EVALUATION OF COEFFICIENT AND EQUIVALENT LINEARIZATION METHOD OF FEMA-440 FOR SOIL-STRUCTURE SYSTEMS

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ABSTRACT

In recent years, Nonlinear Static Procedures (NSPs) have been attracted numerous researchers. The purpose of developing these methods is to predict the nonlinear demands of structures in a simple way with an acceptable accuracy. The whole MDOF structure converts to an equivalent nonlinear SDOF via pushover analysis and the nonlinear displacement of this SDOF system approximates approximately the nonlinear displacement of the whole structure. In order to find the nonlinear displacement of SDOF system varied methods are proposed in which Coefficient Method and Equivalent Linearization Method are the most applicable ones. It could be referred to FEMA-273, FEMA-356, and ATC-40 as the pioneer documents on this issue. It was also presented in FEMA-450 in design procedures. However, the aforementioned procedures were modified in FEMA-440 by analyzing a fixed base SDOF through 20 ground motions recorded at each site class categorized by FEMA-450. Moreover, the effect of Soil Structure Interaction on nonlinear demands in NSPs was discussed more elaborately than previous documents. As it was mentioned, since all the optimizations of FEMA-440 were performed for a SDOF model fixed at its base, its accuracy for soil structure system should be evaluated. In this study all the analysis were done in response spectra format for key influential non-dimensional parameters in soil-structure systems in order to make the result more comprehensive and applicable for wide ranges of ordinary structures and soil conditions.

KEYWORDS: Soil Structure Inertial Interaction, Nonlinear Analysis, Equivalent Linearization, Coefficient Method, FEMA-440.

1. INTRODUTION

In current Performance Based Seismic Design, it is desirable to estimate the nonlinear demands by a simple and accurate procedure. Structural model types and characterization of seismic loads categorized the inelastic analysis in FEMA-440. The selection of one option over another depends on the purpose of the analysis, the anticipated performance objectives, the acceptable level of uncertainty, the availability of resources, and the sufficiency of data. In some cases, applicable codes and standards may dictate the analysis procedure. However, the initial discussion is whether to choose inelastic analysis over conventional elastic analysis. Definitely for those performance objectives that imply greater inelastic displacements, the elastic analysis could not lead to appropriate results. Nonlinear Dynamic Analysis can be inferred as the most precise method in which the detailed structural model subjected to a ground-motion record produces estimates of component deformations for each degree of freedom in the model. The detailed structural model could be replaced by equivalent MDOF system and subjected to ground motions records or response spectra; indicated by Simplified MDOF Dynamic Analysis or Multi-Mode Pushover Analysis respectively in FEMA-440. Further simplification of considering the structure by an equivalent SDOF system could be made. This equivalent system is derived via pushover analysis and if it is subjected to response spectra, it is so called Nonlinear Static Analysis. Although it has more uncertainty with respect to other mentioned methods, the convenience in practice and its acceptable accuracy make this method the most practical one and it is the basis of current rehabilitation documents like FEMA-356 and ATC-40 for calculating inelastic demands. Figure-1 summarizes the relationship among the normal options for inelastic seismic analysis procedures with respect to the type of structural model and characterization of ground motion. Also noted in the figure is the relative uncertainty associated with each option.

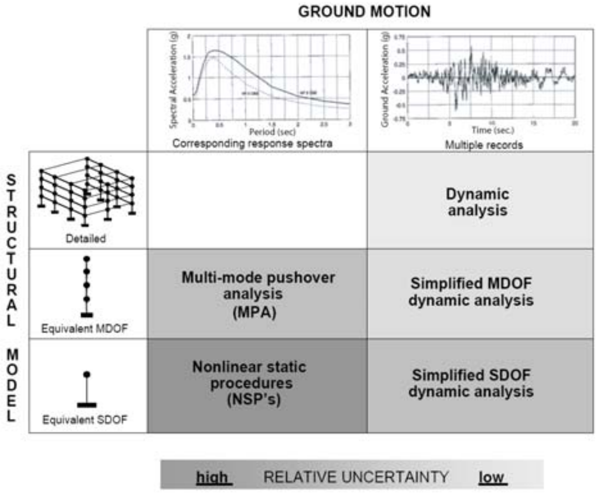


Figure-1 Possible nonlinear seismic analysis for various structural model and ground motion characterization

In nonlinear analysis, the response spectra indicated in Figure-1 should be its corresponding nonlinear spectra. There are also some relations known as $R-\mu-T$ relations that generate approximate nonlinear spectra for different types of hysteretic behavior of the equivalent SDOF, site condition, earthquake magnitude, and etc like what proposed by Vidic et al (1994) or Miranda (1993). Also there are some techniques proposed to find the inelastic displacement by elastic spectra. For instance, FEMA-356 and FEMA-450 are employing Nonlinear Static Procedures so called *Coefficient Method* to estimate the target roof displacement. We can also allude to ATC-40 guideline using Nonlinear Static Procedure with a different approach so called *Equivalent Linearization*.

1.1. Coefficient Method

The basic concept of the Coefficient Method is to convert the elastic linear displacement of equivalent SDOF to its inelastic displacement with some modifying coefficient. Considering a SDOF structure with hysteretic behavior shown in Figure-2, the maximum nonlinear displacement could be computed by equation (1).

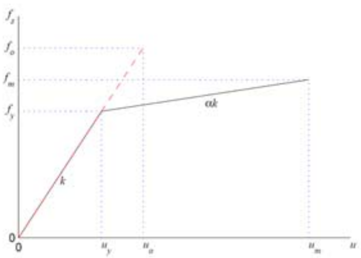
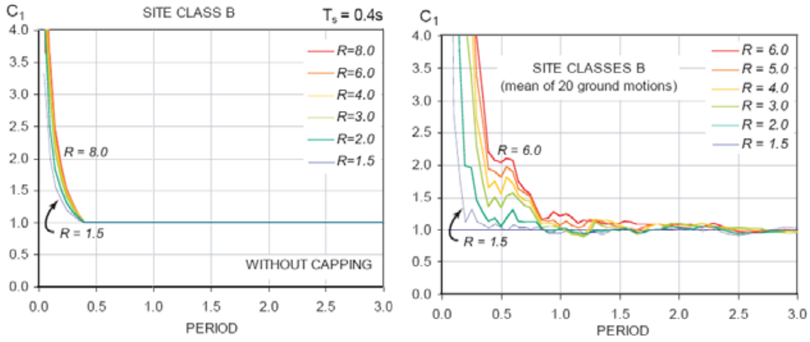


Figure-2 Bilinear behavior of a SDOF system

$$u_m = \frac{\mu}{R}\left(\frac{T}{2\pi}\right)^2 A_o = \frac{\mu}{R} u_o \tag{1.1}$$

Where $A_o = S_a(T, \xi = 5\%)$ is the corresponding response spectra at the period of the structure, T. Equation (1.1) indicates that the maximum inelastic displacement could be calculated by multiplying the elastic displacement to the ratio of the ductility, μ, and the Strength Reduction Factor, R, of the structure. This ratio is defined as Inelastic Displacement Ratio, known as C_1 in Coefficient Method. Elaborate investigation on this ratio were performed in Chopra and Chintanapakdee (2004), and Ruiz-Garcia and Miranda (2003 and 2006). The investigations revealed that the relation proposed in FEMA-356 is not accurate enough and cannot predict the inelastic demands sufficiently well. The Figure-3a shows this ratio defined in FEMA-356 and Figure-3b is its relevant values which are the average time history results of 20 ground motions recorded on site class B for different Strength Reduction Factors.



(a) (b)

Figure-3 Comparison between C_1 defined by FEMA-356 and average values of 20 ground motions

Therefore, by statistical analysis of a SDOF structure with wide ranges of periods at diverse levels of strength reduction factor, the coefficients were redefined in FEMA-440. The analyses were carried out for 20 ground motions recorded at each site class: B, C, D, E, and also near field records. In this technique the maximum inelastic displacement is estimated by equation (1.2), where the C_o is the modification factor to relate spectral displacement of an equivalent SDOF system to the roof displacement of the MDOF building system, C_2 is the modification factor to represent the effect of pinched hysteretic shape, stiffness degradation, and strength deterioration on the maximum displacement response, and C_3 is the modification factor to represent increased displacements due to dynamic P-Δ effects.

$$u_m = C_o C_1 C_2 C_3 \left(\frac{T}{2\pi}\right)^2 A_o \qquad (1.2)$$

For SDOF structures with elastoplastic behavior the coefficients C_o, C_2, and C_3 are unit and C_1 is calculated by equation (1.3), where α depends on the site class of where the structure is located and it is 130, 90, and 60 for site classes B, C, and D respectively.

$$C_1 = 1 + \frac{(R-1)}{\alpha T^2} \qquad (1.3)$$

It was also stated that for soft soil sites, site class E, equation (1.3) could be used with $\alpha = 60$ although the above equation is not appropriate enough for this site class. Ruiz-Garcia and Miranda (2006) proposed following equation for this site type in which the first two items are almost the right terms of equation (1.3) and the third and forth terms are generated to consider the relative minimum values produced at the predominant ground motion period, T_g, and second dominant ground motion period that estimated about $0.33T_g$. The θ_i values are constant coefficients that determined by regression analysis at specific strength reduction factor.

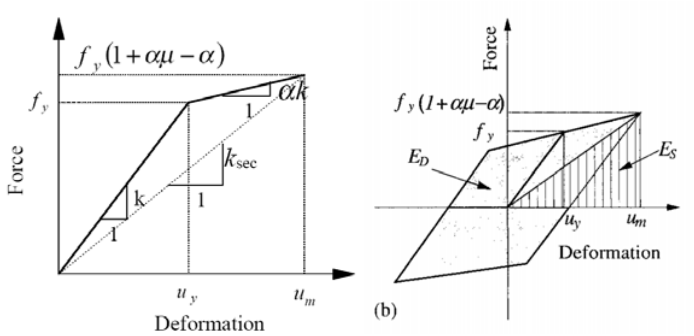
$$C_1 = \theta_1 + \frac{(R-1)}{\theta_2 \left(T/T_g\right)^2} + \frac{\theta_3}{\left(T/T_g\right)} \exp\left[-4.5\left\{Ln\left(\frac{T}{T_g} - 0.05\right)\right\}^2\right] + \frac{\theta_4}{\left(T/T_g\right)} \exp\left[\theta_5\left\{Ln\left(\frac{T}{T_g} + 0.67\right)\right\}^2\right] \qquad (1.4)$$

The most important difference of equation (1.4) and (1.3) is the role of predominant ground motion period, T_g, where the structural periods are normalized to it in order to decrease the dispersion of statistical results.

1.2. Equivalent Linearization

In Equivalent Linearization Method the inelastic equivalent SDOF convert to its equivalent elastic linear SDOF and then the displacement demands is determined by the elastic linear system. That is, an equivalent period and damping is defined in a way that the inelastic displacement would be close to elastic displacement of equivalent system. Assuming bilinear behavior of structure shown in Figure-4a the equivalent period was defined as the Secant period in ATC-40 and the equivalent viscous damping ratio defined by $\frac{1}{4\pi}\frac{E_D}{E_{SO}}$. These equivalent parameters are rewritten in equation (1.5). The κ is defined to represent hysteretic behavior of the structure.

$$T_{eq} = T\sqrt{\frac{\mu}{1+\alpha\mu-\alpha}} \qquad \xi_{eq} = \xi + \kappa\frac{2}{\pi}\frac{(\mu-1)(1-\alpha)}{\mu(1-\alpha\mu-\alpha)} \tag{1.5}$$



(a) (b)

Figure-4 the basic approach of Equivalent Linearization of ATC-40

Through comprehensive studies by Chopra and Goel (2000) or Fragiacomo et al (2006) unacceptability of ATC-40 regulation was declared, and consequently by an optimization statistical analysis explained in the index-D of FEMA-440, modified relations proposed by FEMA-440 committee for calculating equivalent period and damping ratio. It should be mentioned that like Coefficient Method, the response of fixed base SDOF system is used for this modification. Equivalent periods and damping ratios consistent with target ductility ratio are rewritten in equation (1.6) and (1.7). The coefficients A to L depend on hysteretic behavior of a SDOF and its post yield stiffness ratio. To sum up, in this technique instead of nonlinear analysis of a system with period T and damping ratio ξ, elastic linear analysis of an equivalent system with effective period T_{eff} and effective damping ratio ξ_{eff} could estimate the displacement demand with enough accuracy in engineering purposes. Since the ductility demand is unknown in equation (1.6) and (1.7), this technique has a trial procedure. The difference of effective period and effective damping definition of some previous researches are depicted in Figure-5.

for $\mu < 4$: for $4 \le \mu \le 6.5$: for $\mu > 6.5$:

$$\xi_{eff} = A(\mu-1)^2 + B(\mu-1)^3 + \xi \qquad \xi_{eff} = C + D(\mu-1) + \xi \qquad \xi_{eff} = E\left[\frac{F(\mu-1)-1}{F(\mu-1)^2}\right]\left(\frac{T_{eff}}{T}\right)^2 + \xi \tag{1.6}$$

$$T_{eff} = \left[G(\mu-1)^2 + H(\mu-1)^3 + 1\right]T \qquad T_{eff} = \left[I + J(\mu-1) + 1\right]T \qquad T_{eff} = \left\{K\left[\sqrt{\frac{(\mu-1)}{1+L(\mu-2)}} - 1\right] + 1\right\}T \tag{1.7}$$

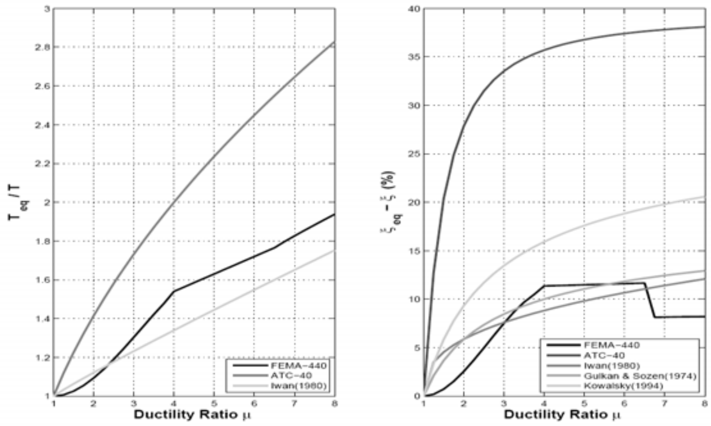


Figure-5 Difference of equivalent period and damping definition in some previous studies

1.3. Including Soil Structure Interaction in Nonlinear Static Procedures

For the elastic structure, the pioneer Veletsos and Nair (1975) showed that the effects of inertial interaction could be approximated by increasing the fundamental period and changing the damping of a fixed base structure. This approach is applied in seismic design codes and standards. This equivalent period and damping is indicated in relations (1.8):

$$T_{ssi} = T\sqrt{1+\frac{k}{k_x}\left(1+\frac{k_x h^2}{k_\theta}\right)} \qquad \tilde{\xi} = \xi_f + \frac{\xi}{\left(T_{ssi}/T\right)^3} \tag{1.8}$$

In equation (1.8), k is the structural stiffness, k_x is the dynamic frequency dependant horizontal stiffness of soil and k_θ is its relevant stiffness in rocking direction, h is the effective height of the structure, and ξ_f is the radiation damping depended on slender ratio of structure and T_{ssi}/T. Indeed, the displacement demands are determined by the equivalent fixed base oscillator with period T_{ssi} and damping ratio $\tilde{\xi}$. However, in rehabilitation procedures like FEMA-356 or ATC-40 the SSI effects were limited to consider its effect on increasing natural period of the system. In FEMA-440 a modification is done for considering its equivalent damping ratio that for inelastic behavior of structure it is somehow different from what is determined by relation (1.8). The basis of this modification is to replace the inelastic structure of period T with its effective period T_{eff}, representing elastic linear response of structure, and find this equivalent damping ratio by equation (1.8). Therefore, this equation is changed to equation (1.9) where the $\left(T_{ssi}/T\right)$ term changes to $\left(\tilde{T}_{eff}/T_{eff}\right)$ that could be estimated by equation (1.10). As can be seen, there is a trial procedure since the assumed equivalent ductility, $\tilde{\mu}$, in this step should be checked with its value at the end of the procedure.

$$\xi_o = \tilde{\xi}_f + \frac{\xi}{\left(\tilde{T}_{eff}/T_{eff}\right)^3} \tag{1.9}$$

$$\frac{\tilde{T}_{eff}}{T_{eff}} = \left\{1+\frac{1}{\tilde{\mu}}\left[\left(\frac{T_{ssi}}{T}\right)^2 - 1\right]\right\}^{0.5} \tag{1.10}$$

Similar to design codes, the basis of rehabilitation documents on this issue is to convert the soil structure system to its equivalent fixed base oscillator and use the relations and procedures derived originally with ignoring flexibility of the foundation like equations (1.6) and (1.7). Actually, the procedure for deriving equations (1.6) and (1.7) for Equivalent Linearization Method or equation (1.3) for Coefficient Method is somehow complicated and time consuming and consequently it is worth having this approach to refrain from analyzing the soil structure systems again to find their corresponding effective periods or effective damping or inelastic displacement ratios, provided this approach have enough accuracy and have convenience in practice. In using these equations cautious should be taken that soil structure equivalent period and damping, T_{ssi} and ξ_o must be used instead of T and ξ. Moreover, the ductility ratios indicated in the equations is the ductility ratio of the equivalent fixed base oscillator and not the pure ductility of structure. In other words, the displacements are comprised of structural displacements and soil movements. Furthermore, the certainty of the whole analysis should be checked; that is, how much the traditional replacement oscillator method works for nonlinear structure soil systems. The procedure of Equivalent Linearization Method of FEMA-440 is illustrated in Figure-6.

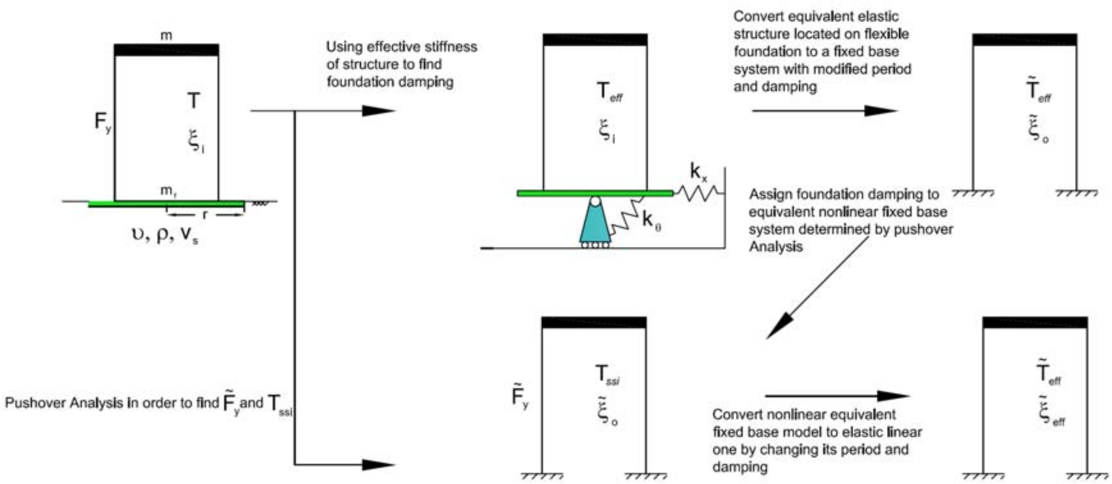


Figure-6 illustration of Equivalent Linearization method of FEMA-440

It could be referred to Bielak (1978) who was firs studied the SSI effects for nonlinear structures. He stated that the resonant structural deformation could be significantly larger than would result if the supporting soil were rigid; nevertheless that could not clarify how soil effects affect the ductility demand or displacement demand of structure. It has been believed that yielding is a kind of energy dissipation process and thus leads to decrease the importance of interaction. However, latter studies highlight this effect more even for nonlinear structures. In Aviles and Perez-Rocha (2003, 2005) it was declared that the SSI effect for yielding systems is still notable and ignoring it in some structural periodic ranges could lead to unconservative results. The analyses were performed by defining nonlinear replacement oscillator with equivalent ductility $\tilde{\mu}$ in which relates to structural ductility by equation (1.11):

$$\tilde{\mu}=\left(\frac{T}{T_{ssi}}\right)^2(\mu-1)+1 \tag{1.11}$$

This issue was attracted by Ghannad and Ahmadnia (2006) and Ghannad and Jahankhah (2007) in which the effect of SSI and Site Effect on strength reduction factors were studied simultaneously.

2. SOIL STRUCTURE MODEL

A simplified model shown in Figure-7 is employed to compute exact objective parameters. The structure is an elasto-plastic SDOF system with stiffness k and period T. m and h are lumped mass and height of the structure, which can be extended to the effective mass and height of MDOF structures. The mass moment of inertia is labeled I. Moreover, the foundation is assumed as a circular disk.
The soil beneath the foundation is considered as a homogeneous half space and is modeled by Fundamental Lumped Mass Parameters based on the concepts of Cone Models, extended by Wolf (1994), representing the

soil with a three DOF system. This model, with fixed parameters, is capable of including frequency dependency of dynamic soil stiffness.
The response of a soil structure system generally depends on the size of the structure, its dynamic properties, soil profile, and the applied excitation. The influence of these factors can be described by the following non-dimensional parameters; a non-dimensional frequency as an index for structure to soil stiffness ratio defined as $a_o = \frac{2\pi h}{Tv_s}$, where T is the period of the structure in its fixed base condition. The practical range of a_o for ordinary building type structures is from zero for the fixed base structures; to about three for cases with dominant SSI affects; aspect ratio of the structure, defined as $S = \frac{h}{r}$; ductility demand of structure $\mu = \frac{u_m}{u_y}$, where u_m and u_y are the maximum displacement due to specific base excitation and the yield displacement, respectively; structure to soil mass ratio index, $\tilde{m} = \frac{m}{\rho r^2 h}$, where ρ is the unit weight of the soil. $\tilde{m}$ is taken to be 0.47; the ratio of the mass of the foundation to that of the structure is defined as $\tilde{m}_f = \frac{m_f}{m}$. We assume it to be 0.1 in all parts of our analysis; poison's ratio of soil, υ, chosen 0.5, in that our time histories were recorded on soft soil sites; material damping of the soil and the structure. We set damping ratio of the structure to 5% as is usual, but the damping ratio of the soil to be zero, mass moment inertia I and I_f taken as $\frac{1}{4}mr^2$ and $\frac{1}{4}m_f r^2$ respectively for simplicity.

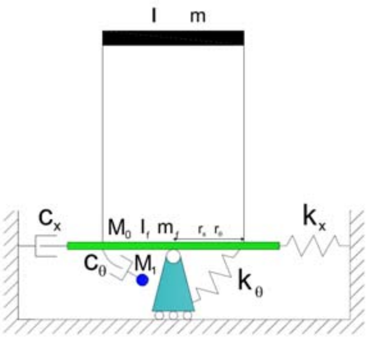
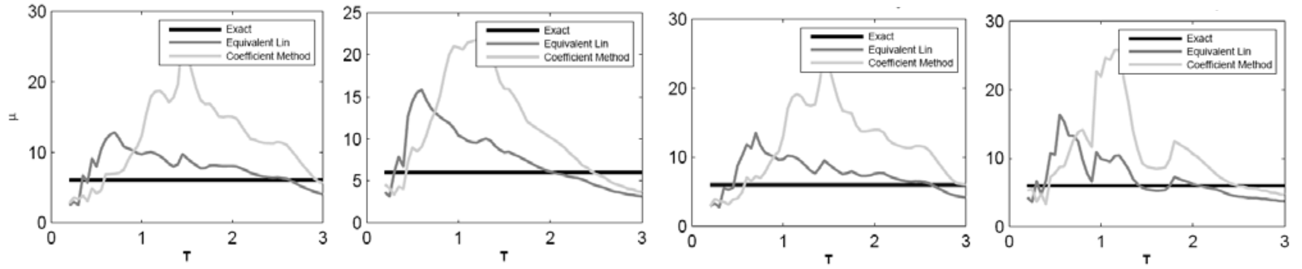


Figure-7 Soil-Structure System

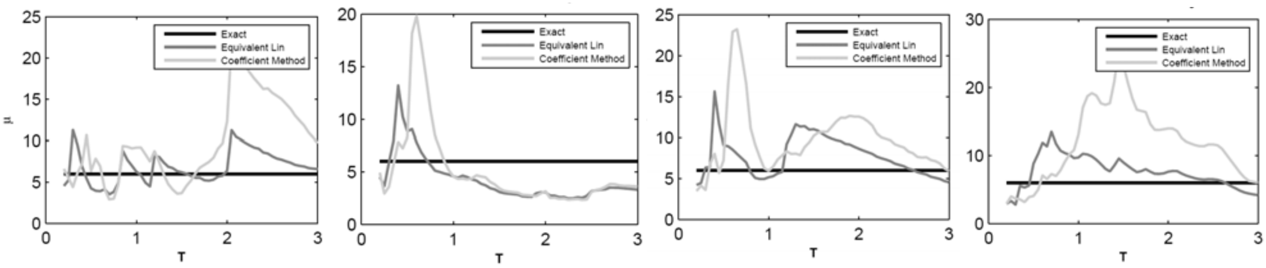
3. EVALUATION OF EQUIVALENT LINEARIZATION AND COEFFICIENT METHOD OF FEMA-440 IN SOIL-STRUCTURE SYSTEMS

In Figure-8 the analysis is performed for different sets of S and a_o for at structural ductility $\mu = 6$. As it can be seen, Equivalent Linearization Method leads to conservative result at short period ranges of the spectra in whole sets of non-dimensional sets of parameters. But this is somehow different for Coefficient Method where this conservative result, which is seemed to be unaccepted, happens at different periodic ranges for each set of S and a_o. Indeed, this behavior occurs when the soil-structure system's period is close to predominant ground motions. To clarify this, Figure-9 depicted the results for different records that indicate that Coefficient Method leads to unacceptably conservative results. Predominant ground motion periods for these records are 2.77sec, 1.1sec, 1.0sec and 1.49sec respectively. The ground motions are for Loma Prieta (1989) earthquake recorded at different stations.
Also Figure-10, the average ductility ratios of 13 ground motions recorded on soft soil sites, shows that, as mentioned before, Equivalent Linearization Method leads to conservative results for short period range and almost appropriate results for other ranges.



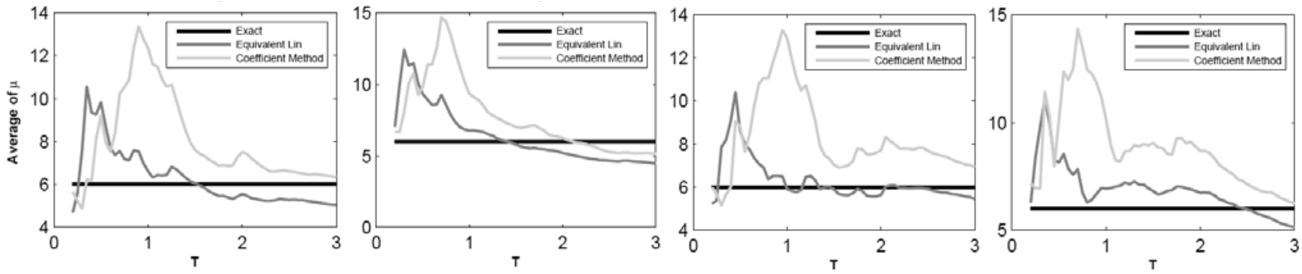
(a) $S = 1, a_o = 1$ (b) $S = 1, a_o = 2$ (c) $S = 2, a_o = 1$ (d) $S = 2, a_o = 2$

Figure-8 Comparison between Equivalent Linearization and Coefficient Method for ground notion recorded at the station Larkspur Ferry Terminal for different sets of S and a_o



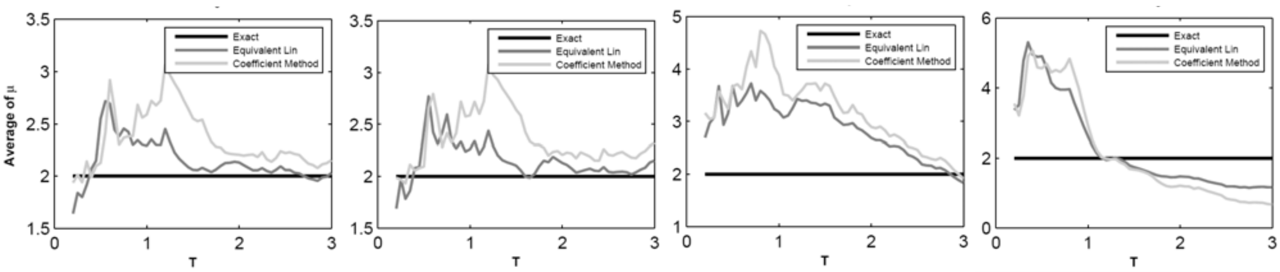
(a) $S = 2, a_o = 2$ (b) $S = 1, a_o = 3$ (c) $S = 2, a_o = 3$ (d) $S = 2, a_o = 1$

Figure-9 Comparison between Ductility response spectra; (a) station 58375; (b) station 1590 (USGS); (c) station 1002 (USGS); (d) station 1662 (USGS)



(a) $S = 1, a_o = 2$ (b) $S = 1, a_o = 3$ (c) $S = 2, a_o = 2$ (d) $S = 2, a_o = 3$

Figure-10 average structural ductility demands of 13 ground motions recorded on soft soil sites for $\mu = 6$



(a) $S = 1, a_o = 1$ (b) $S = 2, a_o = 1$ (c) $S = 2, a_o = 3$ (d) $S = 1, a_o = 3$

Figure-11 average structural ductility demands of 13 ground motions recorded on soft soil sites for $\mu = 2$

It should be added that, in FEMA procedures, an approximate relation is introduced to determine response of the equivalent damped system (damping is produced from foundation or nonlinear behavior of structure) from the response of 5% damping ratio elastic spectra. Nevertheless; elastic time history analysis is done to find those results. This leads to refrain from the errors coming from the approximate relation, and concentrates the errors on SSI effects.

4. CONCLUSION

The procedure of Equivalent Linearization and Coefficient Method of FEMA-440 are investigated for soil-structure systems. The investigation declares that Equivalent Linearization gives conservative results in low structural period ranges, but acceptable results for medium and long structural period ranges. However, the Coefficient Method does not lead to proper results. This method overestimates enormously when the period of soil-structure system is close to predominant site period. Therefore, recommendation is made to determine a equation for C_1 for soft soil condition sites, which is dependent on the predominant ground motion period, and also including SSI effects. In this case, the equivalent nonlinear oscillator for inelastic soil structure system is not required and the responses are derived by elastic linear analysis of soil- structure system having enough accuracy in engineering purposes.

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