SEISMIC DESIGN OF STEEL CONCENTRICALLY BRACED FRAME SYSTEM WITH SELF-CENTERING FRICTION DAMPING BRACES

S. Zhu[1] and Y. Zhang[2]

[1] *Assistant Professor, Dept. of Civil and Structural Engineering The Hong Kong Polytechnic University, Kowloon, Hong Kong*

[2] *Associate Professor, Dept. of Civil & Environmental Engineering, University of Maryland, College Park, MD, USA*

Email: ceszhu@polyu.edu.hk, zyf@umd.edu

ABSTRACT :

This paper concerns the seismic design procedure for a new type of steel concentrically braced frame system with a special type of bracing element termed self-centering friction damping brace (SFDB). This new seismic resistant system demonstrates a few desirable characteristics such as minimal residual drift and potential to be damage free after frequent and design basis earthquakes. A displacement-based design (DBD) approach using inelastic design spectra is adopted for the design of SFDB frame. Two design examples using the proposed design approach are presented in this study. The statistical responses indicate that the SFDB frames can achieve their target performance, and that SFDB frame has the potential to withstand several design level earthquakes without the need for repair or replacement of SFDBs if properly designed.

KEYWORDS: Damper, Design, Seismic response, Shape memory alloy, Steel frame

1. INTRODUCTION

New trends in seismic design have resulted in proposals of several innovative seismic protection strategies, among which buckling-restrained braces (BRB) and the concept of self-centering system have recently received a great deal of attention recently. Recently, an alternative seismic resisting system with self-centering hysteretic behaviors has attracted considerable interests (e.g., Stanton et al 1997; Ricles et al. 2001; Christopoulos et al. 2002a). A flag-shaped hysteresis loop is typical of such self-centering systems with energy dissipation capability. Self-centering systems can be achieved by utilizing post-tensioning, special energy dissipating dampers or special material such as shape memory alloys. Researchers in the US have studied a family of post-tensioned systems with self-centering capabilities, including steel frame system with post-tensioned moment connections. In general, these systems use gap-opening behavior at selected critical joints between main structural members, along with associated energy dissipation elements, to provide nonlinear softening behavior, ductility, and energy dissipation without significant inelastic deformation and related damage to the main structural members. Elastic restoring forces provided by post-tensioning at these joints return the structure to it original position, eliminating residual drift. Self-centering systems have the ability to control damage and to reduce (or even eliminate) residual structural deformation, after strong earthquake events. It is worth noting that residual structural deformation is emphasized as a fundamental complementary parameter in the evaluation of structural (and non-structural) damage in performance-based seismic design and assessment approaches (Pampanin et al. 2003).

Special metals such as superelastic shape memory alloys (SMA) inherently possess the self-centering hysteretic behavior. Zhu and Zhang (2008) proposed a special type of bracing element termed self-centering friction damping brace (SFDB). The SFDB is capable of re-centering itself based on the use of superelastic SMA wire strands and features enhanced energy dissipation capacity through friction. SFDB would be typically installed in a CBF building as part of the bracing system which resists lateral seismic loads. A detailed description of the mechanical configuration of SFDB can be found in Zhu and Zhang (2008). Previous research by Zhu and Zhang (2008) indicates SFDB frames are capable of achieving a seismic control level comparable to that of the buckling-restrained braced frames, while having significantly reduced residual inter-story drift after earthquakes.

The prominent seismic behavior of SFDB frames, which is quite different from conventional CBF, warrants the need to develop a special design procedure for SFDB frame buildings. This paper presents a displacement-based seismic design approach for SFDB frames, in which the SFDB devices and other structural elements are proportioned to achieve a pre-determined target performance level under design basis earthquakes (DBE). The DBD approach involves the use of inelastic design spectra which are derived a prior. The proposed DBD method was applied to two design examples—a 3-story and a 6-story SFDB frame located in Los Angeles, California—in order to evaluate its effectiveness. The results of nonlinear dynamic analyses of these two prototype buildings suggest that SFDB frames designed using this DBD procedure can fairly well achieve their target performance levels, especially in terms of displacement parameters. The minimized residual drifts of SFDB frames and the reusability of SFDB devices after frequent or design basis earthquakes, which are very appealing as they would considerably reduce post-earthquake repair or replacement cost, are demonstrated by the statistical results.

2. DISPLACEMENT-BASED DESIGN PROCEDURE

Performance-based earthquake engineering (PBEE) has been extensively researched in recent years. In PBEE, performance objectives are expressed as a set of performance levels such as immediate occupancy, life safety or collapse prevention, associated with earthquake ground motions of specific intensities. In order to incorporate performance design criteria, however, conventional force-based seismic design method results in significantly increased design effort such as the addition of displacement check afterwards and successive iteration of the assumed elastic characteristics (Priestley 2000). Consequently, a more rational alternative design procedure, displacement-based design (DBD) procedure, has been developed and advocated in the past decade (Priestley 2000; Kowalsky et al. 1995; Sullivan et al. 2003; Fajfar 2000; Chopra and Goel 2001).

It is noted that the displacement/drift levels in a SFDB frame building, are not only related to non-structural damage, but also govern the ductility level and response behavior of the SFDBs, the main lateral resisting element in frame system. For example, at very large displacement the occurrence of slip at friction surface II would impair the self-centering capability of the SFDB. Consequently it is very likely to result in residual drift in the braced frame structure and significantly increase the repair cost after earthquakes. For this reason, the drift target in the design of SFDB frames is selected as the one of primary performance objectives in this study. Considering the inefficiency of the conventional force-based design method in explicit displacement control, the DBD approach using inelastic design spectra (Chopra and Goel 2001) is adopted as the basis of the proposed seismic design procedure for SFDB frames in this study.

The flow chart of the DBD approach is shown in Figure 1. First a planar MDOF model is defined based on basic design parameters such as story height (i.e., h_i), tributary mass at each floor level (i.e., m_i), etc. Then the target performance level needs to be specified for the SFDB frame. The ductility demand of SFDB (i.e., μ) and maximum inter-story drift ratio (i.e., θ_d) under the design basis earthquake (DBE) are two primary target performance indices in this study. The maximum brace ductility affects the self-centering capability and reusability of SFDBs, and the maximum inter-story drift ratio is a significant measure of structural and non-structural damage. Therefore the selection of limiting values for these two performance indices should consider the performance objectives along with a certain safety margin. A displacement profile of frame building can be approximately estimated based on the maximum inter-story drift ratio and building height.

Assuming the seismic response of the frame building is dominated by the fundamental mode, the planar model can be converted to one equivalent SDOF system with the effective displacement, Δ_e and the effective mass, m_e. The design base shear can determined through the inelastic design spectrum corresponding to specified ductility level, μ. The derivation of inelastic design spectra will be discussed in more details in next section. The length of SMA wires in SFDB can be subsequently determined from the displacement profile and the target ductility of SFDB; while the yield strength of SFDBs, i.e. the cross-sectional areas of SMA wires, are determined from the vertical distribution of lateral seismic forces. The beam and column sections are designed based on the brace strength of SFDB. Finally, the seismic performance levels of the designed SFDB frame under maximum considered earthquakes (MCE) and frequent earthquakes (FE) need to be checked through nonlinear analysis procedures such as pushover analysis or nonlinear time-history analysis. The modification of the design needs to be made if the performance objectives under the MCE and FE are not satisfied.

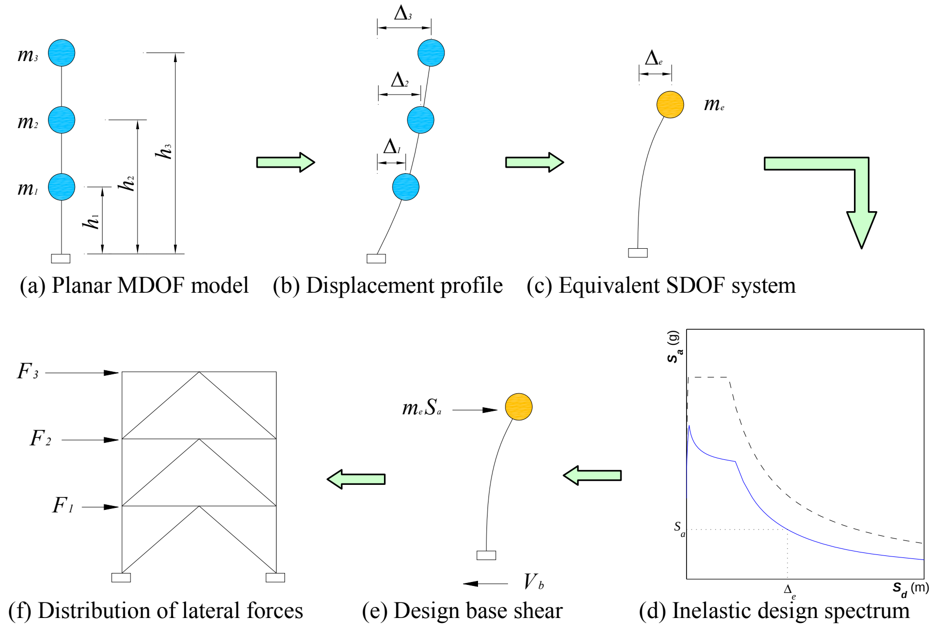


Figure 1 Flow chart of displacement-based seismic design procedure

3. INELASTIC DESIGN OF SPECTRA

3.1. Inelastic SDOF Model

It is assumed that seismic-induced story shear forces are mainly resisted by the bracing elements, i.e., SFDBs, and the contribution from other elements to the lateral resistance is negligible. Therefore inelastic SDOF systems with flag-shaped hysteresis which represents typical self-centering hysteresis behavior of SFDB were studied in this paper. The nonlinear seismic analyses of these SDOF systems utilized the analytical model for SFDB's hysteresis behavior developed by Zhu (2007). This analytical model of SFDB is based on one constitute model for superelastic behavior of SMA wires—the modified Wilde model with the consideration of the effect of friction forces at two different sliding surfaces.

Two different hysteretic behaviors were considered for SDOF system – high damping and low damping to simulate two cases corresponding to SFDB with or without friction dissipation at sliding surface I. The SFDB without friction damping at sliding surface I, designated as SFDB-NF, was studied as a comparative case in order investigate the friction effect. Two essential parameters—initial elastic period, T_0 and strength reduction factor, R are considered in this study, and they govern the initial stiffness and 'yield' strength of hysteresis behavior respectively. The range of the initial elastic period is 0.2 sec $\leq T_0 \leq$ 3.0 sec, which is typical of one- to 20-story steel braced frames; the considered values of the strength reduction factor R are 2, 3, 4, 5, 6, 7 and 8 respectively. One suite of ground motions previously developed by Somerville et al. (1997) was employed in the time history analysis. This suite is corresponding to DBE earthquake level (10% probability of exceedance in 50 years) at downtown Los Angeles, California, and it contains 20 records designated as LA01 – LA20 respectively.

3.2. Inelastic Response Spectra

Seo (2005) proposed smooth-median nonlinear response spectra using contant-R method. Here the smooth median value of sampled data is defined as the geometric mean (Seo 2005)

$$\tilde{x} = \exp\left(\frac{1}{n}\sum_{j=1}^{n}\ln x_j\right)$$

where x_j, j = 1,2, …, n represent the sampled data. The research by Seo (2005) found that the function of contant-R smooth-median nonlinear response spectra can be directly used to derive the function of constant-μ smooth-median nonlinear response spectra.

Therefore the smooth-median nonlinear response spectra were calculated in this study using constant-R method. It is observed that at with the same strength reduction factor, R, the SDOF system with the SFDB-type hysteresis always leads to smaller $\tilde{\mu}_R$ than that of SFDB-NF. This improved control effect on the displacement and brace ductility demand is due to the enhanced energy dissipation in SFDB through friction effect which increases the damping ratio of the SDOF system, especially for small values of R. For example, when $R = 2$, $\tilde{C}_R$ of SFDB system is less than 1 for the period range considered in this study, i.e. 0.2 sec $\leq T_0 \leq$ 3.0 sec, implying that the peak nonlinear displacement is smaller than that of an elastic SDOF system with 5% critical damping ratio.

3.3. Regression Analyses

The regression analyses were carried out on the $\tilde{\mu}_R$ spectra to establish a mathematical function which can be conveniently used in the design procedure. The regression result, denoted as $\hat{\mu}_R$, is usually a function of T_0 and R. The expression of the regression function which is similar to the format proposed by Seo (2005) is adopted in this study to estimate the $\tilde{\mu}_R$ spectra for SDOF systems with SFDB-NF-type hysteresis.

$$\hat{\mu}_R = R^{\exp(0.1653/T_0^{0.83})}$$

The coefficient values in this function were obtained through regression analysis. Interested readers can refer to Zhu (2007) for more details about the regression analysis. As shown by Seo (2005), the regression function $\hat{\mu}_R$ for smooth-median response spectra $\tilde{\mu}_R$ can be directly inverted without knowledge of the dispersion to estimate corresponding $\tilde{R}_\mu$ spectra— constant-μ smooth-median spectra. The resulting function is denoted as $\hat{R}_\mu$. Functions $\hat{R}_\mu$ can also be used to develop functions to estimate the constant-μ smooth-median spectra of nonlinear displacement coefficients, $\tilde{C}_\mu$. The resulting functions for the SFDB-NF system are listed as follows:

$$\hat{R}_\mu = \mu^{\exp(-0.1653/T_0^{0.83})}$$

$$\hat{C}_\mu = \mu/\hat{R}_\mu = \mu^{1-\exp(-0.1653/T_0^{0.83})}$$

Similarly, the regression functions for SDOF system with SFDB-type hysteresis behavior were obtained as follows:

$$\hat{\mu}_R = R^{\exp(0.1353/T_0^{0.8})} - 0.75$$

$$\hat{R}_\mu = (\mu + 0.75)^{\exp(-0.1353/T_0^{0.8})}$$

$$\hat{C}_\mu = \mu/\hat{R}_\mu = \mu/(\mu + 0.75)^{\exp(-0.1353/T_0^{0.8})}$$

With the knowledge of functions $\hat{R}_\mu$ and $\hat{C}_\mu$, the inelastic response spectrum corresponding to pre-determined ductility level, μ can be constructed from the elastic design spectrum through the following relationship

$$S_a = \frac{S_{ae}}{R_\mu} \qquad\qquad S_d = C_\mu S_{de}$$

It should be noted that S_a is the acceleration corresponding to the 'yield' strength of the SDOF system, which is used to determine the required design strength in order to achieve the target performance. Therefore it does not stand for the peak acceleration response during earthquake which is usually larger than S_a due to strain hardening of SFDB at large deformation.

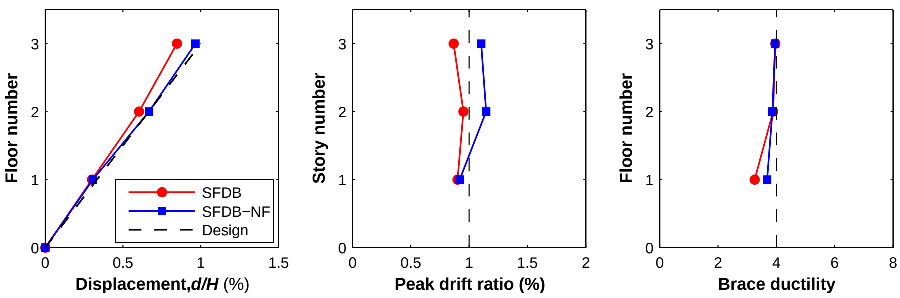
4. DESIGN VALIDATION

4.1. Prototype design buildings

A 3-story and a 6-story office building are designed using the above-mentioned design procedure. These frame buildings are designed for a location in downtown Los Angeles with site class D (firm soil). Details of these buildings can be found in Zhu (2007). For the 3-story building, the seismic mass is 8.73×10^5 kg for the 1st and 2nd floor, 7.88×10^5 kg for the 3rd floor; while for the 6-story building, the seismic mass is 9.06×10^5 kg for the 1st through 5th floor, and 8.19×10^5 kg for the 6th floor. In this study both SFDB frame and SFDB-NF frame were designed to achieve the close performance objectives. The design drift ratio is specified as 1% under the DBE earthquake for both the 3-story and 6-story buildings, and the design ductility levels of braces are assumed to be equal to 4. In comparison with the ductility of SFDBs/SFDB-NFs corresponding to the slip of friction surface II (i.e. μ = 8.0), such a target ductility performance would lead to minimal residual displacements of frames after DBE earthquakes. The performances under FE and MCE need to be checked through the seismic analysis. In this study, no significant structural damage is allowed under FE, and the peak roof drift ratio and maximum inter-story drift ratio should be less than 3% and 4% respectively under MCE.

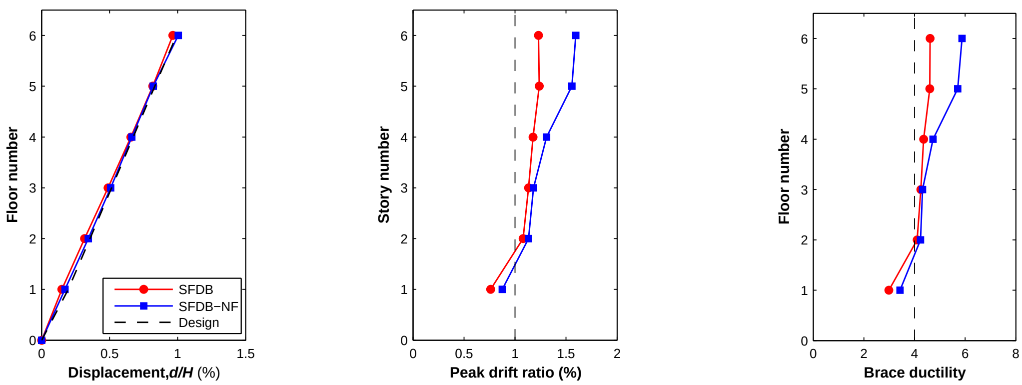
4.2. Results and discussions

Three suites of earthquake ground motions developed by Somerville et al. (1997) were used in the nonlinear time history analyses to evaluate seismic performance of the designed frames. They are corresponding to seismic intensity levels at downtown Los Angeles. Each suite of ground motions contains 20 records, designated as LA01 - LA20 (for DBE), LA21 – LA40 (for MCE) and LA41 - LA60 (for FE), respectively.



(a) Normalized displacement (b) Peak story drift ratio (c) Ductility level of braces

Figure 2 Smooth-median response of 3-story frames under DBE earthquakes



(a) Normalized displacement (b) Peak story drift ratio (c) Ductility level of braces

Figure 3 Smooth-median response of 6-story frames under DBE earthquakes

Figures 2 and 3 show the smooth-median response of the 3- and 6-story building respectively under the DBE earthquakes. The smooth-median response is calculated based on the time history analysis results under 20 ground motions in the DBE suite. The target design performances are also shown as dashed lines in these figures. The peak floor displacement is normalized by the building height. It can be seen that the SFDB or SFDB-NF frame buildings designed using the proposed DBD approach can achieve the target displacement pattern pre-specified in the design procedure. In terms of the peak inter-story drift ratios and ductility levels of the SFDB braces, the 3-story building, including both the SFDB and SFDB-NF frames, can meet the design target performance reasonably well. For the 6-story buildings, the proposed DBD method tends to underestimate the peak inter-story drift ratios and brace ductility demands for both the SFDB and SFDB-NF frames. Particularly, for the 6-story SFDB-NF frame, apparent non-uniform distribution of inter-story drift ratios and brace ductility demands can be observed along the building height, and smooth-median results from the time history analyses are seen to exceed the design estimation by up to 60%. This can be explained by the fact that although the displacement response can be estimated with satisfactory accuracy from the equivalent SDOF system, other response quantities including maximum inter-story drift ratios may be affected by higher mode effects, concentration of inelastic deformation and P-Δ effect (Gupta and Krawinkler 2000; Pampanin et al. 2003, Moehle 1992). Compared to the 3-story building, the concentration of inter-story drift due to inelastic behavior and the effect of higher mode participation is more prevalent in the 6-story building, and as such the discrepancy between the design value and analysis results is more obvious.

Table 1 Statistical results of seismic response under different earthquake levels

		3-story SFDB	3-story SFDB-NF	6-story SFDB	6-story SFDB-NF
Median (standard deviation)					
Peak roof drift ratio (%)	FE	0.28 (0.54)	0.47 (0.59)	0.32 (0.25)	0.46 (0.31)
	DBE	0.85 (0.64)	0.97 (0.64)	0.96 (0.48)	1.00 (0.47)
	MCE	2.42 (1.06)	2.30 (0.89)	2.31 (0.95)	2.35 (0.93)
Maximum story drift ratio (%)	FE	0.35 (0.66)	0.61 (0.75)	0.52 (0.56)	0.81 (0.76)
	DBE	1.04 (0.70)	1.23 (0.75)	1.39 (0.57)	1.71 (0.52)
	MCE	3.08 (1.69)	2.83 (1.19)	3.33 (1.20)	3.28 (1.20)
Maximum acceleration	FE	4.60 (2.58)	7.99 (3.78)	4.38 (2.93)	6.96 (5.65)
	DBE	7.10 (2.51)	10.39 (3.93)	8.42 (3.37)	12.27 (5.36)
	MCE	11.65 (2.30)	16.71 (3.82)	13.68 (2.38)	21.10 (4.37)
# of cases in which (out of 20)					
Apparent residual story drift ratio (>0.1%)	FE	1	1	1	1
	DBE	2	2	3	5
	MCE	14	16	17	15
Sliding surface II slips	FE	1	1	1	1
	DBE	3	4	3	8
	MCE	15	17	17	16
Yielding of column bases	FE	3	3	0	1
	DBE	12	10	3	7
	MCE	20	20	17	19
Yielding of beams	FE	1	1	1	1
	DBE	6	7	8	9
	MCE	16	18	18	17

Table 1 shows the statistical seismic responses of SFDB and SFDB-NF frame under different seismic intensity levels. Pushover analyses of these designed frames show that the slip of sliding surface II occurs at the roof drift around 2%. As mentioned before, the slip of sliding surface II may impair the self-centering capability of

SFDBs and consequently may lead to some residual displacement after earthquakes. Table 1 indicates that under FE and DBE earthquakes, the median values of roof drift ratios and maximum inter-story drift ratios for SFDB and SFDB-NF frames are all less than 2%. The slip of sliding surface II does not occur in most cases of FE and DBE suites, and accordingly the residual inter-story drift and displacement is minimal under most ground motions of FE and DBE levels. Additionally, the 'yield'-like plateau of SFDB or SFDB-NF frames is caused by the solid phase transformation in superelastic SMA wires instead of plastic deformation. Therefore, the SFDBs or SFDB-NFs can withstand several frequent or design basis earthquakes without the need for repair or replacement as long as the slip of sliding surface II does not occur. This is a big advantage of SFDB over other bracing element, such as buckling-restrained braces which tend to yield even under frequent earthquakes (Sabelli et al. 2003; Zhu and Zhang 2008) before the occurrence of plastic deformation in beams and columns, there is essentially no damage in the designed SFDB or SFDB-NF frames. As shown in Table 1, no yielding of beams and columns can be observed in most cases of FE suite and about half of cases in DBE suite. Therefore SFDB frames have the potential to achieve a damage-free structural system under FE and even some DBE earthquakes. These desirable characteristics of SFDB frames—reusability of SFDB braces, reduced residual displacement and potential damage free—would lead to considerably reduced repair cost and service interruption after frequent and design-level earthquakes, particularly in seismic active regions.

The median response of peak roof drift and inter-story drift of four designed frames is less than 3% and 4% respectively. Thus the designed SFDB and SFDB-NF frames also satisfy the performance requirement under MCE earthquakes. The performance levels of the four buildings under the MCE earthquakes are close to each other. But under MCE suite, the activation of sliding surface II, and therefore noticeable residual inter-story drifts can be observed in most cases. The median responses of maximum floor acceleration for both SFDB and SFDB-NF frames are also shown in Table 1. The SFDB frames have smaller acceleration demands than the SFDB-NF frames under all three seismic intensity levels owing to the lower brace strength used in the design of SFDB frame. Cautions need to be exercised on the high acceleration demands observed in SFDB-NF frames which may cause undesirable damage to acceleration sensitive building contents and components.

5. CONCLUSIONS

This paper presents a seismic design approach for steel concentrically braced frame (CBF) buildings with a special bracing element termed self-centering friction damping brace (SFDB). Through a displacement-based design (DBD) procedure using inelastic design spectra, SFDB frame is proportioned according to the target performance level under design basis earthquakes. The proposed DBD method was applied to the design of a 3-story and 6-story building located in Los Angeles, California, respectively. The design examples of CBF buildings with self-centering braces without friction, denoted as SFDB-NF, are also included as reference cases in this chapter in order to evaluate the benefit of friction damping in SFDB. The design examples demonstrate that owing to the contribution of friction force at sliding surface I, SFDB frames are believed to yield more economical design than the SFDB-NF frames.

Nonlinear dynamic analyses results show that SFDB frames designed using the DBD procedure can achieve the target displacement parameters with a high degree of accuracy. However, caution should be exercised on design of medium-rise buildings since the proposed DBD method tends to give underestimated values of maximum story drift ratios and brace ductility demands which can be attributed to the exclusion of concentration of story drift due to inelastic behavior and higher mode contribution in this simplified design method. Their effects generally increase with the displacement ductility ratio and the number of stories. The results of nonlinear time history analyses also show that through proper selection of target performance levels, SFDBs has a potential to establish a new type of CBF systems with self-centering capability that can withstand several frequent or even some design basis earthquakes without the need for replacement. The superior performances of SFDB frames—reusability of braces, minimal residual displacement and potential damage free—would lead to considerably reduced repair cost and service interruption after frequent and design-level earthquakes, particularly in seismic active regions.

REFERENCES

Chopra, A. K. and Goel, R. K. (2001) Direct displacement-based design: use of inelastic vs. elastic design spectra. *Earthquake Spectra*, **17:1**, 47-64.

Christopoulos, C., Filiatrault, A. and Folz, B. (2002a). Seismic response of self-centering hysteretic SDOF systems. *Earthquake Engineering and Structural Dynamics*, **31**,1131–1150.

Christopoulos, C., Filiatrault, A., Folz, B. and Uang, C-M. (2002b). Post-Tensioned Energy Dissipating Connections for Moment-Resisting Steel Frames. ASCE *Journal of Structural Engineering*, **128:9**,1111-1120.

Clark, P. W., Aiken, I. D., Kasai, K., Ko, E. and Kimura, I., (1999). Design procedures for buildings incorporating hysteretic damping devices. *Proc., the 69th SEAOC Annual Convention*, Sacramento, CA.

Fajfar, P. (2000). A nonlinear analysis method for performance based seismic design. *Earthquake Spectra*, **16**, 573-592.

FEMA (2003). NEHRP recommended provisions for seismic regulations for new buildings and other structures. FEMA-450, Federal Emergency Management Agency, Washington D.C.

Gupta, A. and Krawinkler, H. (2000). Estimate of seismic drift demands for frame structures. *Earthquake Engineering and Structural Dynamics*, **29**, 1287-1305

Kowalsky, M. J., Priestley, M. J. N. and MacRae, G. A. (1995). Displacement-based design of RC bridge columns in seismic regions. *Earthquake Engineering and Structural Dynamics*, **24**, 1623-1643.

Moehel, J. P. (1992). Displacement-based design of RC structures subjected to earthquakes. *Earthquake Spectra*, **8:3**, 403-428.

Pampanin S., Christopoulos, C. and Priestley, M. J. N. (2003). Performance-based seismic response of frame structures including residual deformations. Part II: multi-degree of freedom systems. *Journal of Earthquake Engineering*, **7:1**, 119-147.

Priestley, M. J. N. (2000). Performance based seismic design, *Proc., 12th World Conference on Earthquake Engineering*, Paper No. 2831, Auckland, New Zealand, January-February 2000.

Ricles, J. M., Sause, R., Garlock, M. M. and Zhao, C. (2001). Posttensioned seismic-resistant connections for steel frames. *Journal of Structural Engineering*, **127:2**, 113-121.

Sabelli, R., Mahin, S.A., and Chang, C. (2003). Seismic demands on steel-braced buildings with buckling-restrained braces. *Engineering Structures*, **25**, 655-666.

Seo, C. Y. (2005) Influence of ground motion characteristics and structural parameters on seismic responses of SDOF systems, Ph.D. dissertation, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA.

Stanton, J., Stone, W. C. and Cheok, G. S. (1997). A hybrid reinforced precast frame for seismic region. *PCI Journal*, **42:2**, 20-32.

Sullivan, T., Calvi, G.M., Priestley, M.J.N. and Kowalsky, M. (2003). The limitations and performances of different displacement-based design methods. *Journal of Earthquake Engineering*, **7**(No. Special Issue **1**): 201-241.

Zhu, S. (2007). Seismic behavior of framed structural systems with self-centering friction damping braces. *Ph.D. thesis*, Lehigh University, Bethlehem, PA

Zhu, S. and Zhang, Y. (2008). Seismic analysis of concentrically braced frame system with self-centering friction damping braces. *ASCE Journal of Structural Engineering*, **134:1**, 121-131