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# FIRST MODE PROCEDURE IN RESPONSE SPECTRUM METHOD

**G. Magliulo[1], V. Pentangelo[2] and E. Cosenza[3]**

[1] *Assistant Professor, Dept. of Structural Engineering ,University of Naples "Federico II", Naples, Italy*
[2] *Ph D. Student, Dept. of Structural Engineering, University of Naples "Federico II", Naples, Italy*
[3] *Full Professor, Dept. of Structural Engineering, University of Naples "Federico II", Naples, Italy*
*Email: [1] gmagliul@unina.it, [2] vincenzo.pentangelo@unina.it, [3] cosenza@unina.it*

**ABSTRACT :**

The response spectrum method, as usually applied, provides the maximum positive values of individual response, but the critical combination of these responses may be characterized by negative values and may even not involve any of these maxima. In the present study the combination of these responses has been evaluated according to the three following procedures: 1) "first mode method", which gives the signs of the first mode to the maximum positive values of individual responses; 2) "diagonal method", which assumes all positive the maximum values of individual responses; 3) "rectangular envelope", which assumes every possible combination of the signs of the seismic responses (e.g. for column biaxial bending and axial load eight combinations are considered). The results show that the "first mode procedure" leads to the lowest quantity of reinforcement, i.e. it is favourable from an economic point of view; furthermore it is simple and very reliable.

**KEYWORDS:** Response spectrum method, envelopes of seismic response vector, first mode procedure, non linear dynamic analysis, orthogonal effects.

## 1. INTRODUCTION

In the last years the response spectrum method has been largely applied for analysis and design of structures, being provided by many international codes, as ASCE 07 (ASCE, 2005), EC8 (CEN, 2003a) and new Italian Seismic Codes (Norme Tecniche per le Costruzioni, 2008; OPCM 3431, 2005).
In the paper a very simple response spectrum based procedure for the evaluation of the seismic response is presented, assuming the ratio between the seismic components acting along the two orthogonal horizontal directions $\gamma$ equal to 1. It is applied to different structures (2D- and 3D- frames) and its results are compared to the corresponding ones obtained by the "rectangular envelope" procedure (Menun and Der Kiureghian, 2000a; Menun and Der Kiureghian, 2000b).

## 2. DESIGN OF THE ANALYZED STRUCUTURES

In this Section the elastic design of two very simple analyzed structures is reported: a 2-storey plane frame and a 2-storey space frame, whose geometry is shown in figures 1 and 2 respectively.

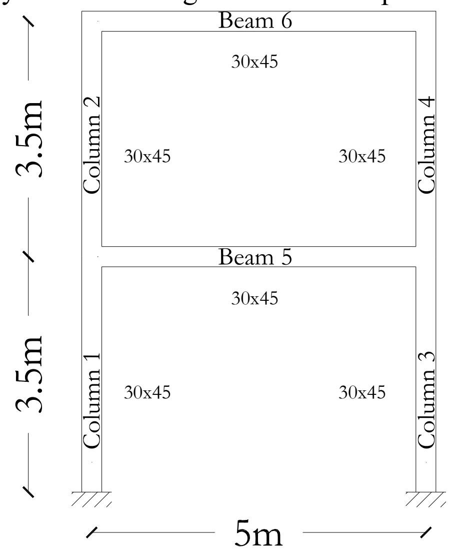


Figure 1 Plane frame: geometry and labels

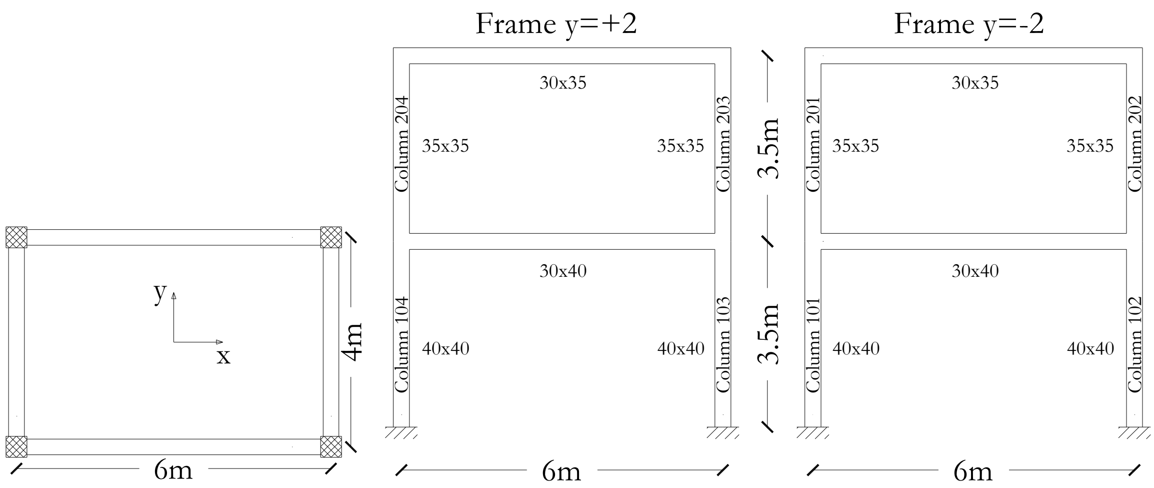


Figure 2 Space frame: plan and frames

Elastic analyses are performed by the computer program SAP2000 (CSI, 2004). Beams and columns are modelled as massless one dimension finite elements; mass is concentrated at floor levels, assumed rigid in their own planes. Consequently for each floor the plane frame is characterized by one DOF, while the space frame by three DOFs. The mass of the plane frame at the first floor is equal to 28 t, at the second one is equal to 24 t; for the space frame the corresponding masses are 37 t and 28 t respectively. In this last case the polar moment of inertia is computed considering an uniform mass distribution on the floor area. The periods associated to the two modal shapes of the plane frame are: 0.38 sec and 0.12 sec, while the periods of the first three modal shapes of the space frame are: 0.36 sec (X dir.), 0.33 sec (Y dir.) and 0.19 sec (rot).
Such frames are designed according to EC0 (CEN, 2002), EC1 (CEN, 2002), EC2 (CEN, 2004) and EC8 (CEN, 2003a), by modal response spectrum analysis. A soil B type 1 design spectrum with a stiff soil design acceleration $a_g$ =0.35g (OPCM 3431, 2005), is considered. The design is performed according to the DCM rules; for the plane frame the behaviour factor is equal to 3.6, while it is equal to 2.88 in the case of space frame.
Concrete characteristic cylindrical strength equal to $f_{ck}$= 30 N/mm$^2$ and steel characteristic yielding strength equal to $f_{yk}$=450 N/mm$^2$ are adopted.
As provided by the code, all nodes of the frame structure, unless top floor ones and those at foundations, satisfy the following design condition:

$$\sum M_{Rc} \geq \sum M_{Rb} \tag{2.1}$$

where $\sum M_{Rc}$ and $\sum M_{Rb}$ are the sum of design values of moments of resistance of columns and beams respectively framing the joint; this is satisfied considering both the signs of seismic action and, in the case of space frame, along both the orthogonal directions.
Considering that the usual response spectrum method provides the maximum positive values of individual responses, the combination of these responses is evaluated according to the three following procedures:
1) “First mode method”, which gives the signs of the fundamental mode in the considered direction to the maximum positive values of individual responses;
2) “Diagonal method”, which assumes all positive the maximum values of individual responses;
3) “Rectangular envelope” (Menun and Der Kiureghian, 2000a; Menun and Der Kiureghian, 2000b) which assumes every possible combination of the signs of the seismic responses (e.g. for column biaxial bending and axial load eight combinations are considered).
In the case of procedures 1) and 2), the orthogonal effects are evaluated by the 30% rule, according to the Eqn. (2.2), while the SRSS one (Eqn (2.3)) is adopted in the case of procedure 3); both the rules are prescribed by Euroceode 8 (CEN, 2003a).

$$E_x + 30\%E_y;\ E_x - 30\%E_y;\ -E_x + 30\%E_y;\ -E_x - 30\%E_y;\ 30\%E_x + E_y;\ 30\%E_x - E_y;\ -30\%E_x + E_y;\ -30\%E_x + E_y \tag{2.2}$$

$$E=(E^2_X+E^2_Y)^{1/2} \tag{2.3}$$

$E_x$ and $E_y$ are the peak responses due to a single component of ground motion defined by response spectrum applied along X and Y directions respectively; in the case of procedure 1) such responses are characterized by the signs of the fundamental mode in the considered direction.
For each procedure a variant of the design (called NCD), not following the capacity design rule at the beam-column ends, is also considered; this allows to compare the results of the three procedures without taking into account the overstrength given by such rule.

### *2.1. Plane frame reinforcement*

Column longitudinal reinforcement, resulting by the design performed according to the three described procedures, is listed in Table 2.1 and Table 2.2 (for NCD); one side end sections reinforcement is reported.

Table 2.1 Columns longitudinal reinforcement

| Col. | Length (m) | Dist (m) | Procedure 1) | | Procedure 2) | | | Procedure 3) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | ($cm^2$) | ($cm^2$) | ($cm^2$) | ($cm^2$) | Δ (%) | ($cm^2$) | ($cm^2$) | Δ (%) |
| 1 | 3.50 | 0.00<br>3.50 | 7.55<br>2.75 | 7.55<br>6.10 | 7.55<br>5.00 | 7.55<br>7.65 | 0<br>25 | 8.95<br>5.00 | 8.95<br>7.05 | 19<br>16 |
| 2 | 3.50 | 0.00<br>3.50 | 5.20<br>7.85 | 5.90<br>7.85 | 5.20<br>8.75 | 7.35<br>8.75 | 25<br>11 | 6.10<br>8.75 | 7.95<br>8.75 | 35<br>11 |
| 3 | 3.50 | 0.00<br>3.50 | 7.55<br>2.75 | 7.55<br>6.10 | 8.95<br>2.75 | 8.95<br>7.05 | 19<br>16 | 8.95<br>5.00 | 8.95<br>7.05 | 19<br>16 |
| 4 | 3.50 | 0.00<br>3.50 | 5.20<br>7.85 | 5.90<br>7.85 | 6.10<br>7.85 | 7.95<br>7.85 | 35<br>0 | 6.10<br>8.75 | 7.95<br>8.75 | 35<br>11 |

Table 2.2 Columns longitudinal reinforcement (NCD case)

| Col. | Length (m) | Dist (m) | Procedure 1) | | Procedure 2) | | | Procedure 3) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | ($cm^2$) | ($cm^2$) | ($cm^2$) | ($cm^2$) | Δ (%) | ($cm^2$) | ($cm^2$) | Δ (%) |
| 1 | 3.50 | 0.00<br>3.50 | 7.55<br>2.75 | 7.55<br>*5.21* | 7.55<br>5.00 | 7.55<br>*5.21* | 0<br>0 | 8.95<br>5.00 | 8.95<br>*5.21* | 19<br>0 |
| 2 | 3.50 | 0.00<br>3.50 | 5.20<br>7.85 | *5.21*<br>7.85 | 5.20<br>8.75 | *5.21*<br>8.75 | 0<br>11 | 6.10<br>8.75 | 6.10<br>8.75 | 17<br>11 |
| 3 | 3.50 | 0.00<br>3.50 | 7.55<br>2.75 | 7.55<br>*5.21* | 8.95<br>2.75 | 8.95<br>*5.21* | 19<br>0 | 8.95<br>5.00 | 8.95<br>*5.21* | 19<br>0 |
| 4 | 3.50 | 0.00<br>3.50 | 5.20<br>7.85 | *5.21*<br>7.85 | 6.10<br>7.85 | 6.10<br>7.85 | 17<br>0 | 6.10<br>8.75 | 6.10<br>8.75 | 17<br>11 |

In Tables 2.1 and 2.2, for each procedure, two columns of values are reported: in the left one the design reinforcement is considered without the minimum reinforcement prescribed by the Eurocode 8 (CEN, 2003a), which is reported in the right column by italic style; this allows to also compare the results of the three procedures without taking into account the minimum reinforcement. Finally, for procedures 2) and 3) the increment of reinforcement with respect to the procedure 1) considering the minimum reinforcement is also reported in percentage (Δ); for both procedures the maximum Δ is equal to 35% (Tab. 1), value which decreases to 19% in the NCD case (Tab. 2). The reported data also allow to state that: a) procedure 1) leads to the lowest quantity of reinforcement, i.e. it is the cheapest; b) even though the structure is symmetric and symmetrically loaded, procedure 2) provides unsymmetrical reinforcement (columns 1 and 3 do not have the same reinforcement as well as columns 2 and 4).

### *2.2. Reinforcement of space frame*

Total longitudinal reinforcement at initial and terminal sections of columns of the 2-storey space frame is listed in Table 2.3. The results of the three design procedures are reported as in Table 1; the NCD case provides reinforcement which satisfies the capacity design rules, consequently it is not considered.

Table 2.3 Columns longitudinal reinforcement

| Col. | Length (m) | Dist (m) | Procedure 1) | | Procedure 2) | | | Procedure 3) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | ($cm^2$) | ($cm^2$) | ($cm^2$) | ($cm^2$) | Δ (%) | ($cm^2$) | ($cm^2$) | Δ (%) |
| 101 | 3.50 | 0.00 | 19.20 | 19.20 | 19.20 | 19.20 | 0 | 32.00 | 32.00 | 67 |
| | | 3.50 | 6.80 | *16.00* | 11.20 | *16.00* | 0 | 14.80 | *16.00* | 0 |
| 102 | 3.50 | 0.00 | 19.20 | 19.20 | 21.60 | 21.60 | 13 | 32.00 | 32.00 | 67 |
| | | 3.50 | 6.80 | *16.00* | 10.40 | *16.00* | 0 | 14.80 | *16.00* | 0 |
| 103 | 3.50 | 0.00 | 19.20 | 19.20 | 22.40 | 22.40 | 17 | 32.00 | 32.00 | 67 |
| | | 3.50 | 6.80 | *16.00* | 6.80 | *16.00* | 0 | 14.80 | *16.00* | 0 |
| 104 | 3.50 | 0.00 | 19.20 | 19.20 | 20.00 | 20.00 | 4 | 32.00 | 32.00 | 67 |
| | | 3.50 | 6.80 | *16.00* | 8.40 | *16.00* | 0 | 14.80 | *16.00* | 0 |
| 201 | 3.50 | 0.00 | 13.20 | 13.20 | 13.20 | 13.20 | 0 | 20.40 | 20.40 | 55 |
| | | 3.50 | 15.20 | 15.20 | 16.80 | 16.80 | 11 | 23.20 | 23.20 | 53 |
| 202 | 3.50 | 0.00 | 13.20 | 13.20 | 14.00 | 14.00 | 6 | 20.40 | 20.40 | 55 |
| | | 3.50 | 15.20 | 15.20 | 16.40 | 16.40 | 8 | 23.20 | 23.20 | 53 |
| 203 | 3.50 | 0.00 | 13.20 | 13.20 | 14.40 | 14.40 | 9 | 20.40 | 20.40 | 55 |
| | | 3.50 | 15.20 | 15.20 | 15.20 | 15.20 | 0 | 23.20 | 23.20 | 53 |
| 204 | 3.50 | 0.00 | 13.20 | 13.20 | 14.00 | 14.00 | 6 | 20.40 | 20.40 | 55 |
| | | 3.50 | 15.20 | 15.20 | 15.60 | 15.60 | 3 | 23.20 | 23.20 | 53 |

Each section is reinforced by eight bars, four at the corners and four at the middle point of each side, which have the same diameter; the cover is assumed equal to 4 cm.
As in the case of plane frame, procedure 1 leads to the lowest quantity of reinforcement: the increment given by procedure 2) is equal to 17%, while it is 67% applying procedure 3). Procedure 2) provides unsymmetrical reinforcement.

## 3. NON LINEAR DYNAMIC ANALYSIS

### *3.1. Modelling*

The presented structures, designed according to procedure 1, are analyzed by non linear dynamic analyses, performed by means of the computer program CANNY99 (Li, 1996).
Non linearity concerns flexural rotations, while all the other deformations are assumed elastic. Both beams and columns are characterized by lumped plasticity models; in the latter case for each section two independent non linear springs are assigned, one for each orthogonal direction. No axial force-bending moment interaction is considered at plastic hinge.
Bending moment springs are characterized by a tri-linear skeleton curve, defined by cracking and yielding moment and corresponding rotations; the post-yielding stiffness is assumed equal to zero. Such moments and the corresponding curvatures are computed considering for concrete under compression a parabola-rectangle diagram. It is characterized by maximum and ultimate strength equal to the medium value for concrete 30/37 according to EC2 (CEN, 2004), i.e. 38 N/mm$^2$; a strain value at the end of the parabola equal to 0.2% and an ultimate strain equal to 0.35% are assigned. The concrete Young modulus and concrete maximum tensile strength are also computed according to EC2 (CEN, 2004). An elastic-perfectly plastic steel stress-strain diagram is considered; it is characterized by an yielding strength equal to 530 N/mm$^2$, computed as mean of tests on more than 200 bars, made by steel called FeB44K, performed at the laboratory of Structural Engineering Department of University of Naples "Federico II". A steel maximum strain equal to 1% and a

Young modulus equal to 200000 N/mm$^2$ are assigned.
The cracking rotation is computed multiplying the corresponding curvature by L/6, where L is the length of the element. The yielding and the ultimate rotations are evaluated as provided by Eurocode 8 (CEN, 2003b) equations (A.10b) and (A.1) respectively, where the already reported average values are assigned to concrete maximum ($f_c$ = 38 N/mm$^2$) and steel yielding ($f_y$ = 530 N/mm$^2$) strength.
The hysteretic model is Takeda type, even though in CANNY99 (Li, 1996) the pinching effect is also taken into account and a small value of the unloading stiffness is assigned, i.e. in each cycle it is reduced by 50% with respect to the previous one; a less degrading model is assumed for columns with respect to beams: in CP7 model (Li, 1996) for beams the value of all the 3 coefficients $\gamma$, $\xi$ and $\lambda$ is always 0.5, while for columns $\xi$ is changed and assumed equal to 1.0 (Faella et al., 2000).

### *3.2. Seismic Input*

Both the horizontal components of a set of 7 earthquakes (Table 3.1), i.e. 14 natural records, are used for non linear dynamic analyses; according to the selection procedure presented in (Iervolino et al., 2006), they satisfy the Eurocode 8 (CEN, 2003) provisions: the mean of zero period spectral response acceleration values (calculated from individual time histories) should not be smaller than the value of $a_g \cdot S$ (for the site in question); in the range of periods between 0.2$T_1$ and 2$T_1$, where $T_1$ is the fundamental period of the structure, in the direction where the accelerogram is applied, no value of the mean 5% damping elastic spectrum, computed from all time histories, should be less than 90% of the corresponding value of the 5% damping elastic Eurocode 8 (CEN, 2003a) response spectrum; if the response is obtained from at least 7 non linear time histories analyses, the average of response quantities should be used as the design value of the action effect $E_d$ in relevant verifications.

Table 3.1 Earthquakes used for non linear dynamic analyses

| Earthquake label | Earthquake name | Earthquake Country | Date |
|---|---|---|---|
| 000187 | Northern and central Iran | Iran | 16/09/1978 |
| 000196 | Montenegro | Yugoslavia | 15/04/1979 |
| 000199 | Montenegro | Yugoslavia | 15/04/1979 |
| 000230 | Montenegro (aftershock) | Yugoslavia | 24/05/1979 |
| 000291 | Campano lucano | Italy | 23/11/1980 |
| 006263 | South Iceland | Iceland | 17/06/2000 |
| 006334 | South Iceland (aftershock) | Iceland | 21/06/2000 |

According to the European Strong-Motion Database, the labels of the two horizontal components of the earthquakes listed in Table 3.1 are characterised by X or Y as they are recorded along N-S or E-W direction respectively. In the case of the plane frame, the two components of each earthquake are separately applied along the frame direction, consequently the input is represented by 14 accelerograms; in the case of the space frame, the analyses are performed applying, for each earthquake, the X component along the longitudinal direction of the building (Figure 2) and the Y component along the orthogonal direction. The non linear response of the space frame is also evaluated varying the input angle of incidence, i.e. rotating by $\pi/6$ rad both the components of each earthquake from 0 to 2$\pi$ rad; consequently 12 angles of incidence are considered.

### *3.3. Analysis Results*

*3.3.1. Plane frame*

Results are given in terms of flexural rotations.
In particular a single analysis provides a couple of values, max and min, corresponding to two possible signs of rotations for every section of structural element; the average of 14 values for each sign provides the required rotation. Obviously, in the case of columns, the maximum rotations are computed along both the directions and, for each of them, the maximum absolute value is computed among positive and negative maximum.
Ultimate rotation, i.e. $\theta_u$, is obtained according to Eurocode 8 (CEN, 2003b); then it is multiplied by 0.75 because the Ultimate Limit State is considered (CEN, 2003b).
All the maximum values are divided by the corresponding available rotations at Ultimate Limit State, obtaining four ratios for each structural element, i.e. two of them for each element end section. In Table 3.2 the maximum ratio for each beam and for each column for both the design conditions, with and without (NCD case) the application of capacity design, is listed.

Table 3.2 Beams and columns: demanded/available chord rotation ratio at Ultimate Limit State

| Beam | | NCD case | Column | | NCD case |
|---|---|---|---|---|---|
| 5 | 0.29 | 0.31 | 1$^{st}$storey | 0.30 | 0.29 |
| 6 | 0.33 | 0.23 | 2$^{nd}$storey | 0.16 | 0.16 |

Table 3.2 shows that the analysed frame, designed according procedure 1), is largely verified at Ultimate Limit State, even though such procedure leads to the lowest quantity of reinforcement.

*3.3.2. Space frame*

In the case of space frame the demanded rotation is obtained as the average of 7 values, because seismic load is given by a couple of accelerogram acting simultaneously along structural axes. Consequently, also considering the variability of input incidence angle θ (12 values in [0, 2 π[),7 x 12 =84 analyses are performed.
In Table 3.4, for each input incidence angle, the maximum demanded/available chord rotation ratio at Ultimate limit State among all the beams and all the columns is listed; the Table is referred only to $0 \le \theta < \pi$, because of the structural symmetry. In Table 3.4 is also listed the variation in percentage with respect to the case cheraterised by the input components parallel to the structural axes ($\theta = 0$).

Table 3.4 Demanded/available chord rotation ratio at Ultimate Limit State varying the input incidence angle

| θ (rad) | Beams | Variation (%) | Columns | Variation (%) |
|---|---|---|---|---|
| 0 | 0.295 | - | 0.210 | - |
| π/6 | 0.290 | -1,69 | 0.207 | -1,43 |
| π/3 | 0.263 | -10,85 | 0.205 | -2,38 |
| π/2 | 0.307 | 4,07 | 0.212 | 0,95 |
| 2π/3 | 0.299 | 1,36 | 0.203 | -3,33 |
| 5π/6 | 0.283 | -4,07 | 0.198 | -5,71 |
| **MAX** | **0.307** | **4.07** | **0.212** | **0.95** |

Observing Table 3.4, it can be stated: 1)varying θ, differences are negligible; this is probably due to the structural symmetry; 2) as in the case of plane frame, the analysed building, designed according procedure 1), is largely verified at Ultimate Limit State, even though such procedure leads to the lowest quantity of reinforcement.

## 4. CONCLUSIONS

The "first mode procedure", which gives the signs of the fundamental mode in the direction of analysis to the maximum positive values of individual responses obtained from modal combination rules (CQC or SRSS), seems to be a valid tool for the application of the Response Spectrum Method mainly in consideration of: 1) the difficulty of applying elliptical and supreme envelope in the elastic analysis of structures; 2) the problems which the "diagonal method" can cause, i.e. unsymmetrical reinforcement in symmetric and symmetrically loaded structures; 3) the savings in amount of reinforcement with respect to the application of other procedures.

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