structure to introduce mass uncertainty. Experiments were conducted and the results validate the effectiveness of the proposed $H^\infty$ controller in dealing with stiffness and mass uncertainties.

KEYWORDS: Active Mass Damper, Robust Control, Linear fractional transformation, Seismic Response, $H^\infty$ Control.

1. INTRODUCTION

The structural control system is commonly classified by its device type resulting in three general control types: passive, active and semi-active. Active control systems have the ability to adapt to different loading conditions and to control vibration modes of the structure (Housner et al. 1997). The most commonly investigated active control devices is Active Mass Damper (AMD) which is developed by introducing an active controlled actuator in Tuned Mass Damper (TMD). In 1989, the Kyobashi Seiwa Building in Tokyo, Japan was constructed using an AMD making it the first building in the world to use active structural (Kobori et al. 1991). However, there are still some practical and important problems to be solved such as system instability due to structural modeling error.

The $H^\infty$ approach is advantageous since it can address both attenuation of disturbances and perturbation of unstructured parameters (Chen and Patton 1999). Doyle et al. (1989) developed state-space formulas for all controllers that solve a standard $H^\infty$ problem, thereby making a significant breakthrough in $H^\infty$ control. $H^\infty$ design methods may be found in many references, such as Kwakernaak (1987), Doyle et al. (1989), Wang et al. (1995a, 195b, 1998), and Zhou et al. (1996). $H^\infty$ control methods provide controllers with robustness to external disturbances and system parameter uncertainty, such as modeling errors and system parameter perturbations. Hence, these controllers can guarantee stability and optimized vibration suppression performance despite insufficient or inaccurate knowledge of the structural system parameters. Recent formulations of the $H^\infty$ control problem in terms of linear matrix inequality (LMI) allow computationally efficient and systematic design of robust controllers.
This paper presents the application of the $H^\infty$ control theory to designing controllers for structures with AMD taking into account of mass and stiffness uncertainties. For synthesizing the $H^\infty$ controller we use a linear matrix inequality (LMI) technique (Iwasaki and Skelton 1994; Gahinet and Apkarian 1994). More precisely, the $H^\infty$ control problem can be formulated as a minimization problem of a non-convex function, subject to convex constraints expressed by a system of LMIs. The controller uses the acceleration signal for feedbacks. A two-story building model with an AMD is used to test the designed $H^\infty$ controller on a shaking table. To study the robustness of the $H^\infty$ controller, a diagonal shape memory alloy (SMA) wire brace is installed in the first floor to adjust the stiffness of the structure when the SMA is heated. Masses can be added to the structure to alter its mass matrix. Experiments are conducted to validate the effectiveness of the proposed $H^\infty$ controller in dealing with stiffness and mass uncertainties.

2. MODELING OF AMD CONTROL SYSTEMS WITH UNCERTAINTIES

A building structure implemented with an AMD and subjected to earthquake can be described as:

$$M_s \ddot{X} + C_s \dot{X} + K_s X = -M_s E_s \ddot{x} + E_s u(t)$$  \hspace{1cm} (2.1)

where $M_s$, $C_s$, and $K_s$ are respectively mass, damping and stiffness matrix of the system. $X$ is displacement vector of the system. $u(t)$ is the control force generated by actuator. $\ddot{x}$ is the acceleration of ground. $E_s$ and $E_t$ are coefficient matrices with appropriate dimensions.

In a realistic AMD control system, the three physical parameters of $M_s$, $C_s$, and $K_s$, in equation (2.1) are not known exactly. However, they can be assumed that their values are within certain known intervals. That is

$$M_s = \bar{M} + P_M \delta_M$$
$$C_s = \bar{C} + P_C \delta_C$$
$$K_s = \bar{K} + P_K \delta_K$$ \hspace{1cm} (2.2)

where $\bar{M}$, $\bar{C}$ and $\bar{K}$ are so called nominal values of $M_s$, $C_s$ and $K_s$. $P_M$, $P_C$ and $P_K$ represent the possible perturbations on those parameters. $\delta_M$, $\delta_C$ and $\delta_K$ are diagonal with uncertain values, however, their values belong to [-1, 1]. $I$ is unity matrix.

We note that the quality of $M_s^{-1}$ can be represented as a linear fractional transformation (LFT) in $\delta_M$

$$M_s^{-1} = \bar{M}^{-1} - \bar{M}^{-1} P_M \delta_M (I + P_M \delta_M)^{-1} I = F_u(M_s, \delta_M)$$ \hspace{1cm} (2.3)

Similarly, the parameter $C_s = (I + P_C \delta_C) \bar{C}$ may be represented as an upper LFT in $\delta_C$

$$C_s = F_u(M_C, \delta_C)$$ \hspace{1cm} (2.4)

with $M_C = \begin{bmatrix} 0 & \bar{C} \\ \bar{C} & \bar{C} \end{bmatrix}$ and the parameter $K_s = (I + P_K \delta_K) \bar{K}$ may be represented as an upper LFT in $\delta_K$

$$K_s = F_u(M_K, \delta_K)$$ \hspace{1cm} (2.5)
Then, the input/output dynamics of the control system which takes into account the uncertainty of parameters can be denoted by $G$. The state space representation of $G$ is

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$  \hspace{1cm} (2.6)$$

Let $\Delta = \text{diag}(\delta_m, \delta_c, \delta_s)$, $u_\Delta = [u_m, u_c, u_s]$ and $y_\Delta = [y_m, y_c, y_s]$. Then, the effects of uncertain structural parameters on the system $G$ can be expressed by

$$u_\Delta = \Delta \cdot y_\Delta$$  \hspace{1cm} (2.7)$$

3. FORMULATION OF LMI BASED $H_\infty$ CONTROL FOR AMD-STRUCTURE SYSTEM

In this section, we will transform the computation of optimal controller for AMD-structure system into a general $H_\infty$ control frame. The weighting functions are used to the system performance. The advantages of using weighted functions are obvious in controller design. First, some components of a vector signal are usually more important than others. Second, each component of the signal may not be measured in the same units. Also, we might be primarily interested in rejecting errors in a certain frequency range. Hence, some frequency-dependent weights must be chosen to get high performance controller (Zhou et al. 1996).

A detailed block diagram representation of the system is depicted in Figure 2. The frequency domain weighted function $W_p$ shapes the spectral content of the earthquake excitation. $W_p$ is used to weight the measurement noise $v$. The block $T_m$ is a constant matrix that dictates the regulated response transformed from the measurement vector. The matrix weighting function $W_1$ and $W_2$ are frequency dependent, with $W_1$ weighting regulated response and $W_2$ weighting the control signal. $K$ is the controller to be designed which can generate a control signal $u$ according to the measured response $y$. The input excitation $w$ consists of earthquake excitation $\dot{x}_r$ and measurement noise $v$. The output $z$ comprises frequency weighted regulated response and control signal. The regulated response denotes the quantities of the design interest which can be floor acceleration, floor displacement or base shear force etc. The rectangle with dash line in Figure 2 represents the augmented system model $P$.

$H_\infty$ control methods provide controllers with robustness to external disturbances and system parameter uncertainty, such as modeling errors and system parameter perturbations. Hence, these controllers can guarantee stability and optimized vibration suppression performance despite insufficient or inaccurate knowledge of the structural system parameters. The formulations of the $H_\infty$ control problem in terms of Linear Matrix Inequality (LMI), which allows computationally efficient and systematic design of robust controllers (Skelton et al. 1998), is used here. Using the linear fractional transformation (LFT), the vibration control of structures with AMD can be represented by

$$\dot{x}_r = A_x x_r + B_x u + D_x w$$
$$z = C_x x_r + B_x u + D_x w$$
$$y = C_y x_r + D_y w$$  \hspace{1cm} (3.1)$$

where $x_r$ is state variables of augmented system model $P$. Let the transfer matrix from $w$ to $z$ be denoted by
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$T_p$. The $H_\infty$ norm of the control system is defined as $\|T_p\|_\infty = \sup_{\omega} \sigma(T_p(j\omega))$ and the $H_\infty$ control problem here is to design a controller $K$ to make the closed-loop system stable and $\|T_{pK}\|_\infty$ minimal. The LMI formulation of the $H_\infty$ control problem results in an efficient optimization method that can handle large-scale systems. LMI optimization problems are convex, leading to computationally efficient global optimal solutions.

Consider the system with order $n_k$ that has the state space representation of (3.1). The following theorem is useful to design a controller $K$ of order $n_k$ which is less than or equal to $n_c$ with the following state space representation,

$$\begin{align*}
\dot{x}_c &= A_c x_c + B_c y \\
u &= C_c x_c + D_c y
\end{align*}$$

(3.2)

Where $x_c$ is the state vector of controller, $u$ is the control signal, $y$ is the output of the generalized plant shown in equation (3.1), $A_c$, $B_c$, $C_c$ and $D_c$ are real matrices of appropriate dimensions.

Assembling the equations (3.1) and (3.2) yields to

$$
\begin{bmatrix}
\dot{x}_c(t) \\
\dot{x}_i(t)
\end{bmatrix} =
\begin{bmatrix}
A_c + B_c D_c y & B_c C_c \\
B_c C_c & A_i
\end{bmatrix}
\begin{bmatrix}
x_c \\
x_i
\end{bmatrix} +
\begin{bmatrix}
B_c D_r + D_r \\
B_r
\end{bmatrix} w
$$

(3.3)

or simply

$$
\begin{align*}
\dot{x}(t) &= A_c x(t) + B_c w(t) \\
z(t) &= C_c x(t) + D_r w(t)
\end{align*}
$$

(3.4)

The notation to be used as follows: Given a real matrix $N$, the orthogonal complement $N^\perp$ is defined as the (possibly non-unique) matrix with maximum row rank that satisfies $N^\perp N = 0$ and $N^\perp N^\perp > 0$ (Skelton et al. 1998). Hence, $N^\perp$ can be computed from the singular value decomposition of $N$ as follows: $N^\perp = U_2^T$ where $T$ is an arbitrary nonsingular matrix and $U_2$ is defined from the singular value decomposition on $N$

$$
N = [U_1 \hspace{1cm} U_2]
$$

(3.5)

To design an $H_\infty$ controller using LMI for system expressed by equation (3.4), one must find a matrix pair $(X_p, Y_p)$ with dimension of $n_p \times n_p$ such that the following statements are satisfied for a given scalar $\gamma > 0$:

$$
\begin{bmatrix}
X_p \\
Y_p
\end{bmatrix} \geq 0, \quad \text{rank} \begin{bmatrix}
X_p & \gamma I \\
\gamma I & Y_p
\end{bmatrix} \leq n_p + n_c
$$

(3.6)

$$
\begin{bmatrix}
B_r \\
B_r
\end{bmatrix} \begin{bmatrix}
A_r x_r + x_r A_r^T + D_r D_r^T & x_r C_r + D_r D_r^T \\
C_r x_r + D_r D_r^T & D_r D_r^T - \gamma^T I
\end{bmatrix} \begin{bmatrix}
B_r \\
B_r
\end{bmatrix}^T < 0
$$

(3.7)

$$
\begin{bmatrix}
C_r \\
D_r
\end{bmatrix} \begin{bmatrix}
Y_r A_r + A_r^T Y_r + C_r^T C_r & Y_r D_r + C_r^T D_r \\
D_r^T Y_r + D_r^T C_r & D_r^T D_r - \gamma^T I
\end{bmatrix} \begin{bmatrix}
C_r \\
D_r
\end{bmatrix}^T < 0
$$

(3.8)
One can compute a matrix factor \( Y_p \) and \( Y_p > 0 \) according to \((X_p, Y_p)\) such that
\[
Y_p^{-1}Y_p^T = Y_p - \gamma^2 X_p^{-1}
\]
and construct \( Y \) and \( X \) as
\[
Y = \begin{bmatrix} Y_p \\ Y_p \end{bmatrix}, \quad X = \gamma^2 Y^{-1}
\]

Then, the controller is given by
\[
K = -R^{-1}T^T \Phi \Lambda (\Lambda \Phi \Lambda^T)^{1/2} + S^{1/2}L(\Lambda \Phi \Lambda^T)^{1/2}
\]

The proof of equations (3.11) can be found in the reference (Skelton et al., 1998).

4. SHAKING TABLE TEST OF A FLEXIBLE STRUCTURE WITH AMD

To demonstrate the applicability of the robust \( H_{\infty} \) controller presented here, shaking table test of a small-scale two-story structure with AMD as shown in Figure 3 was conducted in the Smart Materials and Smart Structures Laboratory at University of Houston. The structural inter-story height is 490mm. The first floor mass is 1.16 kg and the second one is 1.38 kg. The structure has the natural frequencies of 1.4 Hz and 4.4 Hz. A moving cart driven by a DC motor is installed on the top of structure acting as an AMD control device. The moving cart is driven by a brushless DC motor sliding along a geared rack, which generates control force to the structure. The maximum stroke of the AMD is \( \pm 9.5 \) cm with a total moving mass of 0.65 kg.

The scheme of control system is shown in Figure 4. The structure is controlled by the miniature AMD when it vibrates subjected to shaking table movement excitation. The accelerometer feedback signals on both stories are transferred into computer after amplification and A/D converter. The computer produces a control signal according to the designed control algorithm and feed back signals. The control signal applies on AMD device and generates control force after D/A converter and amplification. The accelerometers used here are manufactured by Quanser Consulting Inc and produce an output of \( \pm 5 \) volts with a range of \( \pm 5g \). Two Universal Power Module are used here as power amplifiers, one of which is used to power shaking table and accept the sensor signals and another one is used to power the AMD. The data acquisition and control board used to collect data and drive the power amplifier is a Q8 extended terminal board manufactured by Quanser Consulting Inc.

To guarantee the safety of structure, the regulated output \( z_1 \) in Figure 2 is selected to be the displacement of the second floor. The weighted function \( W_g \) is chosen as the square root of Kanai-Tajimi spectrum to reflect the frequency content of earthquake
\[
W_g(s) = \frac{1}{\sqrt{\omega_g^2 + 2\zeta_g \omega_g s + \omega_g^2}}
\]

The parameters of Kanai-Tajimi spectrum in this paper are \( S_w = 0.09 \), \( \zeta_g = 0.65 \), \( \omega_g = 18.65 \). For the regulated response \( z_1 \), we are only interested in low frequency response. Hence, the weighted function \( W_1 \) is selected as
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\[ W_i = \frac{5}{(1/40)s + 1} \] (4.2)

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![Figure 1](image1.png) experimental set-up

![Figure 2](image2.png) Scheme of AMD system

The weighted function \( W_i \) and \( W_v \) are set to be \( 5 \times 10^{-4} \) and 1 which means that control signal and noise are weighted in whole frequency region. A 9th order \( H_\infty \) controller is acquired using MATLAB Robust Control Toolbox and it is reduced to 6th order by balanced truncation method to facilitate implementation.

To verify the effectiveness of the designed robust \( H_\infty \) controller, the shaking table test of two-story building model with AMD introduced previously was conducted. The El Centro earthquake record in 1940 with peak value scaled to be 0.2g was input to the shaking table as excitation. The time history of accelerations and displacements for both stories are shown from Figure 5 to Figure 8. It can be seen from these figures that the structural responses, especially the displacements are reduced greatly. The reduction ratios of displacement on first floor and second floor are 51.27% and 62.69%, respectively.

![Figure 3](image3.png) Acceleration history of the first floor

![Figure 4](image4.png) Acceleration history of the second floor

To experimentally investigate the robustness of designed controller, a diagonal shape memory alloy (SMA) wire brace is installed in the first floor to adjust the stiffness of the structure when the SMA is heated. The building model on the shaking table is excited by the same scaled El Centro earthquake with the temperature of SMA wire kept to be 30, 40, 50, 60 and 70°C, respectively. The displacement reduction ratio of second floor with the temperature of SMA wire is shown in Figure 9. It can be seen that the displacement reduction ratio of second floor keeps a high value though the stiffness of the structure varied with temperature of SMA wire,
which verifies the robustness of the designed controller on stiffness uncertainties.

Mass uncertainties are introduced here by adding additional masses on each floor. The following four cases are considered to investigate the influence of mass uncertainties. Case 1: Structural vibration control without additional masses; Case 2: Structural vibration control with 1kg (86% of the first floor mass) additional mass on the first floor; Case 3: Structural vibration control with 1kg (72% of the second floor mass) additional mass on the second floor; Case 4: Structural vibration control with 0.5kg (43% of the first and 36% of the second floor mass) additional mass on both floors.

The building model on shaking table is still excited by the same scaled El Centro and second floor displacement reduction ratio of four cases is shown in Figure 10. It can be seen that the displacement reduction ratio of second floor is nearly unchanged though the structural mass varies a lot, which verifies the robustness of the designed controller on mass uncertainties.

5. CONCLUSIONS

This paper outlines a general approach to the design of an $H_\infty$ controller for Active Mass Damper (AMD) control system. The uncertainties of systems are taken into account by Linear fractional transformation (LFT). To facilitate computation of the $H_\infty$ controller, an efficient solution procedure based on linear matrix inequalities (LMI), the so-called LMI problem, is employed. A controller is designed for a two-story building model with an AMD by LMI method which uses the acceleration signal as feedbacks. The shaking table test of the two-story building model was conducted with 1940 El Centro earthquake record as excitation. The results show that the structural responses are reduced significantly. The stiffness uncertainties are introduced by installing a diagonal shape memory alloy (SMA) wire brace in the first floor. When the SMA wire is heated, it can adjust
the stiffness of the structure. The mass uncertainties are introduced by adding additional masses to the structure. The results show that the displacement reduction ratio of second floor keeps a high level with the changes of structural stiffness and mass. The experiment results verify that the designed control has a good robustness on stiffness and mass uncertainties.

REFERENCES


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<thead>
<tr>
<th>Material</th>
<th>Deflection x</th>
<th>Deflection y</th>
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<tbody>
<tr>
<td>Concrete beams</td>
<td>0.234mm</td>
<td>0.523mm</td>
</tr>
<tr>
<td>Steel beams</td>
<td>0.107mm</td>
<td>0.432mm</td>
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</table>
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Figure 1 Load versus axial strain curves
(a) RHS
(b) RHS

4. EQUATIONS

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\[ X = 2y + 1 \]  \hspace{1cm} (4.1)

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