

# MODE CONTROL SEISMIC DESIGN WITH DYNAMIC MASS

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# **ABSTRACT :**

This paper presents a new response control method for building structures against earthquakes by making use of the inertia element connected between mass points. The device is named as the dynamic mass which generates the force to be proportional to the acceleration difference between mass points. In this paper, two topics are introduced. One of them is the method which the natural period of structure can be elongated keeping its mode shapes. Next topic is the method that can be made all participation factors of higher modes into 0 except the 1st mode. It is very simple to make the suitable distribution of dynamic masses along height which is satisfied to the above situation. As a result, response shear coefficients of all stories become same, because the response for higher modes does not occur.

**KEYWORDS:** Mode Control, Response Control, Dynamic Mass, Seismic Design

# **1. INTRODUCTION**

In general, seismic designs of buildings are performed by adjusting the stiffness and the strength and, in recent years, by artificially giving the damping. The response control by making use of these terms means to design the stiffness factor and the damping factor in the equation of motion. On the other hand, there are studies to control the response by making supplementary masses behave according to the displacement difference between mass points <sup>1]-3]</sup>. These response control methods using the supplementary masses induce characteristic changes of the vibration system like a decrease effect for ground motion input etc., which can not be obtained by only controlling the stiffness and damping.

Rightly, the damping element and the stiffness element each generate the forces to be proportional to the velocity difference and the displacement difference between mass points. After the above expression, the authors newly define "the dynamic mass" that generates the force to be proportional to the acceleration difference between mass points. The high performance "dynamic mass" to be much greater than its actual mass can be realized by using the displacement amplification mechanism such as the rotation mechanism etc. Thus, we can gain a quite new response control method. That is, the three main factors of mass, damping and stiffness in the equation of motion can be adjusted in order to satisfy the engineer's target design performance.

This paper introduces two topics. One is the method which the natural period of structure can be elongated keeping its mode shapes. Next is the method that can be made all participation factors of higher modes 0 except the 1st mode.

# 2. DYNAMIC MSSS

Fig.1 shows the concept of the dynamic mass which is composed of the rotation body with combined the inner wheel and the outer wheel. And mass *m* is concentrated at the outer wheel. Now, we define the amplification ratio  $\beta$  which is the ratio of the radius of the outer wheel to the inner wheel. When the inner wheel is pushed in the direction of the tangent with the acceleration  $\alpha$ , the inertia force of the mass *m* of the outer wheel is  $m\beta\alpha$ . And the reaction force to push the inner wheel becomes  $\beta^2 m \alpha$ . As a result, the mass *m* of the outer wheel can display the magnitude of  $\beta^2 m$  at the position of inner wheel which means the formation of mass amplification device. If  $\beta$  is large enough, this mechanism can be assumed as the dynamic mass that generates the force to be proportional to the acceleration difference between mass points. Now, we express the magnitude of dynamic mass as  $m' (=\beta^2 m)$  in distinction from the real mass *m* of the structure.





Figure 1 Rotation Body

Figure 2 Rotary Damping Tube (RDT)

The practical device of dynamic mass is composed by a little improvement for a viscous damping device called "Rotary Damping Tube" (RDT) which converts an axial movement into the gyration of the inner cylinder with a ball screw, and generates the resistance force from the viscous body filled between the rotating inner cylinder and the fixed cover cylinder. The displacement of the direction of the tangent of the inner cylinder is amplified to about 5 to 40 time of axial displacement. The effect of the inertia mass of the rotating inner cylinder is amplified to the square of the displacement amplification ratio. It becomes 1,000 times or more the mass of the inner cylinder. The dynamic mass device with 1,000 ton has been made for trial purposes, and an experiment was executed successfully<sup>4</sup>.

## 3. SINGLE DEGREE OF FREEDOM SYSTEM WITH DYNAMIC MASS

Consider the single degree of freedom vibration system that the above-mentioned rotation body is built into shown in Figure 3(a), or the single degree of freedom vibration system that has the dynamic mass shown in Figure 3(b). The equation of motion is given by Eqns.3.1, 3.2 and 3.3 below:



Figure 3(a) Single Degree of Freedom Vibration System with Rotation Body



Figure 3(b) Single Degree of Freedom Vibration System with Dynamic Mass (Hereafter, dynamic mass is shown by -∞-.)

$$(m+m')\ddot{x} + c\,\dot{x} + kx = -m\ddot{y} \tag{3.1}$$

$$(m+m')\ddot{x} + c\dot{x} + kx = -(m+m')\eta\,\ddot{y}$$
 (3.2)

$$\ddot{x} + 2h\omega\dot{x} + \omega^2 x = -\eta \, \ddot{y} \tag{3.3}$$

$$\omega^{2} = \frac{k}{m+m'} = \eta \frac{k}{m}, \quad h = \frac{c}{2\omega(m+m')} = \sqrt{\eta} \frac{c}{2\sqrt{km}}, \quad \eta = \frac{m}{m+m'}$$

From Eqn.3.3, it is understood that dynamic mass induces the following characteristic changes to systems.

(1) Elongation of the natural period,

(2) Decrease of damping effect

(3) Decrease effect for acceleration of ground motion.

Considering Eqn.3.1, in case of k=0 and c=0, the absolute acceleration A becomes Eqn.3.4 that is not 0. This means that the ground motion acceleration is transmitted through the dynamic mass. As a result, it appears another vibration mode that represents "acceleration transmitted directly through the dynamic mass". However, this effect doesn't become visible at the left side of Eqn.3.1.

$$A = \ddot{x} + \ddot{y} = -\frac{m}{m+m'}\ddot{y} + \ddot{y} = \frac{m'}{m+m'}\ddot{y} = (1-\eta)\ddot{y}$$
(3.4)

in which,



In order to clearly specify the effect of dynamic mass, the equation of motion for two-mass system, in which a dummy mass may be added to one-mass system as shown in Figure 4, is given by Eqn.3.5 as follows:

$$\begin{bmatrix} m+m' & -m'\\ -m' & m'+m_0 \end{bmatrix} \begin{bmatrix} \ddot{x}_1\\ \ddot{x}_0 \end{bmatrix} + \begin{bmatrix} c & -c\\ -c & c \end{bmatrix} \begin{bmatrix} \dot{x}_1\\ \dot{x}_0 \end{bmatrix} + \begin{bmatrix} k & -k\\ -k & k+k_0 \end{bmatrix} \begin{bmatrix} x_1\\ x_0 \end{bmatrix} = -\begin{bmatrix} m+m' & -m'\\ -m' & m'+m_0 \end{bmatrix} \begin{bmatrix} 1\\ 1 \end{bmatrix} \ddot{y}$$
(3.5)

in which,  $m_0$  is dummy mass,  $m_0 \ll m$ ,  $k_0$  is dummy spring constant,  $k_0 \gg k$ ,  $x_0$  is displacement of dummy mass from the ground,  $x_0 \ll 1$ .



Figure 4 Addition of Dummy Mass

Considering the eigenvalue problem of the left side of Eqn.3.5 under the condition with disregard of the viscous damping term, the eigenvalues  $\omega^2$ , the eigenvectors  $\{u\}$ , the participation factors  $\beta$  and the participation vectors  $\beta\{u\}$  are the following.

$$\omega^{2} = \frac{k}{m+m'} = \eta \frac{k}{m}, \qquad {}_{2}\omega^{2} = \infty$$
(3.6)

$${}_{1}\beta = \frac{\{{}_{1}u\}^{T}[M]\{1\}}{\{{}_{1}u\}^{T}[M]\{{}_{1}u\}} = \eta , \quad {}_{2}\beta = \frac{\{{}_{2}u\}^{T}[M]\{1\}}{\{{}_{2}u\}^{T}[M]\{{}_{2}u\}} = 1$$
(3.8)

$${}_{1}\beta\{{}_{1}u\} = \begin{cases} \eta \\ 0 \end{cases}, \qquad {}_{2}\beta\{{}_{2}u\} = \begin{cases} 1-\eta \\ 1 \end{cases}$$
(3.9)

Now, the symbol  $\eta$  is defined as the decrease effect of input. The 1st mode is "the mode of the response to the decreased input" and the 2nd mode is "the mode of the acceleration that acts directly through dynamic mass".

## 4. TWO DEGREE OF FREEDOM SYSTEM WITH DYNAMIC MASS



Figure 5 2-Mass System with Dynamic Mass

The equation of motion for two-mass system with the dynamic mass shown in Figure 5 is expressed as two-mass system of Eqn.4.1 and as three-mass system with a dummy mass of Eqn.4.2. In the expression of two-mass system Eqn.4.1, an input index vector at the right side of the equation becomes  $\{\eta\}$ . Each element of  $\{\eta\}$  is a value of 1 or less. The 1st mode and the 2nd mode are the modes of the response to the decreased input. In the other expression of three-mass system Eqn.4.2, the vector is  $\{1\}$ , and it has the 3rd mode of the rigid body which directly transmits the acceleration of ground motion in the ratio of the mode shape through the dynamic mass devices.

$$\left[\hat{M}\right]\!\!\left[\hat{x}\right]\!\!\left\{\hat{x}\right\}\!+\left[\hat{C}\right]\!\!\left\{\hat{x}\right\}\!\!\left\{+\left[\hat{K}\right]\!\!\left\{\hat{x}\right\}\!\right\}\!=-\left[\hat{M}\right]\!\!\left\{\eta\right\}\!\!\left[\eta\right\}\!\!\left[\eta\right]\!\!\left\{\eta\right\}\!\!\left[\eta\right]\!\!\left[\eta\right$$



in which,

$$\begin{bmatrix} \hat{M} \end{bmatrix} = \begin{bmatrix} m_2 + m_2 & -m_2 \\ -m'_2 & m'_2 + m_1 + m'_1 \end{bmatrix}, \quad \begin{bmatrix} \hat{C} \end{bmatrix} = \begin{bmatrix} c_2 & -c_2 \\ -c_2 & c_2 + c_1 \end{bmatrix}, \quad \begin{bmatrix} \hat{K} \end{bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 + k_1 \end{bmatrix}$$

$$\{\eta\} = \begin{bmatrix} \hat{M} \end{bmatrix}^{-1} \begin{bmatrix} \hat{M}_0 \end{bmatrix} \{1\}, \quad \begin{bmatrix} \hat{M} \end{bmatrix} = \begin{bmatrix} \hat{M}_0 \end{bmatrix} + \begin{bmatrix} \hat{M}' \end{bmatrix}$$

$$\begin{bmatrix} \hat{M}_0 \end{bmatrix} \text{ is ordinary mass matrix, } \quad \begin{bmatrix} \hat{M}_0 \end{bmatrix} = \begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{M}' \end{bmatrix} \text{ is dynamic mass matrix, } \quad \begin{bmatrix} \hat{M}' \end{bmatrix} = \begin{bmatrix} m'_2 & -m'_2 \\ -m'_2 & m'_2 + m'_1 \end{bmatrix}$$

in which,

$$\begin{bmatrix} M \end{bmatrix} \{\ddot{x}\} + \begin{bmatrix} C \end{bmatrix} \{\dot{x}\} + \begin{bmatrix} K \end{bmatrix} \{x\} = -\begin{bmatrix} M \end{bmatrix} \{1\} \ddot{y}$$
(4.2)  
$$\begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} m_2 + m'_2 & -m'_2 & 0 \\ -m'_2 & m'_2 + m_1 + m'_1 & -m'_1 \\ 0 & -m'_1 & m'_1 + m_0 \end{bmatrix}, \quad \begin{bmatrix} C \end{bmatrix} = \begin{bmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_1 & -c_1 \\ 0 & -c_1 & c_1 \end{bmatrix}, \quad \begin{bmatrix} K \end{bmatrix} = \begin{bmatrix} k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_1 & -k_1 \\ 0 & -k_1 & k_1 + k_0 \end{bmatrix}$$
$$\begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} M_0 \end{bmatrix} + \begin{bmatrix} M' \end{bmatrix}$$
$$\begin{bmatrix} M_0 \end{bmatrix} = \begin{bmatrix} M_0 \end{bmatrix} + \begin{bmatrix} M' \end{bmatrix}$$
$$\begin{bmatrix} M_0 \end{bmatrix} = \begin{bmatrix} m_2 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_0 \end{bmatrix}$$
$$\begin{bmatrix} M' \end{bmatrix} \text{ is ordinary mass matrix, } \begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} m'_2 & -m'_2 & 0 \\ -m'_2 & m'_2 + m'_1 & -m'_1 \\ 0 & -m'_1 & m'_1 \end{bmatrix}, \quad m_0 \ll m_1, \quad k_0 \gg k_1$$

#### 5. RESPONSE CONTROL OF TWO DEGREE OF FREEDOM SYSTEM WITH DYNAMIC MASS

There are some effective uses of the dynamic mass for the response control of the structure against earthquakes.

#### 5.1. Response Control Changing Eigenvalues without Changing Eigenvectors

The natural period can be elongated without changing eigenvectors by adding the dynamic mass proportional to the stiffness of each story of the vibration system. Assuming the following eigenvalue problem from Eqn.4.1, add  $\alpha \cdot \omega^2 [\hat{K}]_{\{,\hat{u}\}}$  to the both sides of the equation.

$${}_{s}\omega^{2}\left[\hat{M}\right]_{\{s}\hat{u}\} = \left[\hat{K}\right]_{\{s}\hat{u}\}$$

$${}_{s}\omega^{2}\left[\hat{M}\right]_{\{s}\hat{u}\} + \alpha {}_{s}\omega^{2}\left[\hat{K}\right]_{\{s}\hat{u}\} = \left[\hat{K}\right]_{\{s}\hat{u}\} + \alpha {}_{s}\omega^{2}\left[\hat{K}\right]_{\{s}\hat{u}\}$$

$${}_{s}\omega^{2}\left(\left[\hat{M}\right] + \alpha\left[\hat{K}\right]\right)_{\{s}\hat{u}\} = (1 + \alpha {}_{s}\omega^{2})\left[\hat{K}\right]_{\{s}\hat{u}\}$$

$$\frac{{}_{s}\omega^{2}}{(1 + \alpha {}_{s}\omega^{2})}\left(\left[\hat{M}\right] + \alpha\left[\hat{K}\right]\right)_{\{s}\hat{u}\} = \left[\hat{K}\right]_{\{s}\hat{u}\}$$

$$\therefore {}_{s}\omega^{2}\left(\left[\hat{M}\right] + \left[\hat{M}^{2}\right]\right)_{\{s}\hat{u}\} = \left[\hat{K}\right]_{\{s}\hat{u}\} \qquad (5.1)$$

in which, dynamic mass matrix  $[\hat{M}'] = \alpha [\hat{K}]$  changes eigenvalue to  ${}_{s} \omega'^{2} = \frac{{}_{s} \omega^{2}}{(1 + \alpha \cdot {}_{s} \omega^{2})}$ 

When the dynamic mass  $[\hat{M}'] = \alpha [\hat{K}]$  is added to the system, the eigenvalues change from  $\omega^2$  to  $\omega^2$  without changing eigenvectors.

#### 5.2. Response Control Adjusting Participation Factor of 2nd Mode to 0

The participation factor of the 2nd mode can be adjusted to 0 by operating eigenvectors by adding the dynamic mass. If the 1st eigenvector of Eqn.4.1 is  $\{\eta\}$ , the 2nd participation factor shown by Eqn.5.2 becomes 0 because of the orthogonality between the eigenvectors. And the response of the 2nd mode disappears.

$${}_{2}\beta = \frac{\{{}_{2}\hat{u}\}^{T} \left[\hat{M}\right] \{\eta\}}{\{{}_{2}\hat{u}\}^{T} \left[\hat{M}\right] \{{}_{2}\hat{u}\}} = 0$$
(5.2)

The required conditions to keep this situation can be solved as shown below.



$$\frac{{}_{2}u_{2}}{{}_{2}u_{1}} = -\frac{m_{1}}{m_{2}}$$

$${}_{2}\omega^{2} = k_{2}\left(\frac{1}{\frac{m_{2} \cdot m_{1}}{m_{2} + m_{1}} + m'_{2}}\right)$$

$$m'_{1} = \frac{k_{1}}{{}_{2}\omega^{2}} = \frac{k_{1}}{k_{2}}\left(\frac{m_{2} \cdot m_{1}}{m_{2} + m_{1}} + m'_{2}\right)$$
(5.3)

in which, the value of  $m'_2$  is arbitrary, including 0.

In this condition, the eigenvector of the 1st mode is  $\{\eta\}$ , and the participation factor of the 2nd mode is 0. The eigenvalue problem of the 1st mode is given by Eqn.5.4.

$$\omega^{2} \left[ \hat{M} \right] \left\{ \eta \right\} = \left[ \hat{K} \right] \left\{ \eta \right\}$$
(5.4)

From Eqn.5.4, considering  $\{\eta\} = [\hat{M}]^{-1} [\hat{M}_0] \{1\}$ , the following relations are obtained.

$${}_{1}\omega^{2} = \frac{1}{\sum_{i=1}^{2} \left(\frac{1}{k_{i}} \sum_{j=1}^{2} m_{j}\right)}$$
(5.5)

$${}_{1}\omega^{2} = \frac{k_{2}(\eta_{2} - \eta_{1})}{m_{2}} = \frac{k_{1} \cdot \eta_{1}}{m_{2} + m_{1}}$$
(5.6)

That is, when the 2nd participation factor is 0 by adding the proper combination of the dynamic mass, the 1st eigenvalue is Eq.5.5. And from Eq.5.6, response shear coefficients of 1st and 2nd stories are the same value.

# 6. MULTI DEGREE OF FREEDOM SYSTEM WITH DYNAMIC MASS



Figure 6 *n*-Mass System with Dynamic Mass

The equation of motion of *n*-mass system with dynamic mass shown in Figure 6 is given by Eqn.6.1. The difference between Eqn.6.1 and a conventional equation of motion, the vector of the right side is not  $\{1\}$  but  $\{\eta\}$ . Each element of  $\{\eta\}$  is a value of 1 or less. This indicates the decrease effect of the input.

$$\begin{bmatrix} \hat{M} \\ \hat{X} \\ \hat{x} \\ + \begin{bmatrix} \hat{C} \\ \hat{x} \\ \hat{x} \\ + \begin{bmatrix} \hat{K} \\ \hat{x} \\ \hat{x} \\ + \begin{bmatrix} \hat{K} \\ \hat{x} \\ \hat{x} \\ + \begin{bmatrix} \hat{K} \\ \hat{x} \\ \hat{x} \\ + \begin{bmatrix} \hat{M}_0 \end{bmatrix} \\ = \begin{bmatrix} \hat{M}_0 \\ \hat{M}_0 \\ \hat{x} \\ \hat{x} \\ + \begin{bmatrix} \hat{M}_0 \\ \hat{x} \\ - \begin{bmatrix} \hat{M}_0 \\ - \begin{bmatrix} \hat{M}_0 \\ \hat{x} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \end{bmatrix} \\ - \begin{bmatrix} \hat{M}_0 \\ - \end{bmatrix} \\ -$$

in which,

in which,





The equation of motion of *n*-mass system with the dynamic mass is also expressed as (n+1)-mass system with a dummy mass. In this expression of (n+1)-mass system, the vector of the right side is  $\{1\}$ , and it has the (n+1)th mode of the rigid body.

7. RESPONSE CONTROL OF MULTI DEGREE OF FREEDOM SYSTEM WITH DYNAMIC MASS

As well as the case of two-mass system, there are some effective uses of the dynamic mass for the response control of the structure to earthquakes.

#### 7.1. Response Control Changing Eigenvalues without Changing Eigenvectors

The natural period can be elongated without changing eigenvectors by adding the dynamic mass proportional to the stiffness of each level of the vibration system. The proof is same as the previous case.

#### 7.2. Response Control Adjusting Participation Factors except the 1st Mode to 0

The participation factors of from 2nd to *n*-th mode can be adjusted to 0 by operating eigenvectors by adding a certain combination of the dynamic mass. The combination of the dynamic mass can be evaluated easily by the following procedure. The detailed procedures can be found in the references <sup>5]-7]</sup>.

$${}_{1}\omega^{2} = \frac{1}{\sum_{i=1}^{n} \left(\frac{1}{k_{i}} \sum_{j=i}^{n} m_{j}\right)}$$
  

$$\eta_{0} = 0$$
  

$$\eta_{i} = \eta_{i-1} + \frac{1}{k_{i}} \sum_{j=i}^{n} m_{j} \qquad (1 \le i \le n)$$

$$(7.1)$$



$$\begin{cases} _{n+1}u \}^{T} = \left\{ \{1 - \eta \}^{T} \quad 1 \right\} \\ D_{i} = \frac{_{n+1}u_{i}}{_{n+1}u_{i-1}} \qquad (1 \le i \le n) \\ m'_{n} = 0 \\ m'_{i} = \frac{m_{i} + m'_{i+1}(1 - D_{i+1})}{\frac{1}{D_{i}} - 1} \qquad (1 \le i \le n - 1) \end{cases}$$

#### 7.3. Example of Response Control with Dynamic Mass

Table 1 shows the process and the result of obtaining the combination of the dynamic mass that adjusts the participation factor of all higher modes of the example vibration system to 0.

Table 2 shows the result of the eigenvalue analysis of the vibration system with the dynamic mass. The participation factors of 2nd to 8th mode are 0. The 9th mode of rigid body (i.e.,  $\omega = \infty$ ) appears, and it has the physical meaning of transmit of the acceleration related to the ground motion.

Figure 7 shows comparisons of time history response analysis results of the original system and the controlled system. The input earthquake motion is El Centro 1940 NS ( $A_{max}$ =510.8 cm/s<sup>2</sup>). The damping is assigned h=0.05 for the first mode in proportion to the stiffness of the original system. By adding the dynamic mass, the first period is elongated from *T*=1.00 [sec] to *T*=1.11 [sec]. The damping factor of the 1st mode has decreased from h=0.05 to h=0.045. As a result of mode control, the response of the higher mode disappears. Response accelerations decrease. Response shear coefficients of all stories are the same. It shows the unique and effective response control.

Table 1 Evaluation of Dynamic Mass T = 11144

	1.00	0.0001	11					
	$_{1}\omega^{2}$	31.788	ł					
i	m <sub>i</sub> (ton)	k <sub>i</sub> (kN/m)	$\sum m_i$ (ton)	$\frac{\Sigma (1/k_i)}{\Sigma m_i}$	$u_i = \eta_i$	$\stackrel{n+1}{=} u_i$	$D_i$	<i>m</i> ' <sub><i>i</i></sub> (ton)
8	750.0	820,000	730.0	0.00091	1.0000	0.0000	0.0000	0
7	760.0	830,000	1,510.0	0.00273	0.9709	0.0291	0.3346	382.09
6	770.0	840,000	2,2800	0.00545	0.9131	0.0869	0.5018	1,031.66
5	780.0	870,000	3,060.0	0.00897	0.8268	0.1732	0.6077	2,004.36
4	790.0	890,000	3,850.0	0.01329	0.7150	0.2850	0.6745	3,267.00
3	800.0	900,000	4,650.0	0.01846	0.5775	0.4225	0.7201	4,793.33
2	850.0	910,000	5,500.0	0.02450	0.4133	0.5867	0.7533	6,693.43
1	900.0	920,000	6,400.0	0.03146	0.2211	0.7789	0.7789	8,985.31
0	-	-	-	-	0.0000	1.0000	-	—

Table 2Eigenvalue Analysis of Controlled System

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Т	1.114	0.621	0.539	0.459	0.381	0.302	0.220	0.135	0
ω	5.638	10.12	11.66	13.70	16.51	20.83	28.53	46.61	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
β	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
$u_8$	1.000	-0.663	-0.663	-0.669	-0.693	-0.733	-0.816	1.000	0.000
$u_7$	0.971	-0.601	-0.580	-0.554	-0.521	-0.442	-0.208	-0.987	0.029
$u_6$	0.913	-0.477	-0.416	-0.325	-0.177	0.137	1.000	0.000	0.087
$u_5$	0.827	-0.293	-0.172	0.015	0.336	1.000	0.000	0.000	0.173
$u_4$	0.715	-0.054	0.145	0.457	1.000	0.000	0.000	0.000	0.285
$u_3$	0.578	0.239	0.535	1.000	0.000	0.000	0.000	0.000	0.423
$u_2$	0.413	0.590	1.000	0.000	0.000	0.000	0.000	0.000	0.587
$u_1$	0.221	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.779
$u_0$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000



Figure 7 Comparisons of Maximum Response



# 8. MODEL VIBRATION EXPERIMENT

A 4-story model vibration experiment that confirms the effect of the mode control effect by the dynamic mass was executed. Figure 8 shows the outline of the vibration model. Participation factors of all higher modes are adjusted to 0 by adding the dynamic mass. As a result of the sine wave excitation experiment, the resonance of higher modes is lost and the response only of the 1st mode is remained. Figure 9 shows the comparison of the absolute acceleration amplification ratio of each mass point between the original system and the controlled system. The absolute acceleration amplification ratio of each mass point of the controlled system agrees with the calculated value very well.



Figure 8 Outline of Vibration Model



Figure 9 Amplification Ratio of Absolute Acceleration

# 9. CONCLUSIONS

The dynamic mass is defined as an element between mass points of vibration systems that generates the force to be proportional to the acceleration difference between mass points. Such an element can be put to practical use with an inertia mass amplification device that uses a displacement amplification mechanism.

Dynamic mass is useful to control response of structures. The characteristic changes are induced to the vibration system, such as the elongation of the natural period, the damping decrease, and the input decrease.

By adjusting the value of the dynamic mass of the each story, the natural period can be changed without changing the eigenvector of the multi degree vibration system and the participation factor of a higher mode can be adjusted to 0. Especially, the combination of the dynamic mass can be evaluated easily, that makes all participation vectors of higher modes zero except the 1st mode.

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