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# PROBABILISTIC METHODOLOGIES FOR PREDICTION OF POST-EARTHQUAKE BRIDGE REPAIR COSTS AND REPAIR TIMES

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**ABSTRACT:**

A new step-by-step probabilistic methodology, based on local linearization of the damage to repair quantity model, evaluates the repair costs and repair times for bridge components at varying degrees of damage. This local linearization repair cost and time methodology (LLRCAT) is compared to existing methodologies that use the Fourway graphical tool, and that are based on Monte Carlo simulation. LLRCAT requires a data structure to organize bridge-specific repair actions, quantities, and costs. Performance groups are defined for bridge components and subassemblies that are repaired together. This information is implemented using six spreadsheets for bridge material and spatial information, structural response, component damage states, repair methods and repair quantities, unit costs, and labor production rates. The results of using LLRCAT are illustrated using repair cost and repair time fragilities for an example multi-span reinforced concrete highway overpass bridge in California.

**KEYWORDS:** damage, loss modeling, performance-based earthquake engineering, repair quantity

## 1. INTRODUCTION

During previous extreme earthquake events, transportation infrastructure, in particular bridges and overpasses, have suffered from structural damage (Zelinsky 1994, Yashinsky 2004). While structural engineers have traditionally focused on individual components (bridges, for example) of transportation networks for design, retrofit, and analysis, it has become increasingly apparent that the economic costs to society after extreme earthquake events are caused at least as much from indirect costs as direct costs due to individual structures (Kiremidjian et al. 2006). Quantifying direct and indirect costs requires a system-level approach (Werner et al. 2004, HAZUS MH 2008) and is best approached probabilistically. All network simulations use a probabilistic relationship between intensity and a limit-state that describes the performance of each structure in the network, called a fragility curve. The limit-state describing structure performance was typically a peak response measure (e.g. drift, strain), damage to individual components, or structural system damage. However, the relationships between response, damage, and losses were not well defined. Ultimately, structure repair cost, repair time, and down time determine the direct and indirect costs to a network. This paper describes a methodology called the local linearization repair cost and time methodology (LLRCAT) for developing probabilistic estimates of repair costs and repair times.

The approach used here for probabilistic repair estimates is the Pacific Earthquake Engineering Research (PEER) Center's performance-based earthquake engineering (PBEE) framework. The LLRCAT methodology is robust, data accessible, easily implementable method and compatible with previous methods for quantifying loss (Mackie and Stojadinović 2006). The method is intended completely for generating probabilities of exceeding different levels of repair costs and repair times, instead of structural engineering quantities like response and damage. To define performance objectives, performance quantities are defined by the probability of exceeding threshold values of socio-economic decision variables (DVs) in the seismic hazard environment under consideration – in this case repair cost ratios and repair times. The threshold, or limit state (LS), value of DV is designated as $dv_{LS}$. The PEER PBEE framework (Cornell and Krawinkler 2000) utilizes the total probability theorem to disaggregate the problem into several intermediate probabilistic models that address sources of randomness and uncertainty more objectively. This disaggregation of the decision-making framework involves the following intermediate variables: repair quantities (Q), damage measures (DMs), engineering demand parameters (EDPs), and seismic hazard intensity measures (IMs).

This paper proposes a new vector-based approach of applying the PEER framework to the problem of post-earthquake highway bridge loss modeling (Mackie et al. 2006, 2007). In this context, the DVs are limited to the post-earthquake repair cost and repair time. The proposed approach is based on linearization of the damage model (relationship between damage and repair quantity, Q-DM). This vector-based approach allows disaggregation of the bridge system into individual components, or performance groups (Porter and Kiremidjian 2001, Yang et al. 2006). The disaggregation approach here requires an intermediate probabilistic model into the framework that relates damage to repair quantities (Q). The vector approach sums over the bridge performance groups $l$ over all of the discrete damage states $m$ applicable to each component, and all of the repair quantities $n$ necessary to repair damage of type $m$ to component $l$. Each repair quantity $Q_{n,l}$ is then treated in a probabilistic manner with a form similar to Eqn. 1.2.

$$P(Q_{n,l}>q^{LS}|IM=im)=\sum_m \int G_{Q_{n,l}|DM_{m,l}}(q^{LS}|dm_{m,l})G_{DM_{m,l}|EDP_l}(dm_{m,l}|edp_l)dG_{EDP_l|IM}(edp_l|im) \tag{1.2}$$

Previous implementations of the PEER framework (Mackie et al. 2006, 2007) were based on a power-law, least-squares, relationship between DM and Q, or DM and DV using Eqn. 1.3, where $E$ and $F$ are coefficients of the best fit relationship between $Q$ and $DM$.

$$\ln(Q)=E+F\ln(DM) \tag{1.3}$$

## 2. METHODOLOGY OVERVIEW

The LLRCAT methodology involves three main pieces: local linearization of the Q-DM model, extension from Q to repair cost and repair time, and a data structure that requires only bridge-specific data. The methodology improves upon the closed-form Fourway method (Mackie and Stojadinovic 2006), piecewise power-law approach, and Monte Carlo simulation. Local linearization overcomes the challenges of the previous approaches, and retains the simplicity of automated, closed-form solutions. Mackie et al. (2008) presents extended details of the methodology.

### *2.1 Previous approaches: Fourway and piecewise methods*

For each structural component of the bridge (or performance group) and repair quantity, a single scalar-type analysis is performed, modeled in IM-EDP-DM-Q space. Solving with the the closed-form or Fourway (Mackie and Stojadinovic 2006) solution strategies produces the expectation and variance of each repair quantity $q_{n,l}$ for the repair item $n$ and performance group $l$. The first moment and second central moment are denoted $E[q_{n,l}]$ and $Var[q_{n,l}]$ , respectively. The expected value and square root of the variance are lognormal distribution parameters due to the assumptions surrounding the Fourway process.

The repair quantity loss model cannot be realistically modeled with the power-law relationship (Eqn. 1.3), because it would grow exponentially beyond the data provided, and cannot represent distinct plateaus of repair data. Formulating the Q-DM relationship is challenging because the repair quantities are not necessarily increasing with greater damage, and no simulations are performed and the outcome is dependent on discerning the first and second moments of Q. Previous attempts (Mackie et al. 2006, 2007) address these challenges by creating four general categories of typical Q-DM model behavior. That method did allow close-form solutions and suited available bridge data, but was prone to unpredictable behavior at demands beyond the last damage state and could not be extended to user-specified Q-DM models without recognition of new piecewise types

### 2.2 New proposed approach: local linearization

This approach uses local linearization of the Q-DM model, a straight-line relationship in linear space (Eqn. 2.1). This is different from the other approach's assumption of a power-law relationship. The first-order Taylor series expansion of $\ln(Q)$ in Eqn. 2.1 about a point in DM space denoted $d_0$ can be written as Eqn. 2.2.

$$Q = e_{lin} DM + f_{lin} \tag{2.1}$$

$$\ln(Q) = \ln(e_{lin} d_0 + f_{lin}) + \frac{e_{lin}}{e_{lin} d_0 + f_{lin}} (DM - d_0) + h.o.t. \tag{2.2}$$

Similarly DM and $d_0$ can be related to $\ln(DM)$ and $\ln(d_0)$ and combined with Eqn. 2.2 to obtain the same form as Eqn. 1.3, but with new parameters $E'$ and $F'$ (Eqn. 2.3), replacing the previous parameters $E$ and $F$.

$$E' = \ln(e_{lin} d_0 + f_{lin}) - \frac{e_{lin} d_0 \ln(d_0)}{e_{lin} d_0 + f_{lin}}, \; F' = \frac{e_{lin} d_0}{e_{lin} d_0 + f_{lin}} \tag{2.3}$$

Implementation of local linearization requires selection of an expansion point $d_0$. The method does not guarantee a non-zero denominator, or that the resulting piecewise Q-DM function is continuous at the end-points of the intervals used. Local linearization works best by continuously updating the location of the linearization point $d_0$ to be at every DM input desired. This results in well-behaved Q-DM models that are actually linear in linear space, and still compatible with the older closed-form solution strategies. Compatibility with the previous closed-form strategy makes it possible to automate the computation for all PG and Q combinations regardless of the form of the repair data.

### 2.3 Comparison to Monte Carlo simulation

The local linearization approach is compared to Monte Carlo simulation for a single IM-EDP-DM-Q analysis, and for an analysis with 2 performance groups and 5 repair quantities. For the single analysis, the distribution of EDP was assumed pre-determined at each of 2 hazard intensity levels with lognormal standard deviation $\sigma_{EDP|IM} = 0.33$, and 0.30 for the damage and loss models, with increasing repair quantity with each successive damage state (Fig. 1).

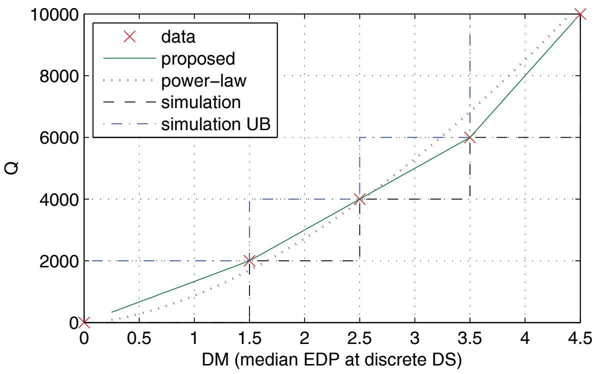



Figure 1: Comparison of Q-DM models, EDP = percent drift ratio

Two types of simulations are considered for comparison: increasing Q at the beginning or end of the damage state interval. These choices are similar to right- and left-endpoint numerical integration. Both methods converge as the number of discrete damage states approaches a continuum, and also diverge if a Q-DM model has few or only one discrete damage state. Two linearization types are possible: linear or power-law relationship between the Q-DM points. Increasing Q at the beginning provides an upper bound on the repair quantity probability, and the power-law approach satisfies the average because it uses a least-squares fit to the data (Fig. 1).

### *2.4 Upper and lower limit damage states*

The LLRCAT method also includes the concept of DS0 and DS∞ damage states. Even though the defined damage states are discrete, the moment-based computation method assumes that a continuous range of damage exists between the discrete states. This allows for closed-form computation of Eqn. 1.2, but also produces an unrealistic, immediate increase in expected repair costs for extremely small earthquake intensities. Therefore, the DS0 damage state provides a lower threshold corresponding to the onset of damage when repair costs begin to accumulate. The repair cost of the bridge is considered $0 below DS0. The upper limit, DS∞, corresponds to the most severe possible damage state for a PG, usually complete failure and replacement of all the elements in the entire PG.

## 3. REPAIR COST AND REPAIR TIME

Repair costs and repair times can be computed once the total repair quantities $Q$ are known. The first step combines all the $N_Q$ repair items across all the $N_{PG}$ performance groups. The expected value and variance of each repair quantity can be computed using Eqn. 3.1. The expected value of the total repair quantity $Q_n$ for each item $n$ is a linear combination of the expected performance group-dependent quantities. The corresponding variances are not simple linear combinations due to the correlation between response quantities and performance groups. The covariance is obtained from the correlation coefficient relating performance groups $l$ and $p$.

$$E[Q_n]=\sum_{i=1}^{N_{PG}} E[q_{n,l}]\ ,\ Var[Q_n]=\sum_{i=1}^{N_{PG}} Var[q_{n,l}]+2\sum_{i=1}^{N_{PG}}\sum_{p>l}^{N_{PG}} Cov[q_{n,l},q_{n,p}] \tag{3.1}$$

### *3.1 Extension to repair cost*

The simple summation of $Q_n$ between performance groups is possible only if the quantities are, or transformed into, normal distributions. Addition of lognormally distributed variables is not as straightforward (Naus 1969). Transformation to a normal distribution introduces significant error when $Var[Q_n]$ is large. The unit cost $C_{un}$ of each item $n$ is treated constant regardless of the quantity $Q_n$. The variance of the cost also depends on $E[Q_n]$ and $Var[Q_n]$. The total expected cost of repair $E[TC]$ is determined by Eqn. 3.2.

$$E[TC]=\sum_{n=1}^{N_Q} Cu_n(E[Q_n]) \tag{3.2}$$

The total cost variance was obtained by summing the individual material quantity variances and adding an additional term for unit cost uncertainty. Because the repair items were assumed to be statistically independent, the covariance term is zero, similar to Eqn. 3.1. The total cost distribution will approach a normal distribution because of the central limit theorem with number of Qs being summed, $N_Q$ Qs varied within $N_{PG}$ PGs.

### *3.2 Extension to repair time*

Repair times are computed based on the labor production rate $PR_n$ of each item $n$, which is considered constant regardless of the quantity $Q_n$. Therefore, the production rate is specified units of crew working days (CWD), and not the normalized quantity of CWD over total output. In addition, the magnitude (or distribution) of $Q_n$ is not used explicitly in the repair time analysis, but rather as a trigger for the presence or absence of the repair activity in the total number of CWD required. Each repair activity is triggered if the probability that $P[Q_n \geqslant tol]>0.5$, where the tolerance is set at a value minimally larger than zero. The total expected repair time $E[RT]$ is then obtained by summing $PR_n$ for all $N_Q$ number of quantities.

The variance of the repair time was assumed to also be an addition of the individual variances of $PR_n$. The mean and standard deviation of production rate are estimated using the PERT criteria (Perry and Grieg 1975). Repair time analysis only captures the correlation between performance groups in a rudimentary manner because the trigger for adding production rates is based on the $Var[Q_n]$ from Eqn. 3.1. Although appropriate to assume the repair quantities were statistically independent for the purposes of repair cost, the true repair times are correlated through production rates of each quantity. The assumption of repair time variance used in this paper does not account explicitly for scheduling dependencies between repair activities.

### *3.3 Comparison to Monte Carlo simulation*

A second comparative analysis determined the CDF of repair cost conditioned on earthquake intensity for each simulation option in Fig. 1. The second analysis is a more complex example using two PGs and 5 repair quantities. The data used for this analysis comes from the first two performance groups of the steel special moment resisting frame (SMRF) building of Yang et al. (2006). The performance groups are for the lateral structural system between levels 1–2 and 2–3. The EDP was measured in terms of drift ratio and the intensity measure was $Sa(T_1)$. The first five repair quantities for these performance groups were selected, along with their corresponding unit costs and an assumed 15% c.o.v. the unit costs. The uncertainty of Q was assumed to be lognormal with lognormal standard deviation of 0.30. The results of these comparisons are shown in Fig. 2. The analyses did not contain DS0 and DS∞ because this data was not present in the building study.

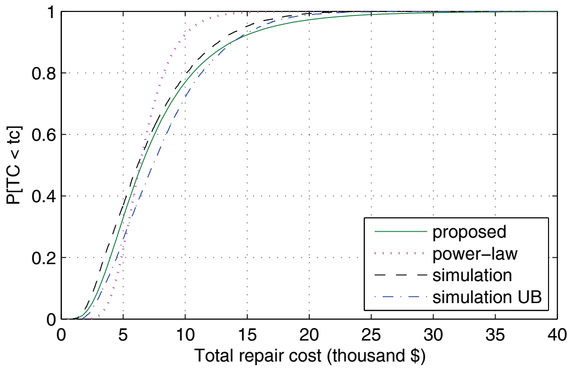



Figure 2: Repair cost CDF for pair of building PGs, $Sa(T_1) = 0.75\text{g}$

## 4. BRIDGE INFORMATION

The methodology requires only bridge-specific data for damage states, repair methods, material quantities, and unit costs to compute estimated repair costs. This data is stored in Excel spreadsheets which are read by Matlab scripts which run the methodology computation. A different spreadsheet is required for unit costs for each Q, production rates for each Q, damage limit states for each performance group EDP, EDP results from structural analysis at various IMs, bridge information, dimensions, quantities for estimation, and repair quantities Q for each damage state and performance group. The examples presented here are based on the Type 1 and Type 11 bridges from Ketchum et al. (2004) with estimating data described in Mackie et al. (2008).

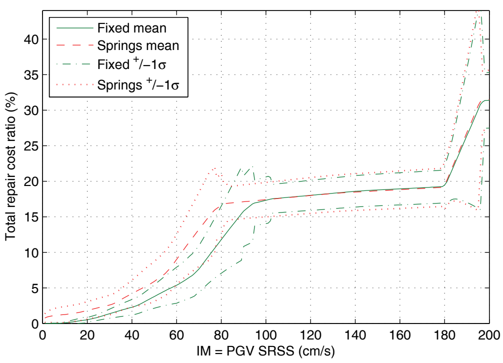
Bridge-specific data for repairs are categorized by performance groups (PG) for each major bridge superstructure, substructure, and foundation component. Performance groups can include both structural and non-structural elements that suffer damage and contribute to repair costs. Performance groups allow grouping of several different bridge components by related repair work. Grouping by PG avoids double counting related repair items by making each PG's repairs independent. For example, the "abutment" PG incorporates both back wall and approach slab repairs since they are typically damages, accessed, and repaired together even though they are different bridge components. Repair costs are presented with a repair cost ratio (RCR), the ratio between repair and replacement cost. Replacement cost can be normalized by a cost index (Caltrans 2007).

The accuracy of the results depends on the quality of bridge damage states, knowing the appropriate repair methods for those damage states, correct repair items, and correct quantities for estimating repair cost and time. This information is best obtained from experts working within state departments of transportation who have experience with real bridge repairs and estimating methods. A method for collecting this data using damage scenarios is documented in Mackie et al. (2008).

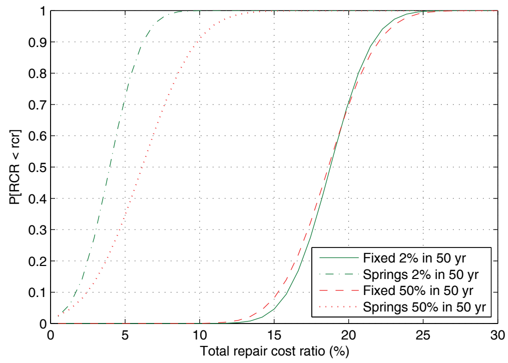
## 5. METHODOLOGY RESULTS

### *5.1 Intensity-dependent repair cost ratios*

The proposed LLRCAT methodology is the ability to assess the intensity-dependent variation in repair cost ratios. Both the first and second probabilistic moments of repair cost ratio are calculated for each intensity level, for two scenarios, fixed and spring column foundations (Fig. 3a). Using a structure-independent PGV allows direct comparison between the scenarios. The use of DS0 can be seen at the low intensity levels. Jumps in the RCR are due to changes in repair method at higher damage states. Fragility curves can also be generated using discrete hazard levels (Fig. 3b).




(a)



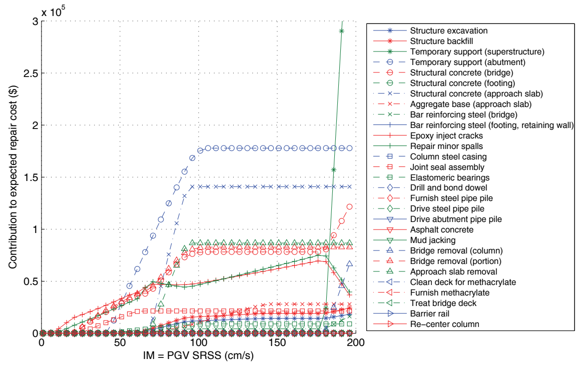

(b)

Figure 3: RCR as (a) function of PGV and (b) fragility curve

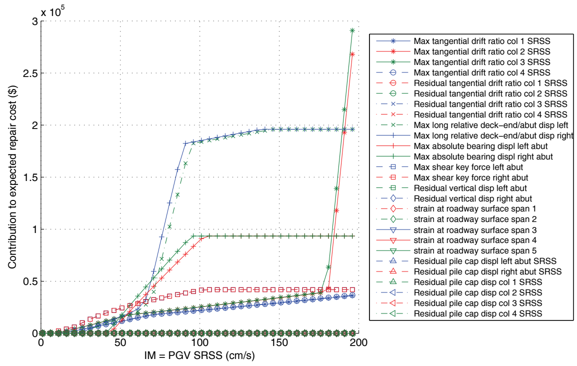
### *5.2 Disaggregation by repair quantity and performance group*

The methodology's assembly-based, vector nature makes it possible to disaggregate contributions to the final repair cost by repair quantity. The total expected cost from each Q is shown in Fig. 4a. The ordinate is plotted in units of cost not normalized as an RCR, and shows only the expected mean cost. The repair quantities contribute different amounts at different IM values. For example, the peak contribution at the range of intensities between 2%- and 50%-in-50-years exceedance probabilities is from temporary support at the abutments. But, at PGV less than 50 cm/s, injecting cracks with epoxy contributes the most.

Disaggregation by performance group describes specifically what components contribute the most to the repair cost (Fig. 4b). This disaggregation is possible for the expected cost, but is difficult for the standard deviation because of the correlations introduced between performance groups (Eqn. 3.1). This disaggregation reveals why repair quantities in several repair methods feature in the expected cost. For example, bridge structural concrete contributes to a significant portion of the overall expected cost at all intensities in Fig. 4a. The reason for this is more readily apparent in Fig. 4b as both the abutment PGs, with damage states defined in terms of the maximum longitudinal relative deck-end/abutment displacement EDPs, have the largest contribution in the range of intensities between 2%- and 50%- (in-50-years) exceedance probabilities. This indicates that the concrete is necessary to repair the back wall.




(a)




(b)

Figure 4: Disaggregation of expected repair cost by (a) repair quantity and (b) performance group

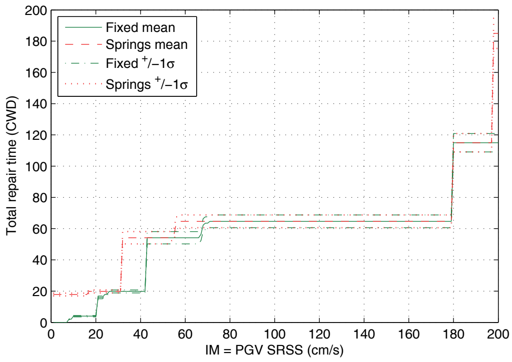



Figure 5: CWD as a function of PGV for two foundation models of bridge Type 1A

### *5.3 Repair time results*

The repair time results are expressed in terms of crew working days (CWD) for two different foundation modeling scenarios, fixed-based and flexible spring-based columns (Fig. 5). The uncertainty arises from activity duration estimates for each repair quantity, not from the uncertainty in the repair quantities themselves. The total number of CWD reflects the effort needed to perform the repair and is not the same as total repair duration because it does not include the effect of schedule dependencies, labor availability, and procurement and installation times. As a byproduct of the methodology, this scenario also gives a small estimated number of CWD for zero intensity events. This is due to the larger initial transverse forces on the abutment shear keys that trigger the shear key repair methods immediately. Once again, this information is more easily obtained by disaggregating the total expected repair time by repair quantity. However, such a disaggregation plot does not provide as much information at the disaggregation of repair cost by repair quantity because the plot shows only when each repair quantity triggers a contribution to the total repair effort. Therefore, the maximum values of a repair time disaggregation by repair quantity plot are merely the mean values from the production spreadsheet. Along similar lines, it is nonsensical to disaggregate the expected repair time by performance group because multiple performance groups may cause an increase in repair quantity that would trigger and increase in CWD.

## 6. CONCLUSIONS

The local linearization repair cost and time methodology improves upon previous closed-form methodologies and is comparable to the results obtained using simulation-based methods. This LLRCAT methodology assesses first and second moments of repair cost and time as a function of earthquake intensity, disaggregates repair cost y repair quantity and performance group, and requires only input of bridge-specific data to run. The introduction of the DS0 and DS∞ lower and upper limit damage states allow additional refinement of repair cost and time probabilities. The disaggregation of repair cost and time provides a higher level of information regarding damage and failure of specific bridge components or performance groups. This information allows for better decision making regarding repair or retrofit. When applied to alternate bridge configurations, the cumulative distribution functions of repair cost can be used to differentiate between design choices in a pre-earthquake planning or design scenario. The repair cost information can be applied to classes of bridges for improved network simulations of economic loss due to earthquakes.

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