ON THE H/V SPECTRUM

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ABSTRACT:

The H/V spectral ratio is overviewed from its origin and on the application for practical disaster prevention. First of all the H/V spectral ratio was found from the strong motion records at various sites in Japan. On the soft ground, horizontal motion is larger than vertical motion. On the other hand, on the hard ground, both horizontal and vertical motions are similar to each other both on the maximum value and waveform. First, horizontal to vertical ratio was derived from each maximum value and was compared with the softness of ground and the amplification factor. As a result, horizontal to vertical ratio of maximum value highly corresponds to these ground characteristics. On the next step, the H/V spectral ratio of microtremor measured at anywhere was confirmed that it is able to estimate the predominant frequency and the amplification factor for any researchers, and the result of estimation is stable for the measured time and season. The H/V spectral ratio shows the amplification characteristics by the multiple reflections of the SH wave at least around F_0, the predominant frequency of sedimentary layer, and shows the characteristics confused by the Rayleigh wave around $2F_0$. In case of less effect of the Rayleigh wave it is possible to estimate not only preliminary peak but also secondary peak of amplification characteristics caused by multiple reflections with the H/V spectral ratio.

1. INTRODUCTION

Damages caused by the recent earthquakes are concluded as a direct result of local geological conditions affecting the ground motion. The best approach for understanding ground conditions is through direct observation of the seismic ground motion, but such studies are restricted to areas with relatively high seismicity. Because of these restrictions in other methods, such as only at high seismicity area and the availability of an adequate reference site, non-reference site methods have been applied to the site response studies. Microtremor is a very convenient tool to estimate the effect of surface geology on seismic motion without other geological information.

The H/V technique, alias QTS, Quasi-Transfer Spectra, fits very well to this description and it has received great attention from all over the world with its simplicity together with quick information about dynamic characteristics of ground and structures. Although several researchers claimed that the theoretical background of this technique is not clear, there have been many successful experimental studies performed. This method is attractive since it gives the ease of data collection and it can be applied in areas of low or even no seismicity.

The H/V technique was developed by the author with relating borehole investigations together with the strong motion records analysis, on the various geological site conditions. It was hypothesized that the vertical component of the ambient noise at the ground surface keeps the characteristics of basement ground, is relatively influenced by Rayleigh wave on the sediments and can therefore be used to remove both of the source and the Rayleigh wave effects from the horizontal components.

It is effective to identify the fundamental resonant frequency of a sedimentary layer, with implied amplification factors that are more realistic than those obtained from sediment to rock site ratios. It has been shown by many

researchers (ex. Ohmachi et. al., 1991; Lermo et. al., 1992; Field and Jacob, 1993, 1995) that how such H/V ratio of noise can be used to identify the fundamental resonant frequency and amplification factor of sediments.

Looking to the examples in the study of Nogoshi and Igarashi (1971) which compared the H/V of Rayleigh wave with that of microtremor and concluded that microtremor was mostly composed of Rayleigh wave, some of theoretical studies (Lachet and Bard, 1994; Konno and Ohmachi, 1998; Bard, 1998) suggested that the peak on H/V can be explained with the fundamental mode of Rayleigh waves. If we think that this approach is true, microtremor should be considered to consist of only Rayleigh wave. On the other hand, if we check the examples given on Nogoshi and Igarashi (1971) carefully (which will be discussed later), we can clearly see that, at the peak frequency of H/V of Rayleigh wave, the energy of Rayleigh wave is very small, nearly zero. Rayleigh wave has its maximum energy at near trough frequency of H/V. Because of this, peak of the H/V of microtremor cannot be explained by the energy of Rayleigh wave. As it is explained by Nakamura (1989), the H/V of microtremor at peak frequency range can be explained with vertical incident SH wave.

2. ORIGIN OF THE H/V TECHNIQUE

First of all the H/V spectral ratio was found from the strong motion records at various sites in Japan. On the soft ground, horizontal motion is larger than vertical motion. On the other hand, on the hard ground, both horizontal and vertical motions are similar to each other both on the maximum value and waveform.
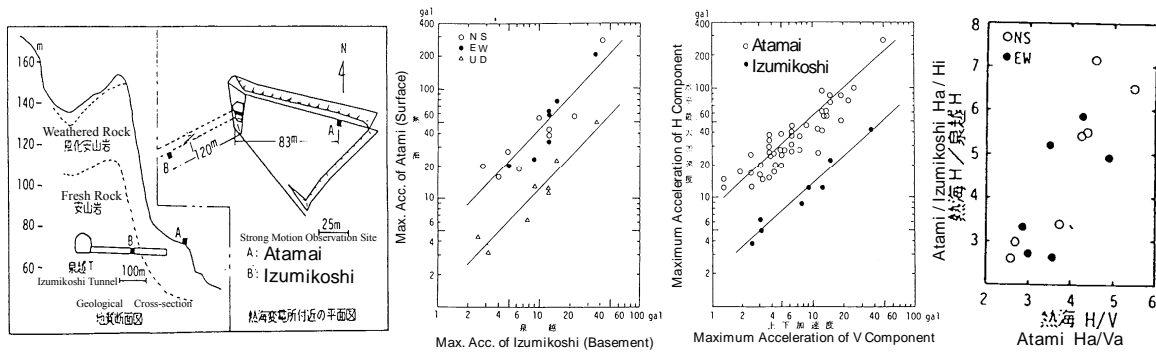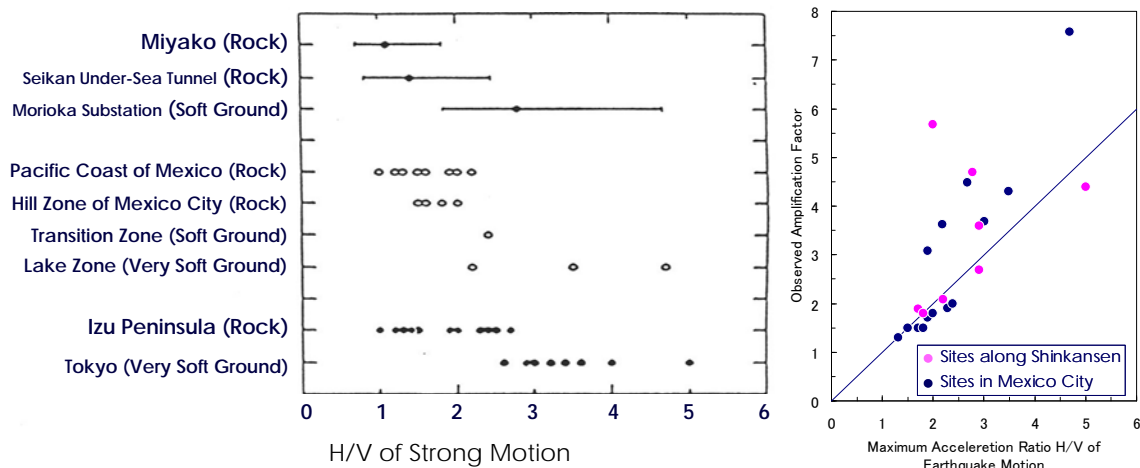

Figure 1   Relation of Max. accelerations on surface and basement (Nakamura and Saito, 1983)


(a) Comparison Site Condition and H/V of PGA   (b) Comparison H/V and H/H of PGA

Figure 2   Comparisons between Sites and H/V

First, horizontal to vertical ratio was derived from each maximum value and was compared with the softness of ground and the amplification factor. As shown in Figures 1 and 2, horizontal to vertical ratio of maximum value highly corresponds to these ground characteristics (Nakamura et al., 1983).
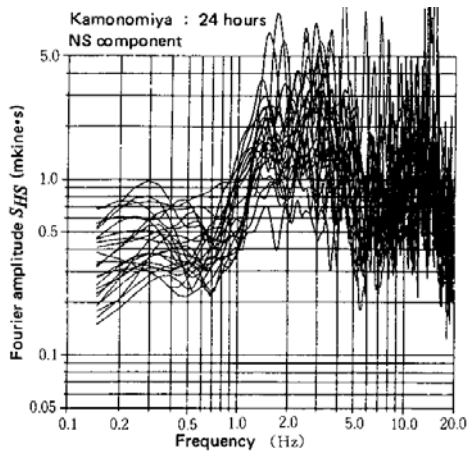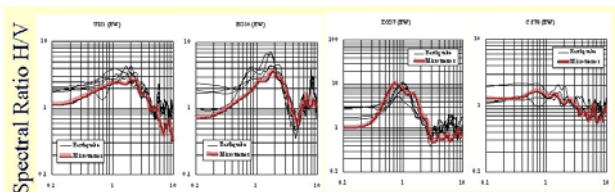
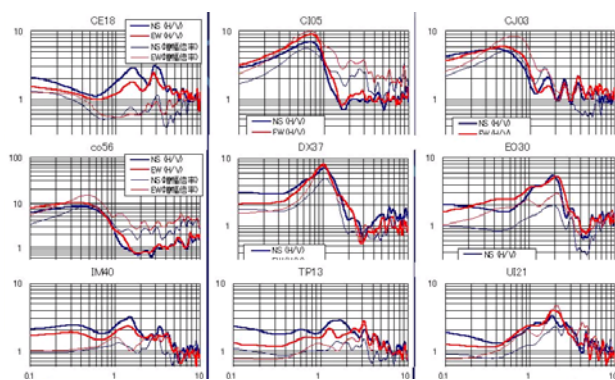(a) A 24-hours Change in Horizontal Spectrum
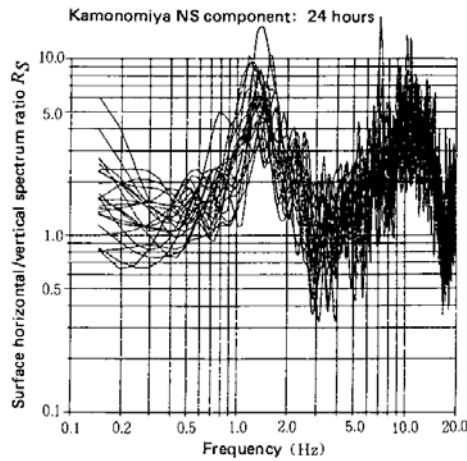
(b) A 24-hours Change in H/V Spectral Ratio

Figure 3   Stability of H/V Spectral Ratio

(a) Comparison H/V of microtremor and Strong Motion

(b) Comparison H/V and H/H of Strong Motion

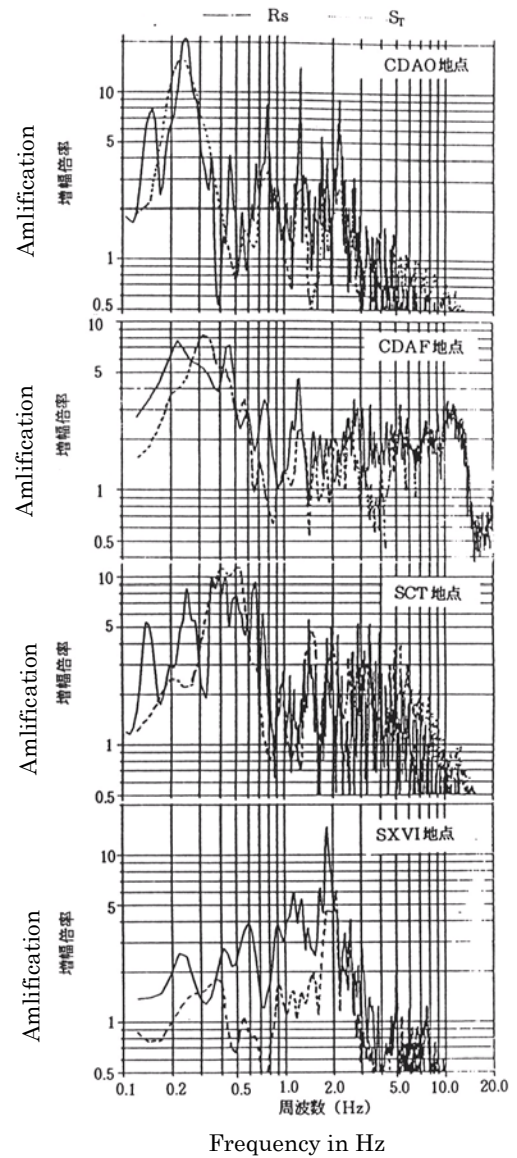Figure 5   Relation between H/V of Strong Motion, H/V of Microtremor and H/H of Strong

Frequency in Hz

Figure 4   Comparisons between Spectral Ratios H/V (Rs) and H/H (St) at Mexico City
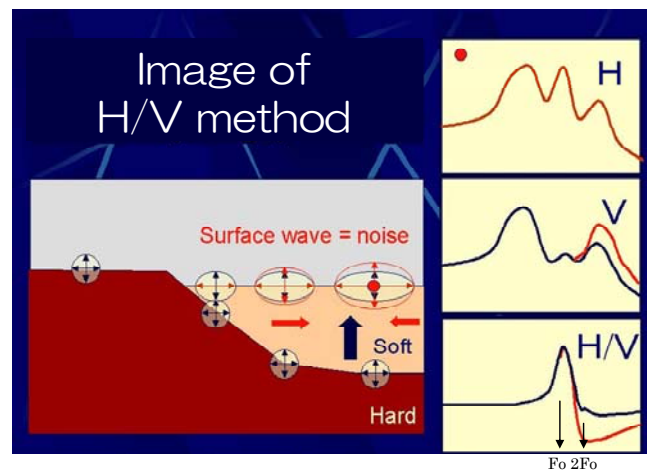

Figure 6   Image of H/V method

On the next step, the H/V spectral ratio of microtremor measured at anywhere was confirmed that it is able to estimate the predominant frequency and the amplification factor. And as shown in Figure 3 the result of estimation is stable for the measured time and season (Nakamura, 1989).

From the background of the H/V technique, it is natural that the H/V spectral ratio of not only microtremor but also the earthquake motion can explain the characteristics, and the original paper (Nakamura, 1989) had pointed that the H/V spectral ratios of earthquake motion are almost similar to the amplification spectra as shown in Figure 4. And then as shown in Figure 5, it is confirmed that the H/V spectral ratios of microtremor and earthquake motion are similar to each other and mostly similar to the amplification spectrum (Sato et al., 2004).

3. BASIC STRUCTURE OF THE H/V SPECTRAL RATIO

3.1. H/V Spectral Ratio Technique
There is no reason to enlarge the amplitude of a particular frequency range and of a particular direction at the hard and uniform ground as imaged in Figure 6. It is natural that the vibration of hard ground is uniform for each frequency range and each direction. The measured results of microtremor and earthquake support it.   On the other hand, under the situation of a soft ground deposition on the hard basement, it is important to grasp the amplification characteristics of the horizontal motion. At the frequency range amplified the horizontal motion, it seems vertical motion is not amplified. Meanwhile, at this kind of ground condition, it is important to consider the Rayleigh wave distributed on the ground surface. Earthquake disaster is caused mainly by the body wave and if the Rayleigh wave causes any damage, it must be a slight damage. That is to say that it is important to understand the phenomenon which the energy of the body wave trapped inside the soft surface layer. The phenomenon is the amplification of earthquake motion by so called the multiple reflections.

From this view point, the Rayleigh wave plays as a noise and it is necessary to reject the effect of the Rayleigh wave. It is possible to understand that the surface layer behaves as a high-pass filter for the Rayleigh wave propagating in the surface layer as illustrated in Figure 7. Rayleigh wave can not propagate in the frequency range under the predominant frequency of surface ground (F_0), and can transmit the energy peak around the frequency of minimum group velocity ($2F_0$, Airy phase). So the effect of multiple reflections of the SH wave is mainly composed around F_0.
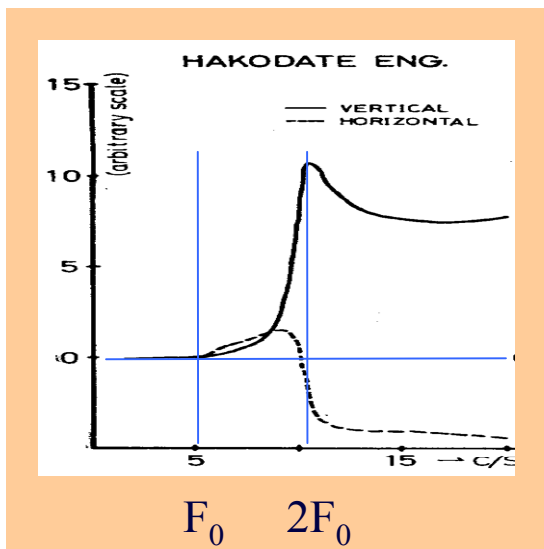
Figure 7   An Example of Horizontal and
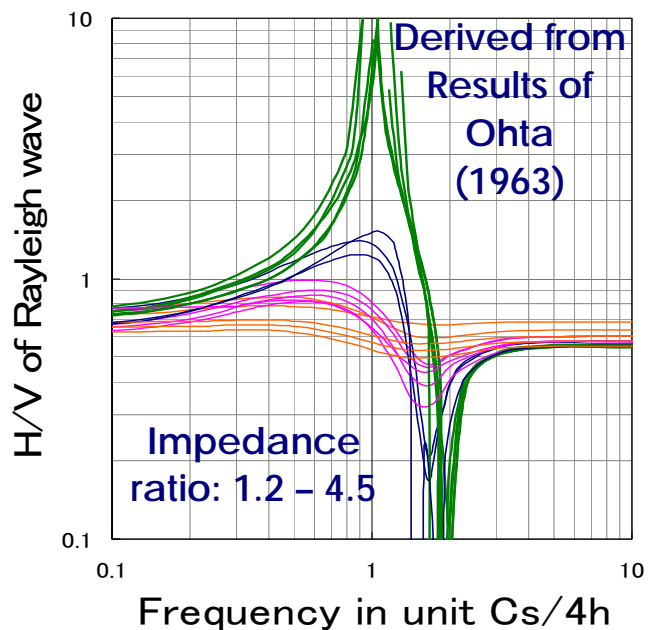
Vertical Components of Rayleigh wave

Figure 8   H/V of Rayleigh waves for two

layered ground

Comparison of the vertical motions between the basement and the ground surface makes sure the existence of the Rayleigh wave. The focused frequency range is around F_0, the predominant frequency of the horizontal motion. The frequency of the vertical motion amplified by the multiple reflections is estimated as V_p/V_s*F_0. In case of the noticed soft ground, while V_s is about 50 m/s to 200 m/s, V_p is around 1000 m/s, near to the V_p of water (=1500 m/s). So the frequency that the vertical motion enlarged is 5 to 20 times for F_0. Corresponds to this, the frequency of maximum energy transmission of the Rayleigh wave is approximately $2F_0$. So it is important to judge that the Rayleigh wave effects much if the vertical motion of the ground surface is relatively larger than the basement at least the frequency range up to three times of F_0.

Many peaks are observed on the spectrum of measured microtremor in actual. In past, the analyzers had read the peak corresponding to the predominant frequency of surface layer based on their experiences and intuitions. It must be only a speculation because of their arbitrariness. It is necessary for extracting peak for the purpose exactly to reject the effect of the Rayleigh wave.

The effect of the Rayleigh wave for the surface to the basement ratio of the horizontal motion R (=A_{hs}/A_{hb}: amplification characteristics) is estimated by the surface to the basement ratio of the vertical motion E (=A_{vs}/A_{vb}). Then the corrected amplification characteristic A_m is estimated as follows;

$$A_m = R/E \tag{3.1}$$
$$= (A_{hs}/A_{hb})/(A_{vs}/A_{vb}) \tag{3.2}$$
$$= (A_{hs}/A_{vs})/(A_{hb}/A_{vb}) \tag{3.3}$$

Using an observational fact that the ratio A_{hb} and A_{vb} is nearly equal to 1 for wide frequency range, A_m could be expressed as follows.

$$A_m = A_{hs}/A_{vs}. \tag{3.4}$$

This is the H/V spectral ratio. At present, however, there are many researchers who believe that the peak of the H/V can be explained by Rayleigh wave. Then the H/V of Rayleigh wave must be discussed.

3.2. H/V of Rayleigh Wave
For different wave-lengths, Ohta (1963) calculated H/V and phase velocity of Rayleigh waves for two layers model for various impedance ratio (varying between 1.2 and 4.5) and Poisson's ratio (varying between 0.25 and
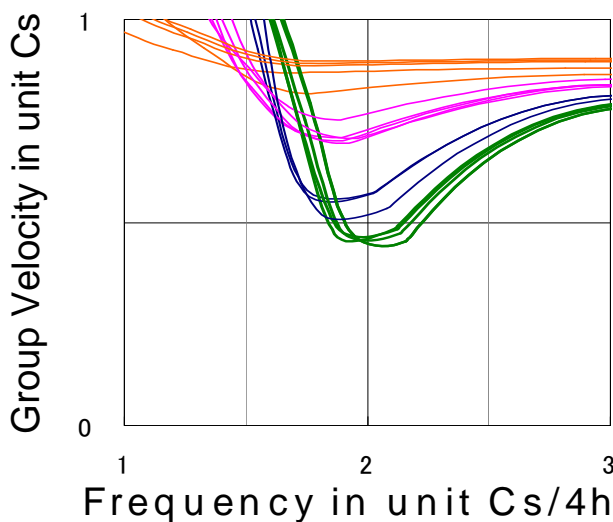

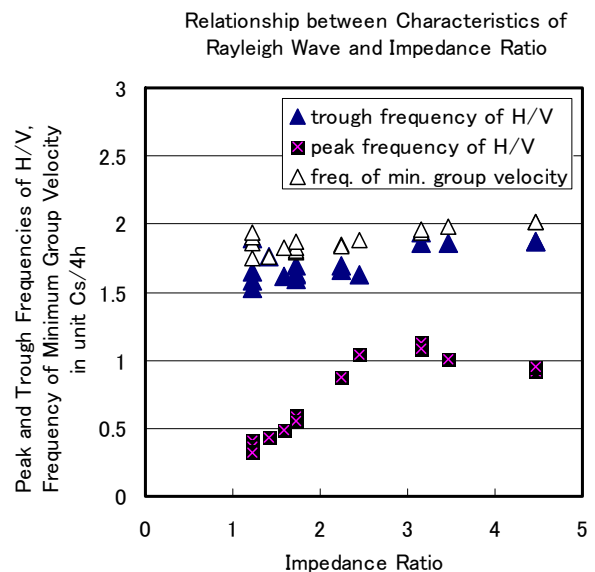Figure 9   Group velocities of Rayleigh waves

Figure 10. Relationship between characteristics of Rayleigh wave and impedance ratio

0.49) both in sedimentary and basement layers. By using these calculated results, the relationship between the H/V of Rayleigh wave and the frequency is shown in Figure 8, and it shows the relations between impedance, peak and frequency. The frequency in this figure is normalized with the predominant frequency of surface layer, $F_0 = C_s/(4h)$. Group velocities calculated from phase velocities are normalized with S wave velocity and drawn with the frequency normalized with F_0. Figure 9 shows the distribution of group velocity versus frequency. As we follow this figure, if the impedance ratio is less than two, the energy of Rayleigh wave exists at the lower frequency range. Figure 10 shows the change of impedance ratio for frequencies of trough and peak of H/V and minimum group velocity of Rayleigh waves. The frequencies corresponding to the trough and minimum group velocity (for almost all impedance ratio values), change between 1.5 and 2. On the other hand, only for the peak frequency varies in a wider range for different impedance ratio. The energy of the Rayleigh wave is almost zero at the peak frequency of H/V, and at the trough frequency of H/V the energy becomes to maximum. When the impedance ratio is greater than 2.5, the Rayleigh wave does not affect the H/V peak of ground motion. And on the other hand, when the impedance ratio is less than 2.5, the Rayleigh wave affects the H/V peak of ground motion.

3.3. Comparison between Microtremor and Rayleigh Wave for H/V

Figure 11, modified after Nogoshi and Igarashi (1971), shows relationship between H, V and H/V for microtremor and for Rayleigh wave. It can be clearly seen from these figures that, the energy of Rayleigh wave does not appear on the peak of H/V of Rayleigh wave. We can easily see that there is no energy around the peak frequency of H/V and amplitude is almost zero for horizontal and zero for vertical components of Rayleigh waves. On the other hand, the energy of Rayleigh wave gets its maximum on later frequencies at minimum group velocity of Rayleigh wave and this is nearly equal to trough frequency which is almost two times of the H/V peak frequency. The peak of vertical component corresponds to Rayleigh wave, but the peak of horizontal component is not caused by Rayleigh waves. These figures show that the peak of the H/V is not caused by Rayleigh waves.

Figure 8 shows that the H/V of Rayleigh wave for a hard ground, corresponding impedance one, is estimated around 0.7, against 1.0 from microtremor measurement on rock sites. It also shows that the H/V spectral ratio of microtremor around the peak is not caused by the Rayleigh waves.
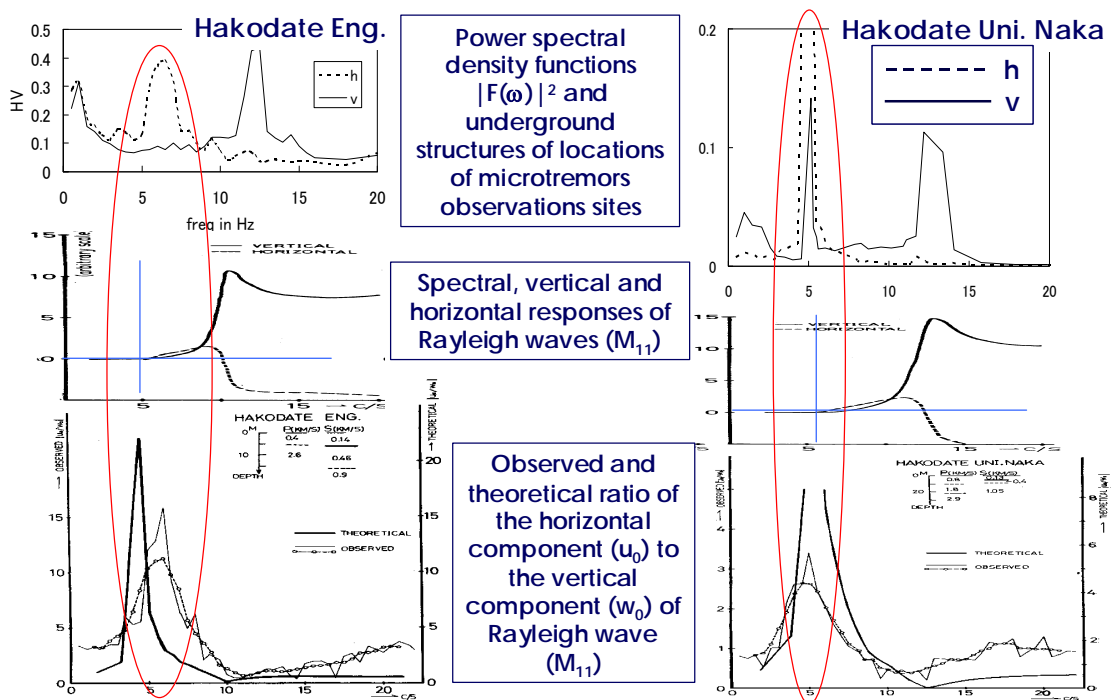

Figure 11. Relation of H, V and H/V for microtremor and for Rayleigh wave (modified after Nogoshi and Igarashi, 1971)

The H/V spectral ratio shows the amplification characteristics by the multiple reflections of the SH wave at least around F_0, and shows the characteristics contaminated by the Rayleigh wave around $2F_0$. In case of less effect of the Rayleigh wave it is possible to estimate not only preliminary peak but also secondary peak of amplification characteristics caused by multiple reflections with the H/V spectral ratio.

3.4. Comparison of the H/V spectral ratio between Earthquake and Rayleigh Wave

Figure 12(a) shows the H/V of strong ground motion of two different earthquakes recorded at the same site. Although the waveforms show different shape, if we take the H/V, they show similar shape. Another example can be found also in a paper of Okuma et al. (1999) which is given in Figure 12(b). The results show again that H/V gives similar characteristics for the different earthquakes recorded at the same sites. If we think that the H/V of earthquake record can be explained by Rayleigh wave, we should see same characteristics for all directions of the site, but this is out of practice and it is proper to think that the H/V of earthquake motion indicate the transfer function of local surface ground, as same as the H/V of microtremor.
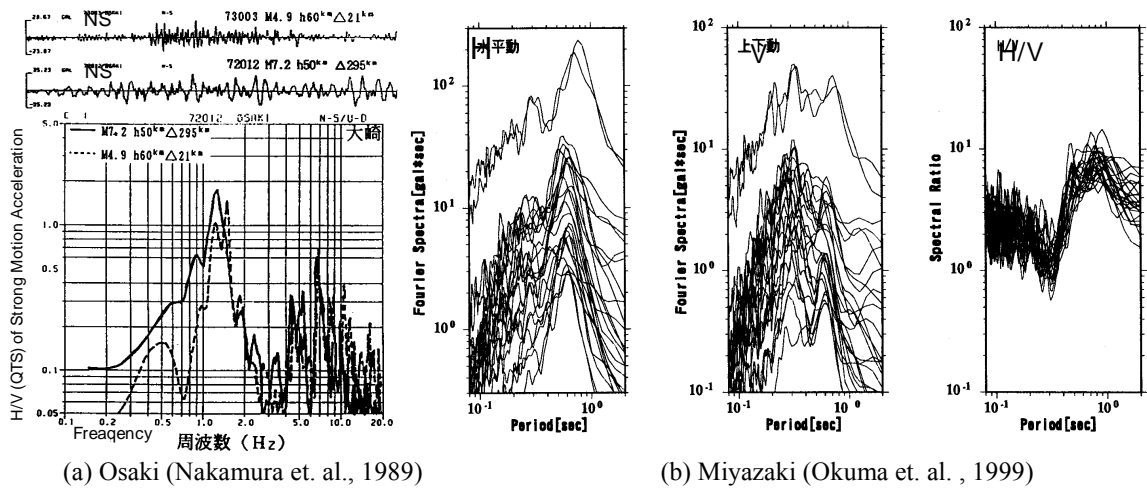

(a) Osaki (Nakamura et. al., 1989)   (b) Miyazaki (Okuma et. al. , 1999)

Figure 12. H/V of strong ground motion for different earthquakes recorded at the same station.

3.5. Numerical Simulation for H/V

The H/V indicates the transfer function of local surface ground. This was confirmed from the numerical simulation performed by Bonnefoy-Claudet et al. (2006) objecting the theory above mentioned contrary to their opinion. This paper is to consider the characteristics of H/V spectral ratio of microtremor experimentally using numerical ground model of two layer; basement and surface grounds with 6.5 of the impedance ratio. The first natural frequency of surface ground is set to 2Hz. Various shape, location and depth of source are adopted for this analysis. In case of weak Rayleigh waves the peak corresponding to the second natural frequency 6Hz appeared. The weaker the Rayleigh waves, the more correct the amplification factor. Consequently, it is natural to consider that the peak of 2Hz is caused by the multiple reflections of S waves except strong affection of the Rayleigh waves near the source. The peak value at 2Hz varies only between 5 and 8 for 6.5 of impedance ratio. This indicates the amplification factor can be estimated by the H/V peak value correctly.

3.6. Summery of the H/V Characteristics

Figure 13 shows the typical shape of the H/V spectral ratio. The characteristic of H/V for microtremor is summarized as follows. The first peak near F_0 consists of S-wave mainly. The first trough near $2F_0$ is caused by Rayleigh wave. Around F_0 there is almost no energy of Rayleigh waves, so the dispersion curves are unstable
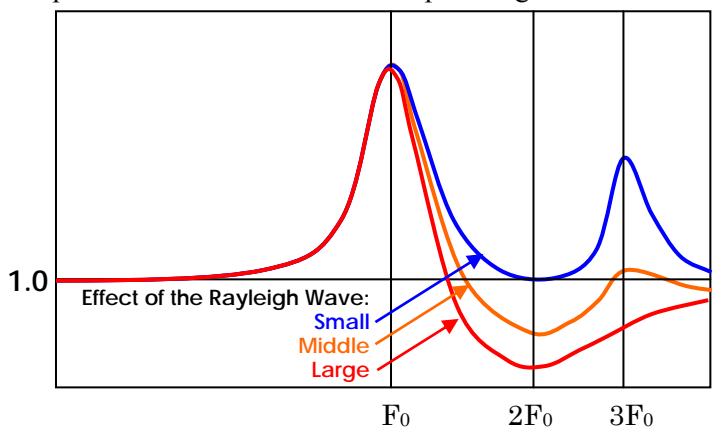

Figure 13   Image of the H/V Spectral Ratio

near F_0. Rayleigh wave are growing from F_0, and reach the first peak near the $2F_0$. It is verified that the H/V spectral ratio of both microtremor and strong motion is useful for estimation of at least fundamental frequency F_0 and its amplification factor. Consequently the excellent papers of Nogoshi and Igarashi (1971) and Suzuki (1933) have no relation to the H/V technique directory.

4. APPLICATIONS TO HAZARD MITIGATION TECHNOLOGY

From above discussions we can now conclude that peak of the spectral ratio H/V is caused by the multiple reflections of S waves. Alias QTS, Quasi Transfer Spectrum, represents the meaning coming from its name. Here, it should also be addressed the depth of the basement and the vulnerability of the surface ground related with QTS.

4.1. Basement Depth
The frequency F_0 related with QTS is;

$$F_0 = C_s/(4h) \qquad (4.1)$$

And the amplification factor A for this frequency is related with impedance ratio. If densities for basement and surface layer are same then,

$$A_0 = C_b/C_s \qquad (4.2)$$

And the depth of basement h is,

$$H = C_b/(4\,A_0F_0) \qquad (4.3)$$

Where, C_b is S wave velocity of basement.

Figure 14 shows the depth of basement along the Shinkansen line estimated from microtremor for the case of basement velocity (C_b) is 600m/sec. The estimated results are compared with boring data and the results showed that, the contact line between alluvium and diluvium is the basement of QTS.
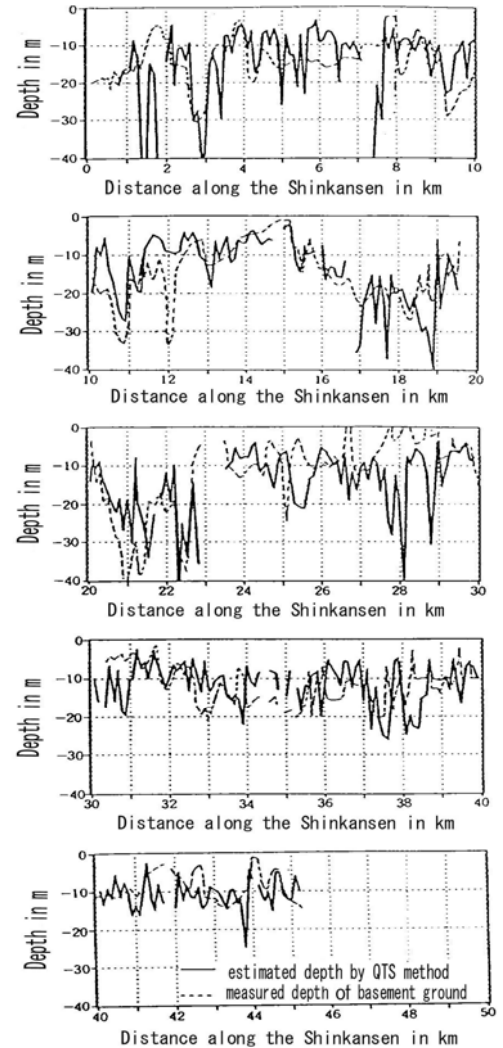
4.2. Vulnerability Assessment
Earthquake damage of structural members occurs at the time of exceeding the limit of the strain caused by deformation, and it causes the collapse if the stability of the structure lacked. Then the vulnerability index focused on the strain was defined (Nakamura, 1997) as imaged in Figure 15. K values in $10^{-6}/(cm/s^2)$ are defined to estimate the strain in the unit of 10^{-6} at particular parts of the structures, by multiplying the maximum acceleration at the engineering basement and the K value.

These K values are defined for ground and various structures, as K_g value for ground, K_j value for embankment, K_s value for rigid frame structure and K_b value for building. As peculiar K value, K_d value for
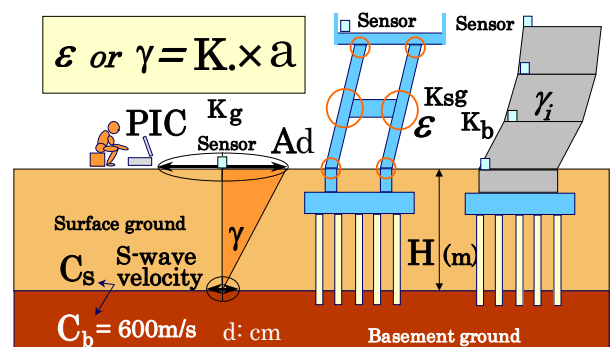

Figure 14 Comparison between Estimated Depth and Measured one (after Nakamura et.al. 1990)

Figure 15 Image of K values

derailment is defined. In this section, K_g is introduced. For other K values, refer the paper of Nakamura (1997).

For ground, shear strain γ of surface ground is noticed. In general, from $\gamma = 1000 \times 10^{-6}$ surface ground begins to show non-linear characteristics and in $\gamma > 10,000 \times 10^{-6}$ large deformation and collapse will occur.

Simplifying the shear strain deformation of surface ground as shown in Figure 16, average shear strain γ of surface ground can be estimated by following formula.

$$\gamma = A_g \times \delta / h \qquad (4.4)$$

Where, A_g is amplification factor, h is thickness of surface layer, and δ is seismic displacement of the basement ground.

Putting S-wave velocities of basement and surface ground as C_b and C_s respectively, proper predominant frequency F_g of surface ground is approximately expressed as follows.

$$F_g = C_b / (4A_g \times h) \qquad (4.5)$$

Acceleration of basement ground α_b is expressed as follows.

Figure 16. Shear deformation of surface ground

$$\alpha_b = (2\pi F_g)^2 \times \delta \qquad (4.6)$$

And γ is expressed by F_g, A_g and C_b as follows:

$$\gamma = (A_g \times \alpha_b / (2\pi F_g)^2) \times (4A_g \times F_g / C_b)$$
$$= (A_g^2 / F_g) \times (\alpha_b / (\pi^2 C_b)) \qquad (4.7)$$

If the efficiency of applied seismic force is assumed to be $e\%$ of static force, effective γ_e is expressed as follows.

$$\gamma_e = K_g(e) \times \alpha_b \qquad (4.8)$$

$$K_g(e) = e \times (A_g^2 / F_g) / (\pi^2 C_b) / 100 \qquad (4.9)$$

The value of C_b is expected to be nearly constant in a broad area and K_g is a proper value for measured point. Thus K_g can be considered as an index to indicate easiness of deformation of measured points which is expected
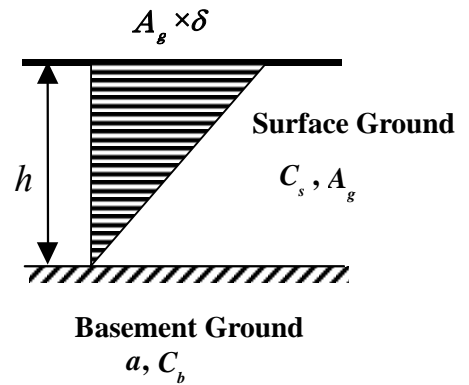

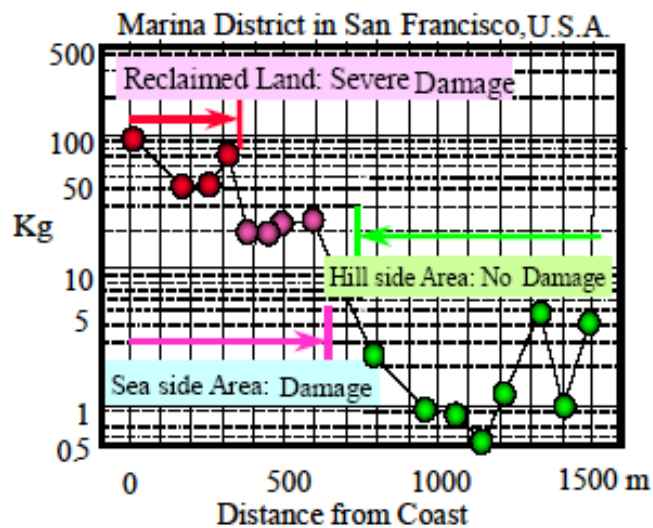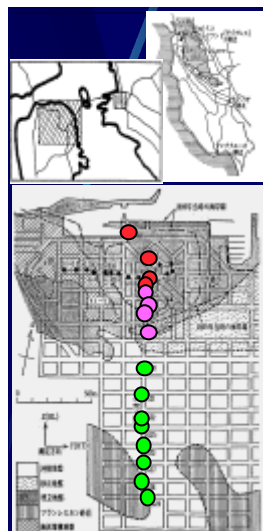Figure 17   Kg values measured after the 1989 Loma Prieta Earthquake

useful to detect weak points of the ground. As we can consider $C_b = 600\ m/s$, we obtain $1/(\pi^2 C_b) = 1.69 \times 10^{-6}$ (s/cm). If we put $e = 60\%$, then $K_g(e)$ is expressed as follows.

$$K_g(e) = A_g^2 / F_g \tag{4.10}$$

The effective strain can be estimated by multiplying $K_g(e)$ value with maximum acceleration of basement ground in Gal (= cm/s^2).

Figure 17 shows the distribution of K_g-values obtained in San Francisco Bay Area after the 1989 Loma-Prieta Earthquake. For Marina district the result along a line from sea coast to hillside is shown. It shows K_g at the sites where grounds deform much are bigger than 20 and K_g at the sites with no damage are very small. Considering the maximum basement accelerations around the area are estimated as 50Gal based on observations, $K_g = 1000 \times 10^{-6}$ separates the areas liquefied or not.

5. CONCLUSION

The H/V spectral ratio, alias QTS, Quasi Transfer Spectrum, is overviewed from its origin to the application for practical disaster prevention. The H/V spectral ratio was found from the strong motion records at various sites in Japan. On the soft ground, horizontal motion is larger than vertical motion. On the other hand, on the hard ground, both horizontal and vertical motions are similar to each other both on the maximum value and waveform. It is proved in the present paper that the peak of H/V ratio (QTS), either for microtremor of for earthquakes cannot be explained with Rayleigh waves, since Rayleigh wave energy is very small for the peak frequency but high on the trough of H/V ratio. Author's explanation in Nakamura (1989) is correct for explaining this peak with SH waves. The H/V spectral ratio shows the amplification characteristics by the multiple reflections of the SH wave at least around F_0, and shows the characteristics contaminated by the Rayleigh wave around $2F_0$. Based on this result, the way to estimate the basement depth and the vulnerability indices called K values were introduced. K values are simply derived from strains of ground and structures.
It seems to be possible to find the weak points previously with K values because of the high correlation between the earthquake damage and K values. There are research groups to establish the database on the vulnerability of structures. The H/V spectral ratio is already popular on the various research fields. It is happy that K values contribute to increase the earthquake-resistant capacity of the city with dissemination of the technique for investigation of the earthquake-resistant capacity.

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