Vulnerability Functions for Wooden Houses Based on Seismic Diagnosis Data and Equivalent Linear Models

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ABSTRACT:

Vulnerability functions of wooden houses, which use the results of seismic diagnosis and an acceleration response spectrum, are formulated using equivalent linear models. They employ a probability density function of seismic performance indices of wooden houses, which are separated into an index of the resistance force and other indices related to the degradation of resistance force. The input seismic motion is evaluated based on an acceleration response spectrum and a natural period is determined by the indices of resistance force and threshold of damage. The application results to the 1995 Hyogo-ken Nanbu earthquake are compared with the damage statistics at the five sites, which have seismic records. The estimated damage ratios coincide fairly well with the actual damage surveyed by local governments.

KEYWORDS: Estimation of Damage, Wooden Building, Seismic Diagnosis, Acceleration Response Spectrum, Equivalent Linear Model

1. INTRODUCTION

A vulnerability function describes relationship between ground motion intensity and building damage ratio in a certain region. The function is often employed by an earthquake disaster information system, which estimates regional damage soon after an earthquake event. As ground motion intensity, peak ground acceleration, peak ground velocity etc. are used, but frequency characteristics of seismic ground motion and natural frequencies of buildings should be considered in order to improve accuracy of the function.

Kohiyama and Yamazaki (2004) propose a new vulnerability function for wooden houses, which considers these frequency characteristics. The vulnerability function uses an acceleration response spectrum, which includes frequency characteristics of seismic ground motion, and a joint probability density function (PDF) of two random variables: variables equivalent to resistance force and natural frequency of wooden houses. The PDF is evaluated based on data of seismic diagnosis (Sakamoto 1995; Housing Bureau, Ministry of Construction 1985) in a target area. The vulnerability function has good accuracy but can evaluate only ratio of minor or more damage, i.e., a threshold is set between no damage and minor damage. This is due to assumption of linear elastic response of buildings in formulation of the function.

In this paper, the vulnerability function is expanded for different thresholds of damage such as moderate and major damage by introducing equivalent linear models of wooden houses. The validity of the function is checked based on the damage statistics and seismic records of the 1995 Hyogo-ken Nanbu earthquake.

2. FORMULATION OF VULNERABILITY FUNCTION BASED ON SEISMIC DIAGNOSIS DATA

The vulnerability function is derived based on the relation between seismic force and restoration force (resistance force) of a building, which is calculated from scores of seismic diagnosis. In the enforcement regulation of the Building Standard Law in Japan, the story drift less than 1/120 rad is allowed for wooden structure with the base shear $C_i = 0.2$. The minimal restoration force of the first story at this deformation level regulated by the law is denoted by $Q_{d0}$. When the seismic load, $Qr_0$, is caused by base shear $C_i = 0.2$, $Q_{d0}$ has
the following relation:

\[ Q_{d0} = Q_{r0} = C_i M g, \]  

(2.1)

where \( M \) is mass of the structure upper than the first story and \( g \) is acceleration of gravity.

Given the first story of the structure has restoration force \( Q_d \) with the drift angle \( 1/120 \) rad and \( Q_d \) is \( R_w \) times larger than \( Q_{d0} \), the restoration force, \( Q_d \), is:

\[ Q_d = R_w Q_{d0}. \]  

(2.2)

The diagnosis score of resistance force, \( I_{DE} \), describes the ratio of the restoration force that the diagnosed structure retains to the force of the minimal low requirement. Actual houses have significant stiffness increase due to the contribution of perpendicular walls and moment resistance of the frames with widow back, etc. These effects of stiffness increase are considered by introducing coefficient, \( \alpha_{DE} \). Hence, it can be assumed that:

\[ R_w = \alpha_{DE} I_{DE}. \]  

(2.3)

When subject to response acceleration \( S_a \) with the shape factor, \( F_{es} \), a seismic load, \( Q_r \), is derived as follows considering Eqn. (2.1):

\[ Q_r = S_a F_{es} M = S_a F_{es} Q_{r0} / (C_i g). \]  

(2.4)

\( F_{es} \) increases the seismic load due to the building shape characteristics such as eccentricity or irregular elevation and the diagnosis score of eccentricity, \( I_{BC} \), corresponds to \( 1/F_{es} \).

The diagnosis score of soils and foundation considers amplification effect of soft soil to seismic ground motion. However, acceleration response, \( S_a \), which is used in the proposed vulnerability functions, includes also the amplification effect. Thus, this effect should be excluded from original score of soils and foundation. The different seismic diagnosis method (Urban Housing Improvement Office, Housing Bureau, Ministry of Construction, 1998) proposes to use two tables separately for increasing factor to amplified seismic load and for fragility of soils and foundation types. Table 2.1 shows the fragility, \( R_b \), determined by the soils and foundation types, which may decrease the resistance force of the structure or the threshold of drift angle of minor damage. Rather than the original score of soils and foundation in the accurate diagnosis method, the score \( R_b \) is used for vulnerability function formulation and this is denoted by \( I_A \).

| Table 2.1 Scores of soils and foundation without soft soil amplification factor, \( R_b \) |
|---------------------------------|----------------|----------------|
| Soil type                       | Good or normal | Rather bad     | Very bad |
| Strip foundation of reinforced concrete | 1.00           | 1.00           | 1.00     |
| Strip foundation of plain concrete    | 1.00           | 0.85           | 0.75     |
| Frame-strengthened stone footing    | 1.00           | 0.85           | 0.75     |
| Strip foundation of cracked concrete | 0.70           | 0.60           | 0.50     |
| Others                            | 0.60           | 0.50           | 0.50     |

Deterioration could also reduce the restoration force. This factor is considered as \( R_d \), which corresponds to the diagnosis score of deterioration, \( I_F \). Note that both \( I_A \) and \( I_F \) are smaller or equal to 1.

Kohiyama and Yamazaki (2004) assume the story drift angle of damage occurrence reduce along with \( I_A \) and \( I_F \), but in this paper, a different assumption is used that restoration force, \( Q_d \), reduces along with \( R_b \) and \( R_d \), independently. In addition, restoration force increases when drift angle is larger than \( 1/120 \) rad and the factor of restoration force increase, \( \kappa_r \), is introduced for evaluation of moderate and major damage (Fig. 2.1).
The condition of damage occurrence is:

\[ \kappa R_b R_d Q_d < Q_r. \]  \(2.5\)

By substituting Eqns. (2.1) to (2.4) in Eqn. (2.5),

\[ \frac{\kappa R_b R_d Q_d}{Q_r} = \kappa \frac{R_b R_d R_w Q_d}{S_a F_{es} Q_0} = \frac{\kappa R_b}{F_{es}} \cdot R_w R_d \frac{C_i g}{S_a} \]

\[ = \kappa I_A I_{BC} \alpha_{DE} I_{DE} I_F \frac{C_i g}{S_a} < 1, \]

i.e., \( I_A I_{BC} I_{DE} I_F < S_a / (\kappa \alpha_{DE} C_i g). \)  \(2.6\)

With respect to acceleration response, \( S_a \) is dependent to a natural period and damping of a structure. The lateral stiffness, \( S_t \), is:

\[ S_t = \alpha_{DE} I_{DE} S_{t0}, \]  \(2.7\)

where \( \alpha_{DE} S_{t0} \) is the stiffness of the first story of a building with the minimal restoration force regulated by the law. As a typical two-storied house, it is assumed that upper and lower mass ratio in a two-degree-of-freedom model is 3:4, story heights are 290 cm, and the fundamental mode shapes a straight line. The fundamental period of this model, \( T \), is:

\[ T \approx \frac{0.834}{\sqrt{\alpha_{DE}} I_{DE}} [s]. \]  \(2.8\)

In the vulnerability function, acceleration response, \( S_a (\alpha_P T) \) is used, where \( \alpha_P \) is a factor of natural period shift. Kohiyama (2006) studies strong motion indices that is correlated with damage of non-linear wooden structures, and maximum response of a certain linear elastic single-degree-of-freedom (SDOF) system shows good correlation. The damping factor of the well-correlated SDOF system is 20%. As for the natural period, a proportional shift is observed for the same threshold of damage; a factor of natural period shift, \( \alpha_P \), are 1.1, 1.5 and 1.9 for threshold of damage 1/60, 1/30 and 1/15 rad, respectively. Note that \( \alpha_P \) is dependent on threshold of damage but independent of \( \alpha_{DE} \). In this study, \( \alpha_{DE} = 3 \) is assumed based on an experiment result of a real scale structure (Isoda et al., 2007).

Suppose a probability density function, \( p_p(I_{DE}, I_{ABCF}) \), is given from seismic diagnosis results of houses in a target area, where \( I_{ABCF} = I_A I_{BC} I_F \). When a seismic motion with the maximum acceleration response \( S_a \) is
applied to the houses, the probability to yield damage, \( P_{f_{pp}} \), is defined as the following vulnerability function:

\[
P_{f_{pp}} = \int_{I_p} p \cdot P_{f_{p}} \left( I_{DE}, I_{ABC} \right) dI,
\]

where

\[
I_p = \left\{ \left( I_{DE} \cdot I_{ABC} \right) | I_{DE} \cdot I_{ABC} < \frac{Sa'(I_{DE})}{\kappa \alpha_{DE} C g} \right\},
\]

and \( Sa'(I_{DE}) \) is an acceleration response spectrum as a function of \( I_{DE} \), to which \( Sa \) is redefined.

3. VALIDITY STUDY AND COMPARISON WITH OTHER FRAGILITY CURVES

3.1. Study Area, Input Motion and Damage Statistics of Wooden Buildings

A validity study is conducted to check accuracy of the proposed vulnerability function using the same data that Kohiyama and Yamazaki (2004) used. The five strong motion records of the 1995 Hyogo-ken Nanbu (Kobe) earthquake are used, which Railway Technical Research Institute and the Committee of Earthquake Observation and Research in the Kansai Area provided. Note that the record of Amagasaki is corrected with respect to saturation (Kagawa et al. 1996). The five areas within a radius of 100 m from the observation points and within the same geological and geomorphological zones as the observation points are selected as the study areas. Numbers and ratios of damaged wooden houses in these five areas are shown in Tables 3.1 and 3.2, respectively; the damage data were surveyed by local governments. The calculation results of damage ratios are also shown in the tables.

Table 3.1 Number of damaged wooden houses surveyed by local governments in the five areas

<table>
<thead>
<tr>
<th>Damage levels</th>
<th>Areas of study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kakogawa</td>
</tr>
<tr>
<td>Major damage</td>
<td>0 in the city</td>
</tr>
<tr>
<td>Moderate damage</td>
<td>13 in the city</td>
</tr>
<tr>
<td>Minor damage</td>
<td>(no data)</td>
</tr>
<tr>
<td>No damage</td>
<td>(no data)</td>
</tr>
<tr>
<td>Total</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 3.2 Ratio of damaged wooden houses

<table>
<thead>
<tr>
<th>Damage Ratios</th>
<th>Areas of study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kakogawa</td>
</tr>
<tr>
<td>Major</td>
<td>0%</td>
</tr>
<tr>
<td>Moderate or more</td>
<td>Almost 0%</td>
</tr>
<tr>
<td>Minor or more</td>
<td>Almost 0%</td>
</tr>
</tbody>
</table>

3.2. Seismic Diagnosis Data Used in the Study

Seismic diagnosis data of wooden houses constructed before 1994 in Hyogo Prefecture was used in the study, which Mokutaikyo evaluated from July 2007 to January 2002. Based on the Housing and Land Survey in 1993, Kakogawa City, Suma Ward, Higashi-nada Ward, Amagasaki City and Takarazuka City had one-storied houses with the ratios of 10.7%, 8.4%, 11.7%, 11.4% and 9.5% among the detached houses, respectively. Hence, the seismic diagnosis data of 902 two-storied houses are used for simplicity.

In calculation of damage ratio by the proposed function, samples of the seismic diagnosis data rather than probability density function were used with weighting factors. Weighting factors are evaluated to adjust the distribution of seismic diagnosis data to the population of existing buildings based on the Housing and Land Survey in 1993. Table 3.3 shows the derived weighting factors. Note that a weighting factor \( x \), e.g. 2.57, in the table means that a single house of seismic diagnosis data is to be dealt as \( x \) houses in calculation of damage.
3.3. Evaluation of Damage Ratios and Comparison with Other Fragility Curves

Fig. 3.1 shows an example of evaluating damage ratios based on seismic diagnosis data and acceleration response spectra. A circle plotted in the graph stands for a sample of seismic diagnosis data of wooden houses. Three curves correspond to acceleration response spectra, which are projected to the $I_{DE}$-$I_{ABCF}$ plane considering three levels of thresholds of damage. Note that each curve is the mean of acceleration response spectra of north-south and east-west directions. In calculation of damage ratio, firstly whether damaged or undamaged is checked on a house basis based on Eqn. 2.10. Then the number of damaged houses is compiled considering weighting factors. Finally, the damage ratio is given by the ratio between the weighted counts of damaged house and a total number of houses. In Fig. 3.1, the circles under the curve represent damaged houses. Damage ratio is calculated considering weighting factors in Table 3.3. Calculated damage ratios are shown in Table 3.4.

![Figure 3.1 Scatter plot of seismic diagnosis data, ($I_{DE}$, $I_{ABCF}$), and curves with acceleration response spectra projected to $I_{DE}$-$I_{ABCF}$ plane for the study area in Takarazuka City](image)

Table 3.4 Estimated damage ratios of the proposed vulnerability function

<table>
<thead>
<tr>
<th>Damage ratios</th>
<th>Areas of study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kakogawa</td>
</tr>
<tr>
<td>Major</td>
<td>0.9%</td>
</tr>
<tr>
<td>Moderate or more</td>
<td>1.0%</td>
</tr>
<tr>
<td>Minor or more</td>
<td>9.0%</td>
</tr>
</tbody>
</table>

The calculated damage ratios are compared with those of fragility curves proposed by Murao and Yamazaki (2000), Sugiura and Yamazaki (2000) and Yamaguchi and Yamazaki (2000). Figs. 3.2(a) to (c) compare actual damage ratios shown in Table 3.2 and damage ratios calculated by the proposed vulnerability function and these fragility curves. Note that the proposed vulnerability function does not use peak ground velocity and these...
results are plotted in the same graph just for the comparison with those of other fragility curves. The root-mean-square (RMS) error evaluated from the five study areas are shown in Table 3.5.

<table>
<thead>
<tr>
<th>Vulnerability function and fragility curves</th>
<th>Root-mean-square errors of damage ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Major</td>
</tr>
<tr>
<td>Proposed vulnerability function</td>
<td>12.0%</td>
</tr>
<tr>
<td>Wooden construction in Nada (Murao and Yamazaki 2000)</td>
<td>13.3%</td>
</tr>
<tr>
<td>Wooden construction in Takarazuka (Sugiura and Yamazaki 2000)</td>
<td>3.9%</td>
</tr>
<tr>
<td>Wooden construction in Nishinomiya (Yamaguchi and Yamazaki 2000)</td>
<td>8.4%</td>
</tr>
<tr>
<td>Kohiyama and Yamazaki (2004)</td>
<td>–</td>
</tr>
</tbody>
</table>

It is observed that the proposed vulnerability function can uniquely evaluate ratios of minor or more damage and, for moderate or more damage, the RMS error of the proposed function is the smallest. It should be noted that, in the literature Murao and Yamazaki (2000), Sugiura and Yamazaki (2000) and Yamaguchi and Yamazaki (2000), fragility curves are also evaluated according to construction year, and RMS errors could be reduced if construction year is considered. Regarding ratio of major damage, the RMS error of the proposed function, 12.0%, is not the smallest. In Fig. 3.2(a), the proposed function overestimates damage ratio in all the target areas. Hence there is room to calibrate the following parameters: a threshold value of story drift angle for major damage (currently, 1/15 rad), a factor of stiffness increase, $\alpha_{DE}$, a factor of restoration force increase, $\kappa$, and a factor of natural period shift, $\alpha_p$. 

![Figure 3.2 Comparison between actual damage ratios and damage ratios calculated by the proposed vulnerability function and the fragility curves in past literatures](image)
6. CONCLUSIONS

A vulnerability function proposed by Kohiyama and Yamazaki (2004) is expanded to evaluate ratio of not only minor or more damage but also moderate or more damage and major damage. The functions employ seismic diagnosis data and an acceleration response spectrum, and they can reflect the regional difference in the distribution of natural periods of wooden houses. In formulation of the vulnerability functions, equivalent linear SDOF models are used, which is proposed by Kohiyama (2006); the natural periods of the models change based on threshold of damage.

The validity of the proposed vulnerability function is checked by using seismic records of the 1995 Hyogo-ken Nanbu (Kobe) earthquake and building damage data surveyed by local governments. It is observed that damage ratios estimated by the proposed vulnerability function coincide fairly well to the actual damage.

However, further investigation on the following parameters is required to improve accuracy of the proposed vulnerability function: a threshold value of story drift angle for major damage, a factor of stiffness increase, \( \alpha_{DE} \), a factor of restoration force increase, \( \kappa \), and a factor of natural period shift, \( \alpha_p \). In addition, it may be useful if database of probability density functions of seismic diagnosis scores are developed or a simplified form of the vulnerability function is proposed to avoid gathering and handling large number of seismic diagnosis data.
ACKNOWLEDGMENTS

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