SEISMIC VULNERABILITY OF R.C. CIRCULAR BRIDGE COLUMNS

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ABSTRACT:
The seismic response of highway viaducts with bearing-supported superstructures and characterized by structural regularity, essentially depends on the behaviour of its piers. Therefore, the vulnerability analysis of the most exposed column often coincides with that of the whole structure. This paper is focused on a simplified procedure that furnishes the expected damage vs ground acceleration for homogeneous families of piers, characterized by same design and site parameters. The procedure preliminarily analyzes different forms of column failures (flexural failure for inadequate confinement of the plastic hinge zone; flexural strength degradation in sections with lap-spliced reinforcement; shear failure; buckling of reinforcements). Subsequently, on the basis of a comparative analysis, the most probable condition of collapse is determined. The algorithm utilizes an approximated closed-form approach, properly obtained, in terms of longitudinal reinforcement, transverse confining steel, and base axial compression.

KEYWORDS: R.c. bridge columns, seismic vulnerability, damage indexes

1. INTRODUCTION

The structural vulnerability analysis is often based on the knowledge of the so-called capacity curves, that relate the acting force F with the deformation response. This approach, when applied to bridges, depends on the behavior of different structural components (piers, abutments, support devices, joints, foundation structures, planimetric and altimetrical bridge configuration, etc.) whose interaction is often difficult to deal with. However, in the cases of regular bridges, the overall response exclusively depends on the behavior of their critical pier and the vulnerability analysis is furnished by that of the most exposed column.

This study proposes a method for assessing vulnerability of r.c. viaducts with single circular bridge columns and bearing-supported superstructure, that constitute a typology widely used in the last fifty years in Italy. In order to simplify the approach, the procedure ignores both the interaction between structure and the underlying foundation ground, and the maximum drift control.

The procedure provides concise indications in the form of performance curves, which correlate the level of maximum expected ground acceleration to the damage of the analyzed column.

2. CHARACTERISTICS OF THE MODEL

The model adopted in the analysis is that of Fig. 1. In the same figure the material constitutive laws are also shown. In particular, in Fig. 1b the relationship assumed for the longitudinal and the transversal reinforcements is represented. The Mander model is utilized for the concrete both for the confined core and for the cortical unconfined zone.

Hereafter, the following parameters are applied: $\rho_c = 4A_{sp}/(D\cdot s)$ effective volumetric ratio of transverse reinforcement in the plastic hinge zone; $\bar{\rho} = \rho_s(20/f'_c)$ relative volumetric ratio of transverse reinforcement, with $f'_c$ in MPa; $\omega = (A_s/\pi R^2)\left(f_{sp}/f'_c\right)$ longitudinal reinforcement’s mechanical ratio of the base section; $c = \chi \cdot R$ non-dimensional curvature of the base section; $f = \delta \cdot R/L$ non-dimensional displacement of the pier cap; $r = (F \cdot L)/(\pi R^3 f'_c)$ non-dimensional flexural base capacity; $\lambda = L_p/L$ non-dimensional plastic hinge
length; \( \nu = \frac{N}{\pi R^2 f'c} \) non-dimensional axial compression; \( \xi_u = x_u / D \) non-dimensional distance between neutral axis and extreme compression fiber at collapse; \( n_B \) number of longitudinal rebars; \( d_{BL} \) diameter of longitudinal rebars; \( \Phi_{BL} = d_{BL} / D \) non-dimensional diameter of longitudinal rebars; \( \kappa_{BL} \) lap-splice parameter; \( L_s = \kappa_{BL} \cdot d_{BL} \) lap-splice length; \( \delta_y, \delta_u \) yield and collapse pier cap displacements; \( \Delta = (\delta - \delta_y) / (\delta_u - \delta_y) \) non-dimensional damage index.

3. COLLAPSE MODES

3.1. Base ultimate curvature

On the basis of the variables defined in paragraph 2, approximate formulations were obtained which are sufficiently reliable in order to determine the collapse in terms of the base curvature ductility. The related typical behaviour is shown in Figure 2, and represents the relationship between the base column resistance and the curvature ductility. This diagram is constructed with reference to the following parameters:

- at yielding: 
  \( r_y = -\left(\frac{2}{3}\right) v^2 + 0.8v + 0.37 \rho \); \( f_y = 10^3 \cdot 0.42v^2 + 0.336v + 0.547 \); \( c_y = 3 \cdot f_y \);

- at collapse: 
  \( r_u = a_i \cdot v^2 + b_i \cdot v + c_i \); \( 10^3 \cdot c_u = a_u \cdot v^2 + b_u \cdot v + c_u \); \( 10^3 \cdot f_u = a_f \cdot v^2 + b_f \cdot v + c_f \),

with \( a_i, b_i, c_i \) function of \( \omega \) and \( \bar{\rho} = 100\rho \),: \( a_r = \left( -0.834 + 0.097 \cdot \bar{\rho} \right) + \left( 0.657 + 0.286 \cdot \bar{\rho} \right) \). \( \omega \)

\( b_r = \left( 0.626 + 0.250 \cdot \bar{\rho} \right) + \left( 0.353 + 0.761 \cdot \bar{\rho} \right) \). \( \omega \)

\( a_f = \left( 3.44 + 10.7 \cdot \bar{\rho} \right) + \left( 1.2 + 19.3 \cdot \bar{\rho} \right) \). \( \omega \)

\( b_f = \left( -4.0 + 16.0 \cdot \bar{\rho} \right) + \left( -0.3 + 21.2 \cdot \bar{\rho} \right) \). \( \omega \)

\( c_f = \left( 2.3 + 16.2 \cdot \bar{\rho} \right) + \left( 0.8 + 10.7 \cdot \bar{\rho} \right) \). \( \omega \)

\( a_c = \left( 44.20 + 65.40 \cdot \bar{\rho} \right) - \left( 44.90 + 126.0 \cdot \bar{\rho} \right) \). \( \omega \)

\( b_c = \left( -46.80 - 140.7 \cdot \bar{\rho} \right) + \left( 44.20 + 248.4 \cdot \bar{\rho} \right) \). \( \omega \)

\( c_c = \left( 15.90 + 97.90 \cdot \bar{\rho} \right) - \left( 11.60 + 126.7 \cdot \bar{\rho} \right) \). \( \omega \)

If the collapse is attained reaching the maximum base curvature, disregarding the second-order effects, and under the hypotheses shown in Fig. 3, the non-dimensional hinge plastic length can be obtained by the positive root of the equation

\[ \left( \frac{c_y}{3} - c_u \right) \lambda^2 - \left( \frac{2}{3} c_y - c_u \right) \lambda + \left( \frac{c_y}{3} - f_u \right) = 0. \]

In this case, the relationship between the pier cap
displacement ductility and the base curvature ductility is: 
\[ \mu_\delta = (1 + \lambda^2 - 2\lambda) + 3\mu_\chi \left( \lambda - \lambda^2 / 2 \right). \]

![Figure 2 Bilinear relationship between non-dimensional flexural base capacity and curvature base ductility for r.c. circular column subjected to constant compression.](image)

The value of \( \lambda \) obtained with this procedure can be affected by some errors (second order effects are neglected, simplified \( M - \chi \) relationship, simplified cracks distribution, etc.), but for the scope of the proposed procedure it can constitute a first approximation for the plastic hinge extension at collapse.

### 3.2 Lap-splice failure of longitudinal bars.

The proposed procedure follows the approach outlined in [1]. A schematic description of principal points of this approach is reported in Figure 4.

![Figure 4 a) Value of the parameter \( p \) in the expression of \( \rho_h \);](image)
b) Sectional equilibrium at lap-splice failure;

c) Modified relationship $r-\mu$ as function of lap-splice failure.

The base assumption is that the relationship between sectional resistance and ductility is modified by lap-splice failure when volumetric ratio of transverse reinforcement is less than $\rho_h = \frac{1.4 \pi d_{bl}^2}{4}$. In this case, it is first necessary to evaluate the theoretical non-dimensional residual flexural strength $r_o$. This value depends on the acting axial force alone, in the hypothesis that longitudinal rebars are completely unbonded and ineffective, being $\zeta_o$ the neutral axis depth without reinforcement ($r_o = \frac{N \cdot \zeta_o D}{\pi R^3 f'_c} = \nu \zeta_o D = 2 \nu \zeta_o$). The real residual capacity $r^*$ is than obtained as function of $\zeta_o$ and the actual transversal reinforcement $\rho_s$, following the expression: $r^* = 2 \nu \zeta_o + \frac{\rho_s}{\rho_h} (r_u - 2 \nu \zeta_o)$. Herein it is assumed that $\zeta_o = \frac{1}{3} \left( \frac{\sin^3 \theta}{\theta - \sin \theta \cos \theta} \right)$ and $\theta$ is furnished as function of $\zeta_o$.

Figure 4 c) shows the modified relationship $r-\mu$ as a function of the lap-splice failure, in the two typical cases: when the yield stress $f_{sy}$ of the longitudinal reinforcement is reached (upper part of the figure), and when $f_{sy}$ cannot be developed (bottom part of the figure).

### 3.3 Shear failure

In the procedure is utilized the model proposed in [2]. The shear strength is given by the non-dimensional relation $r_y = v_u + v_p + v_c$. The first contribution $v_u = \frac{V_u \cdot L}{\pi R^3 f'_c}$ is related to the effective transversal reinforcement. The second one $v_p = \frac{V_p \cdot L}{\pi R^3 f'_c}$ is the contribution obtained from the resistance of the compressed strut connecting the compression parts of the final sections of the column. Finally, $v_c = \frac{V_c \cdot L}{\pi R^3 f'_c}$ is associated with the interlocking effect. The shear strength is then obtained following the expression:

$$v = v_u + v_p + v_c = \left[ \frac{1}{3} \frac{R}{D} \right] + [1.7 \cdot \nu \zeta_o] + [\mu] \quad (1)$$

Figure 5 Column response modification as function of shear failure.
In (1) $\xi_u$ is function of $\xi_u$ (neutral axis non-dimensional depth at collapse), evaluable from the approximate expression $\xi_u = \frac{1}{2} \left[ a_\xi \nu + b_\xi \right]$. Herein $a_\xi$ and $b_\xi$ (function of $\omega$ and $\bar{\rho}$) are assumed as follows:

$$a_\xi = \left( 2.018 - 0.784 \cdot \bar{\rho} \right) + \left( -1.551 + 0.687 \cdot \bar{\rho} \right) \cdot \omega,$$

$$b_\xi = \left( 0.228 - 0.027 \cdot \bar{\rho} \right) + \left( 0.556 + 0.017 \cdot \bar{\rho} \right) \cdot \omega.$$

The parameter $\kappa$ depends on the column slenderness and longitudinal reinforcement, while $\gamma$ (ranging between 0.05 and 0.25) modifies the shear resistance as a function of the ductility increment. The expression (1) allows to compare the flexural and the shear strength in the plane $r - \mu$ (Figure 5), as a function of the ductility level. In this way, the column behavior is modified when shear failure anticipates flexural collapse.

3.4 Failure for buckling of longitudinal rebars

A rebar that undergoes repeated loading into the inelastic range can be subjected to inelastic buckling: flexural collapse is often modified by this phenomenon. In the proposed procedure, this effect is represented following the simple model depicted in [3], where the longitudinal rebar buckling is related to the lateral deflection of the pier cap. This procedure provides the deformation demand $\delta_{ubb}$ of the cap at the onset of rebars’ buckling. The displacement limit $\delta_{ubb}$ is related to the confinement level, to the diameter of longitudinal reinforcement and to the column compression stress. The approach in [3] furnishes the following relation:

$$f_{ubb} = \frac{\delta_{ubb} R}{L^2} = \frac{0.0325}{2} \left[ 1 + 150 \rho \left( \frac{f_{c}}{f_{c}^*} \frac{d_{BL}}{D} \right) \right] \left( 1 - \nu \right) \left[ 0.1 + \frac{1}{L/D} \right]$$

(2)

4. SECOND-ORDER EFFECTS

The proposed approach also incorporates the so-called second order effects. These are included by modifying the acting moment as a function of the lateral displacement of the pier cap: $M = FL + P\delta$. In the previous formula second order effects are represented by the term $\Delta M^{II} = P\delta$, which reduces the lateral strength via the non-dimensional base moment capacity variation:

$$\Delta r = \frac{\Delta M}{\pi R^2 f_{c}^*} = \frac{P\delta}{\pi R^2 f_{c}^*} = \frac{P}{\pi R^2 f_{c}^*} \frac{\delta}{R} = \nu \cdot f_{y} \cdot \left( \frac{L/R}{R} \right)^2 = \nu \cdot f_{y} \cdot \left( \frac{L/D}{D} \right)^2$$

(3)

The resistance-ductility relationship is first obtained assuming that the capacity displacement ductility is non affected by this phenomenon, and then considering the second-order effect as a reduction of the yielding and ultimate resistance through the formulas: $\Delta r_y = \frac{\nu \cdot f_{y}}{4} \left( \frac{L/R}{R} \right)^2$; $\Delta r_u = \frac{\nu \cdot f_{u}}{4} \left( \frac{L/D}{D} \right)^2$.

5. STRUCTURAL PERFORMANCE

Evaluation of seismic non-linear performance is often obtained utilizing Inelastic Demand Spectrum Analysis, that is focused on the direct use of inelastic spectrum, whose construction depends by the relations between reduction factor and structural ductility. In the framework of this methodology the N2 method, first proposed by Fajfar and Fishinger, and than adopted by Eurocode 8 is considered. Its success is also due to the formulation in AD format (acceleration-displacement) which is particularly convenient in order to obtain a valid graphic representation.

Since the method was initially based on the response of a SDOF system, the principal difficulty of the procedure consists in the transformation of the actual MDOF system into the equivalent SDOF one. This passage is necessary in order to perform the analysis and, during the inverse procedure, in order to restore the results to the
actual structure.
However, when applied to the bridge pier, the approach is sufficiently precise, because the structure is actually a SDOF system, and no procedural difficulties arise during the method implementation.
The fundamental aspects of the N2 method can therefore be summarized in the following steps.
1) Starting from the elastic pseudo-acceleration and displacement spectra, elastic spectrum in acceleration-displacement (AD) format is determined with the formula 
\[ S_{de}(T) = S_{ae}(T) \cdot \left( \frac{T}{2\pi} \right)^2, \]
that in the proposed procedure takes the form
\[ S_{de}(T) = a_g \cdot f(T) \cdot \alpha(T), \]
in which the functions \( f(T) \) and \( \alpha(T) \) are evaluated from Eurocode 8.
2) A bilinear force deformation relationship is evaluated for the inelastic SDOF system equivalent to the r.c. circular bridge column, on the basis of the procedures defined in previous paragraphs 3 and 4.
3) The structure elastic stiffness is determined on the same AD plane, and the target displacement \( \delta^* \) of the pier cap, representing the structural performance, is evaluated.
4) The target displacement \( \delta^* \) is transformed into the corresponding spectral displacement (unlimited elastic behaviour). This procedure requires the application of the reduction factor
\[ R_\mu = \mu \left( T > T_C \right) \] or
\[ R_\mu = 1 + \left( \mu - 1 \right) \frac{T}{T_C} \left( T \leq T_C \right). \]
In the short period range it results
\[ S_{de} = \frac{\delta^* \cdot R_\mu}{1 + \left( R_\mu - 1 \right) \frac{T}{T_C}}, \]
and in the long period range “the equal displacement principle” \( S_{de} = \delta^* \) is adopted.
5) The anchorage acceleration, taken as parameter of the Structural Performance, is then determined from the spectral displacement by the equation
\[ a_g = \frac{S_{de}}{f(T) \cdot \alpha(T)}. \]
The fundamental points of the above mentioned procedural steps are well summarized in Fig. 6.

![Figure 6 Elastic and inelastic demand spectra versus capacity diagram for short and medium-long period range](image)

6. RESULTS OF THE PROCEDURE

The approaches analysed in the previous sections allowed to construct an algorithm for the vulnerability assessment of r.c. circular bridge columns.
On the basis of a target drift \( \delta_c \leq \delta^* \leq \delta_u \) of the column cap, the procedure evaluates the anchorage acceleration which conducts to the predefined displacement. Few parameters are involved in the procedure, and the application in non dimensional terms is straightforward. The approach can also be used to plot curves synthesizing the vulnerability level presented by this structural type.
It is possible to obtain a set of abacuses in which the variable parameters are the mechanical ratio of base section
longitudinal reinforcement $\omega$, and the relative volumetric ratio of transverse reinforcement $\rho$. Each abacus refers to a specific soil category, a specific axial force $v = \frac{N}{\pi R^2 f_c}$ and a specific ratio $L/D$. It is also possible to evaluate different values of participating mass in the motion.

The abacuses in Fig. 7 a) (for participating mass equal to 1,2,3 and 4 times the mass which produces the axial stress) are examples of anchorage accelerations which cause pier cap displacements at yielding (black) or at collapse (red) in the case of soil category “A”, non dimensional compression $\nu = 0.1$, ratio $L/D = 2.5$ and assuming the value of $\kappa_{BL} = 40$ for the lap-splice parameter. A closer look to the abacuses, displayed in Fig. 7 b) for the case $n_c=2$, highlights the different collapse modes which the same column can suffer, as a function of the longitudinal reinforcement ratio, according to the level of confinement at the plastic hinge.

In the mentioned diagrams, it is interesting to note that if collapse does not occur in a brittle way, due to shear (very frequent case for strongly reinforced, poorly confined squat columns), the ultimate base curvature collapse (for column with poor longitudinal reinforcement) may be anticipated by failure of lap spliced bars (in the case of reduced confinement) or may occur because of buckling of longitudinal rebars in the strongly confined areas.

6. THE PERFORMANCE CURVES

Concise, interesting relations called “performance curves” can be obtained by following the same procedure. On the basis of the actual design parameters, performance curves associated to an appropriate damage indicator furnish the ground acceleration connected with the damage.

An appropriate damage index can be represented by the parameter $\Delta$ which correlates the pier cap drift $\delta$ to displacements corresponding to yielding ($\delta_y$) and collapse ($\delta_c$).

The proposed procedure adopts the damage parameter defined in the following relation:
\[ \Delta = \frac{\delta - \delta_y}{\delta_u - \delta_y} \]  

Figures 8 illustrate, as an example, the performance curves for the same cases analyzed in Fig. 7 (soil category A) for columns having, at base section, a mechanical ratio of longitudinal reinforcement \( \omega = 0.1 \) and \( \omega = 0.5 \), and a relative volumetric ratio of confinement reinforcement \( \rho \) ranging between 0.025 and 0.5.

Once the soil category is determined, together with the mechanical and geometric structural parameters that characterize elastic behavior (synthesized in the elastic period \( T \)), it can be observed, in agreement with the basic hypotheses of the procedure, that the relation \( S_{de-ag} \) is linear. If the period falls back in the medium-long range \( (T > T_c) \), as it frequently happens by virtue of the principle of “equal displacement”, the relation \( a_g - \delta^* \) will also be linear, and the performance curves will simplify into straight lines which can be easily drawn with the sole determination at yield and collapse condition.

Figure 8 Performance curves relating to a r. c. circular column on “S=A” category soil and \( \omega = 0.1 \ \omega = 0.5 \)

**REFERENCES**