DIRECT HAZARD ANALYSIS OF INELASTIC RESPONSE SPECTRA

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ABSTRACT

In this paper we present results of direct probabilistic and deterministic hazard analyses of inelastic response spectra. Inelastic response spectra of over 3100 horizontal components of ground motions recorded in 64 shallow crsutal earthquakes in active tectonic regions were computed. This comprehensive database of inelastic response spectra was used to develop a ground motion prediction equation (GMPE), “attenuation” model, for inelastic spectra. For given displacement ductility ratio, two-step nonlinear regressions, as well as random effects analyses, were performed on the inelastic spectral ordinates. The GMPE is capable to correlate inelastic spectral ordinates to moment magnitude, fault rupture distance, faulting mechanism, local site condition, and basin depth. The GMPE for inelastic spectra was subsequently used to carry out probabilistic hazard analysis for inelastic spectra. In this computational process, deterministic and probabilistic inelastic responses were directly computed without any assumption and approximation on the relationship between inelastic and elastic response spectra. Given a site and the desired probability of exceedance, one can use the results of this study to directly estimate the inelastic response demanded by earthquake ground motion.

KEYWORDS: Attenuation, Ground Motion Prediction Equation, Inelastic Response Spectra, Ductility

1. INTRODUCTION

There have been major advances in ground motion prediction equations (GMPEs), or “attenuation” models, for peak ground motion values and elastic spectral ordinates. Most significantly is the recent completion of a very comprehensive multidisciplinary research program, Next Generation Attenuation relations for shallow crustal earthquakes in active tectonic regions (NGA-West). The NGA-West was a multi-year program coordinated by the Pacific Earthquake Engineering Research Center (PEER), in collaboration with many individuals and organizations (Power, et al., 2008).

In contrast to elastic response spectra, there are only very few studies on GMPEs for inelastic response (e.g., Lawson, 1996; Chou and Uang, 2000; Bozorgnia et al. 2006; Tothong and Cornell, 2006). In this study we used a very comprehensive database of inelastic response spectra compiled by Hachem and Bozorgnia (2006), and developed GMPEs for inelastic spectral ordinates for constant displacement ductility ratios ranging from 1 to 8. For each displacement ductility ratio, a set of GMPE was developed. Therefore, we made no apriori approximations on deriving inelastic response spectra from that of elastic spectra. The inelastic spectra database used to develop the “attenuation” models is one of the largest inelastic databases ever compiled. This database enabled us to capture scaling of inelastic spectral ordinates with fundamental parameters such as earthquake magnitude, site-to-source distance, style of faulting, local site conditions, basin effects, among others. We subsequently used the developed GMPEs for inelastic spectra in a probabilistic seismic hazard analysis (PSHA) computer package and directly computed probabilistic seismic hazard for inelastic spectra at a site in northern California. In this paper we provide an overview of the process of developing GMPEs and PSHA for inelastic spectra.

2. DATABASE

The ground motion database used for this study is the ground motion database used by Campbell and Bozorgnia (2007, 2008) for their development of GMPE for NGA elastic response spectra. The database includes 3122 horizontal records from 64 earthquakes with moment magnitudes (**M**) ranging from 4.3–7.9 and rupture distances (R_{RUP}) ranging from 0.1–199 km. A complete list of the selected earthquakes and recording stations are given in Appendix A of Campbell and Bozorgnia (2007). A single-degree-of-freedom (SDF) inelastic oscillator with varying initial period T; an elastic-perfectly-plastic (EPP) force-deformation relationship; and 5% linear viscous damping, was subjected to each of the 3122 ground motion records (Hachem and Bozorgnia, 2006). The EPP force-deformation idealization is a major simplification of the real nonlinear behavior; however, it is one step closer to the reality than the traditional linear elastic SDF system and its elastic response spectra.

Different versions of inelastic response spectra were computed for each of the recorded ground motions, as elaborated by Hachem and Bozorgnia (2006). Definitions of the fundamental parameters and various forms of inelastic spectra can be found in, e.g., Bozorgnia and Campbell (2004). In the current study, we used constant ductility inelastic response spectra for displacement ductility ratio μ =1, 2, 4, 6, and 8. For each value of μ and initial period T, GMPE was developed to correlate the inelastic spectral ordinates to the fundamental parameters such as earthquake magnitude, site-to-source distance, etc., as explained below.

3. GROUND MOTION PREDICTION EQUATIONS FOR INELASTIC SPECTRA

In order to be consistent with the GMPE developed in the NGA program for *elastic* response spectra, we used the same functional forms as used in the regression analyses by Campbell and Bozorgnia (2007, 2008). Examination of the residuals of the GMPEs for inelastic spectra also revealed suitability of such functional forms. The details of the functional forms, and bases for each term, are given by Campbell and Bozorgnia (2007, 2008). In summary, the median GMPE is given by:

$$\ln Y_{ij} = f_{mag} + f_{dis} + f_{flt} + f_{hng} + f_{site} + f_{sed} \tag{3.1}$$

where the magnitude term is given by the expression

$$f_{mag} = \begin{cases} c_0 + c_1\mathbf{M}; & \mathbf{M} \le 5.5 \\ c_0 + c_1\mathbf{M} + c_2(\mathbf{M}-5.5); & 5.5 < \mathbf{M} \le 6.5 \\ c_0 + c_1\mathbf{M} + c_2(\mathbf{M}-5.5) + c_3(\mathbf{M}-6.5); & \mathbf{M} > 6.5 \end{cases} \tag{3.2}$$

the distance term is given by the expression

$$f_{dis} = (c_4 + c_5\mathbf{M})\ln\left(\sqrt{R_{RUP}^2 + c_6^2}\right) \tag{3.3}$$

the style-of-faulting (fault mechanism) term is given by the expressions

$$f_{flt} = c_7 F_{RV} f_{flt,Z} + c_8 F_{NM} \tag{3.4}$$

$$f_{flt,Z} = \begin{cases} Z_{TOR}; & Z_{TOR} < 1 \\ 1; & Z_{TOR} \ge 1 \end{cases} \tag{3.5}$$

the hanging-wall term is given by the expressions

$$f_{hng} = c_9\, f_{hng,R}\, f_{hng,M}\, f_{hng,Z}\, f_{hng,\delta} \tag{3.6}$$

$$f_{hng,R} = \begin{cases} 1; & R_{JB} = 0 \\ \left[\max\left(R_{RUP}, \sqrt{R_{JB}^2 + 1}\right) - R_{JB}\right] \Big/ \max\left(R_{RUP}, \sqrt{R_{JB}^2 + 1}\right); & R_{JB} > 0, Z_{TOR} < 1 \\ (R_{RUP} - R_{JB}) / R_{RUP}; & R_{JB} > 0, Z_{TOR} \geq 1 \end{cases} \tag{3.7}$$

$$f_{hng,M} = \begin{cases} 0; & \mathbf{M} \leq 6.0 \\ 2(\mathbf{M} - 6.0); & 6.0 < \mathbf{M} < 6.5 \\ 1; & \mathbf{M} \geq 6.5 \end{cases} \tag{3.8}$$

$$f_{hng,Z} = \begin{cases} 0; & Z_{TOR} \geq 20 \\ (20 - Z_{TOR}) / 20; & 0 \leq Z_{TOR} < 20 \end{cases} \tag{3.9}$$

$$f_{hng,\delta} = \begin{cases} 1; & \delta \leq 70 \\ (90 - \delta) / 20; & \delta > 70 \end{cases} \tag{3.10}$$

the shallow site response term is given by the expression

$$f_{site} = \begin{cases} c_{10} \ln\left(\dfrac{V_{S30}}{k_1}\right) + k_2 \left\{ \ln\left[A_{1100} + c\left(\dfrac{V_{S30}}{k_1}\right)^n\right] - \ln\left[A_{1100} + c\right] \right\}; & V_{S30} < k_1 \\ (c_{10} + k_2 n) \ln\left(\dfrac{V_{S30}}{k_1}\right); & k_1 \leq V_{S30} < 1100 \\ (c_{10} + k_2 n) \ln\left(\dfrac{1100}{k_1}\right); & V_{S30} \geq 1100 \end{cases} \tag{3.11}$$

and the sediment depth (basin response) term is given by the expression

$$f_{sed} = \begin{cases} c_{11}(Z_{2.5} - 1); & Z_{2.5} < 1 \\ 0; & 1 \leq Z_{2.5} \leq 3 \\ c_{12} k_3 e^{-0.75}\left[1 - e^{-0.25(Z_{2.5} - 3)}\right]; & Z_{2.5} > 3 \end{cases} \tag{3.12}$$

In the above equations, Y_{ij} is the median estimate of the geometric mean horizontal component of the "seismic coefficient" $C_Y = F_Y / W$ for an inelastic SDF system with yield strength F_Y and weight W, for recording j of event i; **M** is moment magnitude; R_{RUP} is the closest distance to the coseismic rupture plane (km); R_{JB} is the closest distance to the surface projection of the coseismic rupture plane (km); F_{RV} is an indicator variable representing reverse and reverse-oblique faulting, where $F_{RV} = 1$ for $30° < \lambda < 150°$, $F_{RV} = 0$ otherwise, and λ is the rake defined as the average angle of slip measured in the plane of rupture between the strike direction and the slip vector; F_{NM} is an indicator variable representing normal and normal-oblique faulting, where $F_{NM} = 1$ for $-150° < \lambda < -30°$ and $F_{NM} = 0$ otherwise; Z_{TOR} is the depth to the top of the coseismic rupture plane (km); δ is the dip of the rupture plane (°); V_{S30} is the time-averaged shear-wave velocity in the top 30 m of the site profile (m/s); A_{1100} is the median estimate of PGA on a reference rock outcrop with V_{S30} = 1100 m/s (g); and $Z_{2.5}$ is the depth to the 2.5 km/s shear-wave velocity horizon, typically referred to as basin or sediment depth (km). For each value of ductility, the empirical coefficients c_i are computed through nonlinear regression analysis.

The inter-event standard deviation (τ) and intra-event standard deviation (σ) of the analyses are combined to result in a total standard deviation:

$$\sigma_T = \sqrt{\sigma^2 + \tau^2} \tag{3.13}$$

The performance of GMPEs is assessed by examination of residuals, i.e., the difference between the observed and predicted values. Examples of plots of residuals versus magnitude and fault rupture distance are presented in Figures 1 and 2, respectively. These and other residuals, not shown here, versus other parameters such as rake angle, V_{s30}, and PGA on rock, indicate the suitability of the model as a GMPE for inelastic response spectra.

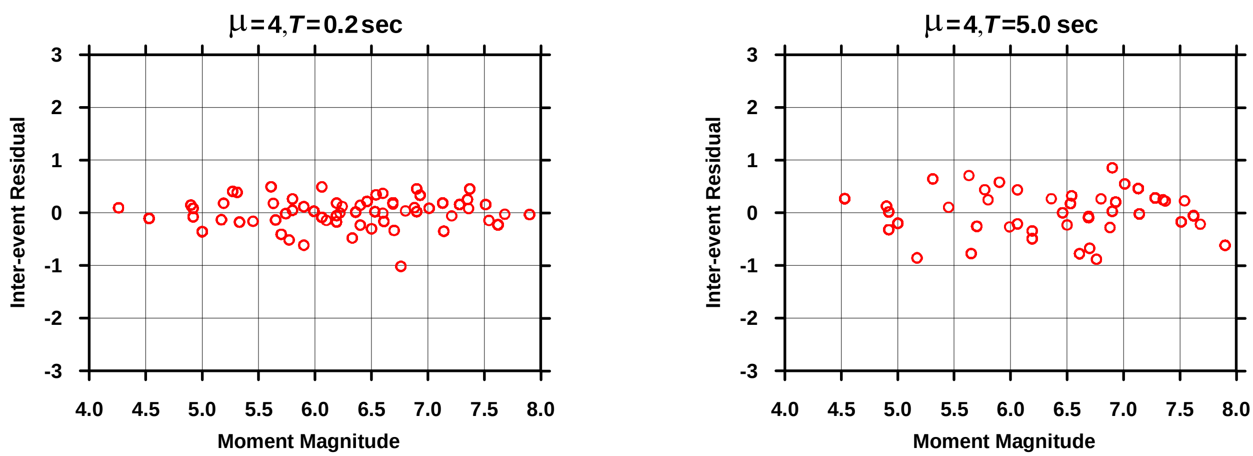

Figure 1. Examples of inter-event residuals for “seismic coefficient” *Fy/W* as a function of earthquake magnitude for ductility ratio 4.0, and periods 0.2 sec (left) and 5.0 sec (right).

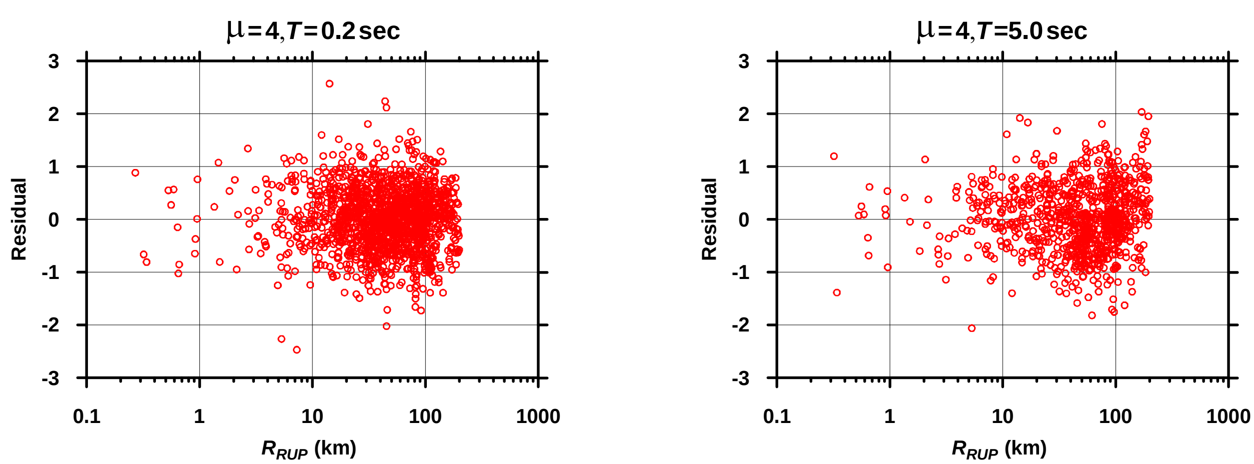

Figure 2. Examples of total residuals for “seismic coefficient” *Fy/W* as a function of $\boldsymbol{R_{RUP}}$ for ductility ratio 4.0, and periods 0.2 sec (left) and 5.0 sec (right).

4. DETERMINISTIC AND PROBABILISTIC PREDICTION OF INELASTIC RESPONSE

Selected median inelastic spectral ordinates for various ductility factors are presented in this section. Figure 3 shows attenuation of median seismic coefficients (F_Y/W) for different magnitudes for initial periods 0.2 and 1.0 sec for ductility factor 4. The magnitude saturation at short period (0.2 sec) is evident in this figure as the seismic coefficients for magnitudes 6.5 and 7.5 approach each other at short rupture distances. Conceptually this behavior also exists in prediction of elastic response spectra in NGA model of Campbell and Bozorgnia (2007, 2008).

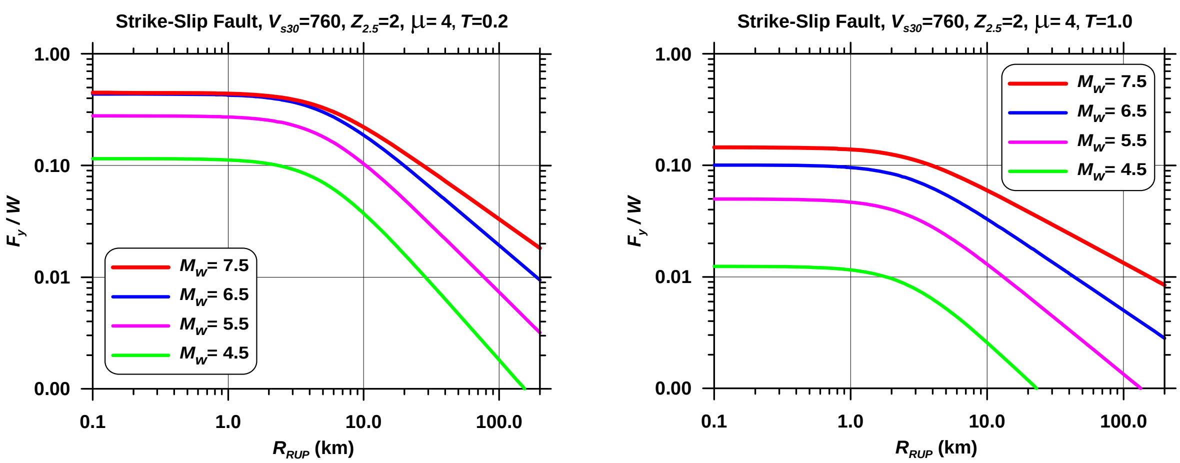

Figure 3. Examples of attenuation of seismic coefficient *Fy/W* for different moment magnitudes and periods 0.2 sec. (left) and 1.0 sec. (right).

Predicted median inelastic spectra for a moment magnitude 7.5 and distances 1 and 10 km for various ductility ratios are plotted in Figure 4. In this figure dashed lines are the results of the Campbell & Bozorgnia (C&B) NGA GMPE. It is noted that the C&B NGA model is based on the rotation-independent geometric mean of elastic response spectra, and the results of the current study are based on the geometric mean of the spectra of the as-recorded ground motions. Even with this difference, the results of the C&B elastic NGA and those of the

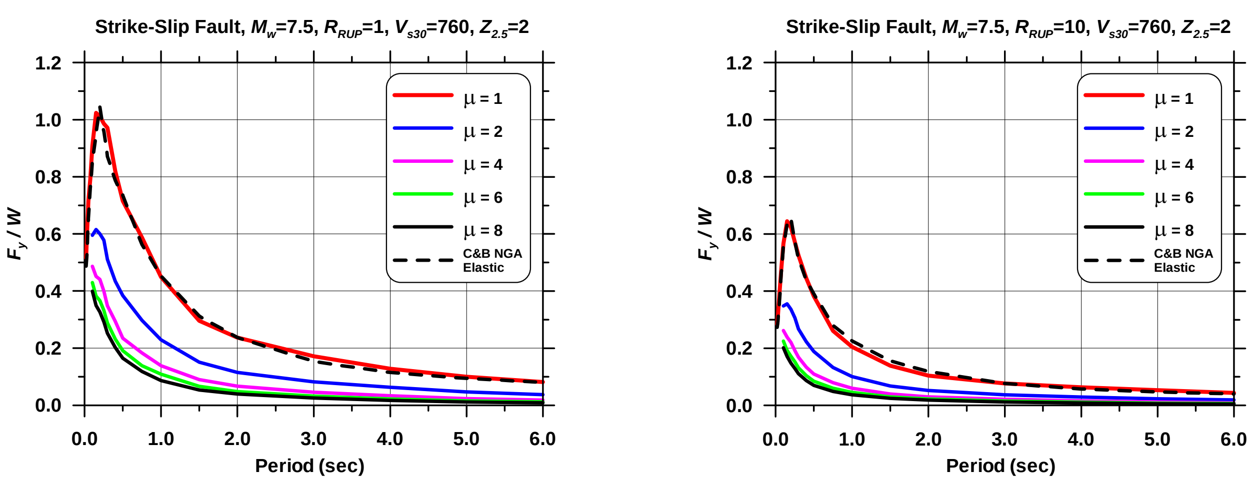

Figure 4. Examples of median seismic coefficient *Fy/W* predicted for a magnitude 7.5 event at rupture distances 1 km (left) and 10 km (right) for different displacement ductility ratios.

current study for μ=1 are very comparable at short periods. The two elastic results, however, can deviate at longer periods of the predicted displacement, due to smoothing and applying constraints on the computed coefficients in the C&B NGA model. As expected, the seismic coefficient decreases by allowing some degree of inelastic deformation. This reduction from elastic strength demand is relatively substantial even when moderate ductility is incorporated into design.

The total standard deviation (σ_T in equation 3.13) as a function of period is plotted in Figure 5. This figure shows that the standard deviation is not sensitive to the ductility level.

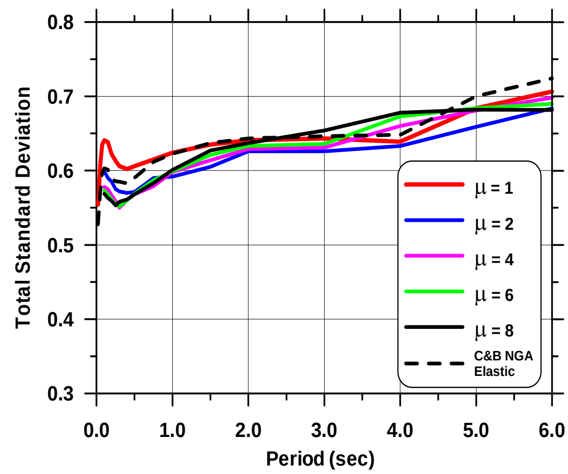

Figure 5. Total standard deviation of the GMPEs for various ductility ratios.

The GMPE developed in this study has the same general functional form as that developed by Campbell & Bozorgnia (2007, 2008). Therefore, any PSHA computer package that already incorporates the C&B NGA model, can easily be modified to use the "attenuation" of inelastic response spectra. One such PSHA computer package is OpenSHA (Field, et al., 2003) which already incorporates the C&B NGA (elastic) model. In this study, OpenSHA was used to carry out the direct PSHA computation for inelastic response spectra. To demonstrate the concept, inelastic PSHA was carried out for a site in northern California located about 20 km east of the Hayward Fault. Example results of the PSHA results are presented in Figure 6. This figure shows probability of exceedance in 50 years for the seismic coefficient *Fy/W*. By choosing the level of probability of exceedance and expected available ductility, one can easily estimate the seismic coefficient demanded by the ground shaking.

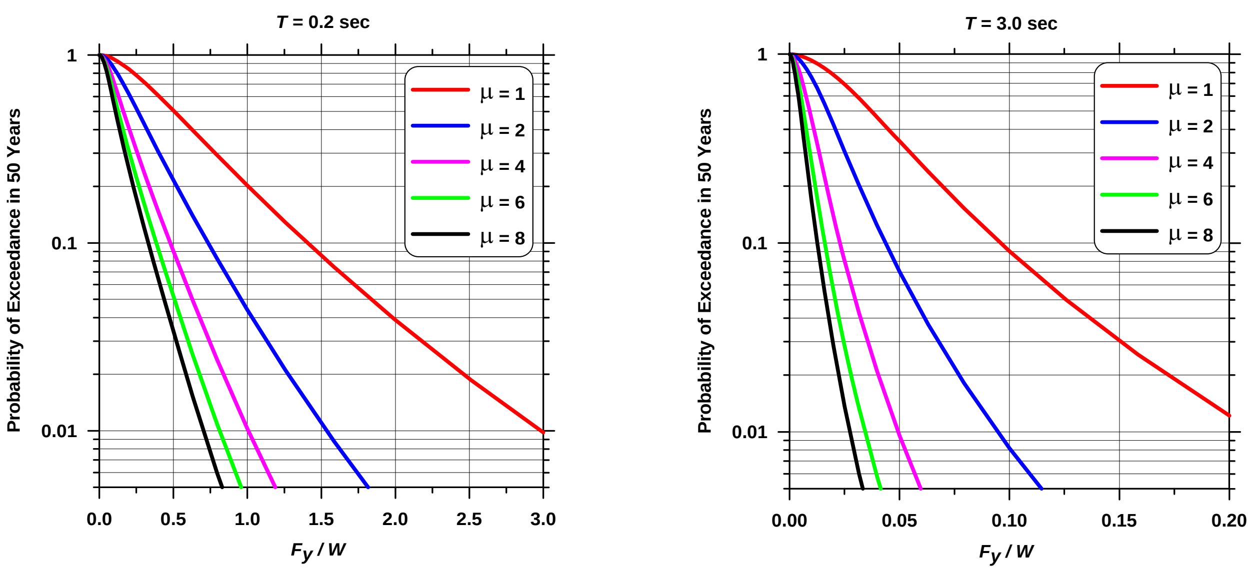

Figure 6. Examples of PSHA results for inelastic response spectral ordinates *Fy/W* for different ductility ratios and periods 0.2 sec. (left) and 3.0 sec. (right).

The same concept can be used to plot hazard curves for the maximum deformation of the inelastic system demanded at a desired probability level.

5. CONCLUDING REMARKS

We computed inelastic response spectra for thousands of ground motions recorded in shallow crustal earthquakes in active tectonic regions. This is one of the largest databases of uniformly processed inelastic response spectral ordinates. Using such a comprehensive database of inelastic spectra, we developed ground motion prediction equations (GMPEs), or "attenuation" models, for inelastic response spectra. The GMPEs are associated with different levels of ductility ranging from unity (i.e., elastic response) to 8. We subsequently used the newly developed GMPEs for inelastic spectra to carry out probabilistic seismic hazard analysis (PSHA). OpenSHA, an open source PSHA computer package, was used for the probabilistic hazard analysis of inelastic response at a site in northern California.

In this study the entire deterministic and probabilistic seismic hazard analyses are based on direct prediction of inelastic response demanded by earthquake ground motions. In this direct computational process we do not need to assume any simplifying approximation on the relationship between elastic and inelastic responses. In fact, such a relationship can be further investigated by using the results of the current study. For example, it is possible to carry out a detailed examination of classical assumptions, e.g. "constant-displacement rule", as a function of the fundamental ground motion parameters.

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