

# Numerical Response Analysis and Shaking Table Tests for Bridge and Building Complex Structural System (Part II: Parameter Optimum and Earthquake Response Analysis)

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### **ABSTRACT :**

The bridge and building complex structural system is proposed as a new seismic structural system in the paper. The bridge and building complex structural system is modeled by a simple two degree of freedom model for earthquake dynamic response analysis, and the main influential parameters of the complex structural system, such as damping, mass ratio and frequency ratio, are theoretically investigated. Based on the analysis of response and parameters of the complex structural system, the governing equation of the optimum frequency ratio and damping ratio are obtained in the paper.

**KEYWORDS:** The bridge and building complex structure, earthquake responses, parameter optimization

# **1. INTRODUCTION**

With the economic development and the progress of sciences and technology, safety, functional applicability and energy saving of structure are required higher and higher. Seismic reduction structure, which can decrease structural seismic response efficiently, has been emphasized widely in engineering circles (Li, G.Q., 1993). There are three major types of seismic reduction structure (Hu, Y.X., 1988, Kelly, J.M., 1997, Liu, W.G., 2003, Architectural Institute of Japan., 2003): active seismic control, passive seismic control and semi-active control. Former researches and applications of seismic reduction technology were concentrated in building structures and bridge structures, and plentiful achievements were created in these two fields. In resent years, cities in China are developing rapidly with more and more bridges and rail transits being constructed. Therefore, the effective utilization of soil resources and seismic behaviors of bridges have been a focal point of researches. In this paper, the bridge and building complex structural system is proposed for the sake of decreasing seismic response of bridge and using soil resources efficiently. At normal state, there's large space between piers under bridge. And the structural system can be established by depositing building structure into the space between piers and connected with the bridge by suspenders or other supports. It can use the untapped space under the bridge in the maximal degree and save urban land resource. By adjusting seismic response of bridge and building complex structure and vibration of building structure with the suspenders or supports, the bridge and building complex structural system will be efficient in seismic resistance.

Structural seismic response is gained in two ways: shaking table test and seismic response numerical emulation analysis (Li, Z.X., 2002). At the present time, shaking table test is one effective test method for simulating seismic action. In this paper, the characteristics, calculation models and equations of motion of the system are analyzed and dynamical parameter optimization analysis is done. Then it is compared with ordinary bridge structure to get further results.

# 2. STRUCTURAL SYSTEM AND CALCULATION MODEL

### 2.1. The Bridge and Building Complex Structural System



The bridge and building complex structural system is established based on passive seismic control theory. It is composed of 4 parts: foundation structure, bridge structure, building structure and the shock absorption layer connecting bridge and building. The foundation structure supports the bridge and building's weight. There is no different with traditional foundation. The bridge structure is also ordinary bridge which is for urban traffic use. And the building structure is set in the space between piers under the bridge deck. So the building's weight is transferred to piers, then to foundation. The total system's core consists in shock absorption layer. The layer has three main functions: connecting bridge and building structure; descending the center of gravity of bridge; absorbing earthquake energy to reduce the bridge and building complex structural system's seismic response. The conception of the system is shown in figure 1. And its characteristics are shown as follows:

- (1) Use the space under bridge deck reasonably to save city limited land resource;
- (2) The seismic response of building structure will be far lower than foundation structure because of low stiffness of shock absorption layer. Building structure's relative displacement caused by earthquake can be controlled in allowable space by setting parameter reasonably;
- (3) The seismic response of bridge deck will be decreased owing to building structure's regulation and damping energy dissipation of the shock absorption layer.





Figure 1 Complex isolated structure model

Figure 2 Analysis model of the complex structure

### 2.2. Simplified Model of Complex Structural System

Considering that the system's displacement concentrates mainly at shock absorption layer by setting shock absorption layer's characteristic, the building structure can be regarded as a simplified particle, and the bridge structure can be regarded as a single degree of freedom system according to its basic vibration mode. The bridge equivalent simplified model was researched in the paper (He, Liu and Feng, 2007). The equivalent mass and stiffness of the substructure can be expressed as equation (2.1) based on the principle that kinetic energy of the single freedom system, and the substructure is equal to their base shear. The system can be simplified into two lumped-mass system model, one lumped-mass is bridge and the other is building structure. The simplified model is shown in figure 2,  $m_2$ ,  $k_2$  represent equivalent mass and the shock absorption layer's stiffness of the bridge.

$$M_{1,eq} = \frac{\left(\sum_{i=1}^{n} m_{i} \varphi_{i}\right)^{2}}{\sum_{i=1}^{n} m_{i} \varphi_{i}^{2}}, K_{1,eq} = \left(\frac{\sum_{i=1}^{n} m_{i} \varphi_{i}}{\sum_{i=1}^{n} m_{i} \varphi_{i}^{2}}\right)^{2} \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij} \varphi_{i} \varphi_{j}$$
(2.1)  
Building

Where  $m_1 = M_{1,eq}$ ,  $k_{il} = K_{1,eq}$  represent equivalent mass and equivalent stiffness of single lumped-mass, and Damper



 $m_i \propto k_{ij}$  represent corresponding mass and stiffness of bridge substructure.

### 3. COMPLEX STREUCRURAL SYSTEM PARAMETER ANANLYSIS

#### 3.1. Motion Equation and Random Response

Considering the damping, the motion equation for two lumped-mass system model in figure 2 can be expressed as follows:

$$\begin{cases} m_{1}\ddot{\sigma}_{1} + c_{1}\dot{\sigma}_{1} + k_{1}\sigma_{1} - c_{2}\dot{\sigma}_{2} - k_{2}\sigma_{2} = -m_{1}\ddot{x}_{1g} \\ m_{2}(\ddot{\sigma}_{1} + \ddot{\sigma}_{2}) + c_{2}\dot{\sigma}_{2} + k_{2}\sigma_{2} = -m_{2}\ddot{x}_{2g} \end{cases}$$
(3.1)

Where,  $m_1 \ m_2$  represent mass of bridge and building respectively,  $c_1 \ c_2$  represent damping of bridge and building respectively,  $k_1 \ k_2$  represent stiffness of bridge and building respectively,  $\sigma_1$  represent relative displacement for bridge to foundation and  $\sigma_2$  represent relative displacement for building to bridge.

When seismic loading is added to the complex aseismic system,  $\ddot{x}_{1g} = \ddot{x}_g$ ,  $\ddot{x}_{2g} = \ddot{x}_g$ , the following parameters can be introduced:

$$\omega_1^2 = \frac{k_1}{m_1}; \omega_2^2 = \frac{k_2}{m_2}; \zeta_1 = \frac{c_1}{2\sqrt{m_1k_1}}; \zeta_2 = \frac{c_2}{2\sqrt{m_2k_2}}; f_1 = \frac{\omega}{\omega_1}; f_2 = \frac{\omega_2}{\omega_1}; \ \mu = \frac{m_2}{m_1};$$

Frequency characteristic for transfer function of the relative displacement  $\sigma_1$  and  $\sigma_2$  can be derived from equation (3.1).

$$\begin{cases} H_{\sigma_{1}}(i\omega) = \frac{f_{1}^{2} - (1+\mu)f_{2}^{2} - i(1+\mu)2\varsigma_{2}f_{1}f_{2}}{\omega_{1}^{2}\Pi} \\ H_{\sigma_{2}}(i\omega) = \frac{-1 - i2\varsigma_{1}f_{1}}{\omega_{1}^{2}\Pi} \end{cases}$$
(3.2)

Where,

$$\Pi = A + iB$$
  

$$A = f_1^4 - f_1^2 \left[ 1 + (1 + \mu) f_2^2 + 4\varsigma_1 \varsigma_2 f_2 \right] + f_2^2$$
  

$$B = -f_1^3 \left[ 2\varsigma_1 + 2(1 + \mu) \varsigma_2 f_2 \right] + f_1 \left[ 2\varsigma_1 f_2^2 + 2\varsigma_2 f_2 \right]$$

The system's absolute acceleration  $\ddot{x}_1$  and  $\ddot{x}_2$  can be expressed as follows:

$$\begin{cases} \ddot{x}_1 = \ddot{\sigma}_1 + \ddot{x}_{1g} \\ \ddot{x}_2 = \ddot{\sigma}_2 + \ddot{\sigma}_1 + \ddot{x}_{2g} \end{cases}$$
(3.3)

And the transfer function for absolute acceleration of the bridge and building complex structural system can be

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derived from equation (3.3).

$$\begin{cases} H_{\ddot{x}_{1}}(\omega) = \frac{f_{2}^{2} - [1 + 4\varsigma_{1}\varsigma_{2}f_{2}]f_{1} + i\left\{-2\varsigma_{1}f_{1}^{3} + f_{1}(2\varsigma_{2}f_{2} + 2\varsigma_{1}f_{2}^{2})\right\}}{\Pi} \\ H_{\ddot{x}_{2}}(\omega) = \frac{f_{2}^{2} - 4\varsigma_{1}\varsigma_{2}f_{2}f_{1}^{2} + if_{1}(2\varsigma_{2}f_{2} + 2\varsigma_{1}f_{2}^{2})}{\Pi} \end{cases}$$
(3.4)

Suppose seismic wave  $\ddot{x}_g$  is the white noise stationary stochastic process and its power spectral density function is  $S_{\ddot{x}_g} = S_0$ , variances of relative displacement  $\sigma_1$ ,  $\sigma_2$  and absolute acceleration  $\ddot{x}_1$ ,  $\ddot{x}_2$  can be obtained by integrating the transfer function for absolute acceleration and relative displacement.

$$\begin{cases} \sigma_{\sigma_{1}}^{2} = \frac{2\pi S_{0}}{\omega_{1}^{3}} \frac{\Pi_{1}}{\Pi_{0}} \\ \sigma_{\sigma_{2}}^{2} = \frac{2\pi S_{0}}{\omega_{1}^{3}} \frac{\Pi_{2}}{\Pi_{0}} \end{cases}; \begin{cases} \sigma_{\tilde{x}_{1}}^{2} = \frac{2\pi S_{0}\omega_{1}\Pi_{1}'}{\Pi_{0}} \\ \sigma_{\tilde{x}_{2}}^{2} = \frac{2\pi S_{0}\omega_{1}\Pi_{2}'}{\Pi_{0}} \end{cases}$$
(3.5)

Where,

$$\begin{split} \Pi_{0} &= 4f_{2} \{ \mu f_{2}(\varsigma_{2} + \varsigma_{1}f_{2})^{2} + \varsigma_{1}\varsigma_{2}[1 - (1 + \mu)f_{2}^{2}]^{2} \\ &+ 4\varsigma_{1}\varsigma_{2}f_{2}[f_{2}(\varsigma_{1}^{2} + \varsigma_{2}^{2} + \mu\varsigma_{2}^{2}) + \varsigma_{1}\varsigma_{2}(1 + f_{2}^{2} + \mu f_{2}^{2})] \} \\ \Pi_{1} &= \varsigma_{1}f_{2}^{2}[\mu^{2} + \mu(1 + \mu)^{2}f_{2}^{2}] + \varsigma_{2}f_{2}[1 - (1 + \mu)^{2}f_{2}^{2}]^{2} + \varsigma_{2}f_{2}\mu(1 + \mu)^{2}f_{2}^{2} \\ &+ 4\varsigma_{1}\varsigma_{2}^{2}f_{2}^{2}(1 + \mu)^{2}[1 + (1 + \mu)f_{2}^{2}] \\ &+ 4\varsigma_{2}^{3}f_{2}^{3}(1 + \mu)^{2}[(1 + \mu) + (\varsigma_{1}/\varsigma_{2})^{2}] \\ \Pi_{2} &= \varsigma_{1}(\mu + 1/f_{2}^{2}) + \varsigma_{2}[(1 + \mu)^{2}f_{2} + \mu/f_{2}] + 4\varsigma_{1}^{2}\varsigma_{2}[(1 + \mu)f_{2} + 1/f_{2}] \\ &+ 4\varsigma_{1}\varsigma_{2}^{2}[1 + \mu + (\varsigma_{1}/\varsigma_{2})^{2}] \\ \Pi_{1}' &= \mu f_{2}^{4}\varsigma_{1} + \varsigma_{2}f_{2}\{[1 - (1 + \mu)f_{2}^{2}]^{2} + \mu f_{2}^{2}\} \\ &+ 4\mu \varsigma_{1}^{3}f_{2}^{4} + 4\varsigma_{1}^{2}\varsigma_{2}f_{2}[(1 - f_{2}^{2})^{2} + f_{2}^{2} + \mu f_{2}^{4}] \\ &+ 4\varsigma_{1}\varsigma_{2}^{2}f_{2}[(1 + \mu)f_{2}^{3} + f_{2}] + 4\varsigma_{1}\varsigma_{2}^{2}f_{2}(1 + \mu)\varsigma_{2}^{3}f_{2}^{2} \\ &+ 16f_{2}^{2}\varsigma_{1}^{2}\varsigma_{2}[(\varsigma_{1}^{2} + \varsigma_{2}^{2})f_{2} + (1 + \mu)^{2}f_{2}^{2}] + 16\varsigma_{1}^{2}\varsigma_{2}^{2}f_{2}^{2}(\varsigma_{1}f_{2} + \varsigma_{2}) \\ &+ 4f_{2}\{\varsigma_{1}^{3}f_{2}^{2} + \varsigma_{1}^{2}\varsigma_{2}[(1 + \mu)f_{2}^{3} + f_{2}]\} \\ &+ 4f_{2}\{\varsigma_{1}\varsigma_{2}^{2}[1 + (1 + \mu)f_{2}^{2}] + \varsigma_{2}^{3}(1 + \mu)f_{2}\} \end{split}$$

Supposed the variances of relative displacement and absolute acceleration of conventional structure are  $\sigma_{\sigma_0}$ ,  $\sigma_{\tilde{x}_0}$  obtained form equation (3.4) when  $f_2 = 0$  and  $\mu = 0$ , the damping effect of bridge and building complex structural system can be expressed as the seismic response ratio of the complex aseismic system to conventional bridge. It contains four parts: the mean square deviation ratio of birdge deck's displacement response of bridge structure  $v_{\sigma_1}$  and corresponding mean square deviation ratio of birdge deck's acceleration response  $v_{\tilde{x}_1}$ ; the mean square deviation ratio of building structure  $v_{\sigma_1}$  and



corresponding mean square deviation ratio of acceleration response  $v_{\vec{x}_1}$ . These parameters are expressed as follows:

$$v_{\sigma_{1}} = \frac{\left|\sigma_{\sigma_{1}}\right|}{\left|\sigma_{\sigma_{0}}\right|}; \quad v_{\sigma_{2}} = \frac{\left|\sigma_{\sigma_{2}}\right|}{\left|\sigma_{\sigma_{0}}\right|}; \quad v_{\ddot{x}_{1}} = \frac{\left|\sigma_{\ddot{\sigma}_{1}}\right|}{\left|\sigma_{\ddot{\sigma}_{0}}\right|}; \quad v_{\ddot{x}_{2}} = \frac{\left|\sigma_{\ddot{\sigma}_{2}}\right|}{\left|\sigma_{\ddot{\sigma}_{0}}\right|}$$

#### 3.2. Parameter Influence Analysis

The relationship between parameters of damping and frequency ratios and damping effect  $v_{\sigma_1}$ ,  $v_{\sigma_2}$  is shown in figure 3. From figure 3 (a) and (b) we can see that the mean square deviation ratio of displacement response of bridge structure increases with frequency ratio and decreases with the increase of damping and the mean square deviation ratio of displacement response of building structure decreases with the increase of frequency ratio or damping when the mass ratio is 1 and bridge's damping ratio is 0.05. In figure 3 (c) and (d) the bridge's damping ratio becomes 0.05 and the damping ratio of bridge is 0.05. Then we can see that the displacement response curve of bridge has a significant wave trough when the mass ratio is small and the wave trough grows to be not significant with the increase of mass ratio. The mean square deviation ratio of displacement response of bridge structure increases with frequency ratio or mass ratio. When mass ratio is small(smaller than 0.5) and frequency ratio is small, the mean square deviation ratio of displacement response of building structure increases with frequency ratio decreases with the increase of building structure increases with frequency ratio. It increases from wave trough to wave crest. And when the curve is over wave crest, the mean square deviation ratio decreases with the increase of frequency ratio is large(lager than 0.5), the mean square deviation ratio decreases with the increase of frequency ratio or frequency ratio.



Figure 3 The influence analysis of mass ratio

The relationship between parameters of damping and frequency ratios and damping effect  $v_{\ddot{\sigma}_1}$ ,  $v_{\ddot{\sigma}_2}$  is shown

in figure 4. From figure 4 (a) and (b) we can see that the mean square deviation ratio of displacement response of bridge structure increases with frequency ratio and decreases with the increase of damping and the mean square deviation ratio of displacement response of building structure decreases with the increase of frequency ratio or damping when the mass ratio is 1 and bridge's damping ratio is 0.05. In figure 4 (c) and (d) bridge's damping ratio becomes 0.05 and the damping ratio of bridge is 0.05. Here we can see that the displacement response curve of bridge has a significant wave trough when the mass ratio is small and the wave trough grows to be not significant with the increase of mass ratio. The mean square deviation ratio of displacement response of bridge structure increases with frequency ratio or mass ratio. When mass ratio is small(smaller than 0.5) and frequency ratio is small, the mean square deviation ratio of displacement response of building structure increases with frequency ratio. It increases from wave trough to wave crest. And when the curve is over wave crest, the mean square deviation ratio decreases with the increase of frequency ratio. When mass ratio is large(lager than 0.5), the mean square deviation ratio decreases with the increase of frequency ratio or frequency ratio. The 14<sup>th</sup> World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China





Figure 4 The influence analysis of mass ratio

We can get some results from figure 3 and figure 4: (1) The higher shock absorption layer's damping ratio  $\zeta_2$  becomes, the lower displacement and acceleration response of bridge deck and building of the complex system will be; (2) When mass ratio is small, the trend curve of the mean square deviation ratio of displacement response to frequency ratio is similar to TMD's. Here the complex aseismic system mainly manifests as TMD. When mass ratio is large, the complex aseismic system's function mainly manifests as energy dissipation of shock absorption layer; (3) When mass ratio is invariable, there is always a reasonable frequency ratio to make building structure's displacement and acceleration responses optimal, and the reasonable frequency ratio decreases with the increase of mass ratio.

#### 3.3. Parameter Selection

From the analysis above we know that the displacement and acceleration responses of the bridge and building complex structural system decrease with the increase of damping ratio when mass ratio is larger than 0.5. So the damping ratio should be chosen as large as it can in engineering design.

Assuming that mass ratio is under 0.5-5 and damping ratio of bridge is 0.05, the optimal frequency ratio to corresponding mass ratio is fitted by nonlinear mathematical programming method when acceleration response of bridge structure is only considered. The fitting date curve is shown in figure 5(a), and the fitting equation is shown in equation (3.6).

$$f_{opt} = 0.26286 + 0.2566e^{-\mu/0.645} + 0.44873e^{-\mu/3.21491}$$
(3.6)

If acceleration responses of both bridge structure and building structure are considered, the optimal frequency ratio for acceleration response of the complex aseismic system is obtained in equation (3.7) based on the principle that base shear of bridge and total mass ratio are minimal.

$$\min_{f_2} \{ (m_1 \ddot{x}_1 + m_2 \ddot{x}_2) / (m_1 + m_2) \}$$
(3.7)

It can be simplified in equation (3.8).

$$\min_{f_2} \{ (\ddot{x}_1 + \mu \ddot{x}_2) / (1 + \mu) \}$$
(3.8)

Considering damping ratio is 0.05, the optimal frequency ratio can be obtained based on equation (3.8) by nonlinear mathematical programming method. And the fitting date curve is shown in figure 5(b), the fitting equation is shown in equation (3.9).

$$f_{opt} = 0.06773 + 0.41408e^{-\mu/1.87448} + 0.43096e^{-\mu/0.4764}$$
(3.9)





Fig. 5 The optimal solution of frequency ratio

### 4. CONCLUSIONS

The bridge and building complex structural system is proposed based on rubber bearing isolation technology and suspension isolation technology. Simplified model and the formulation of the 2-degree of freedom equations of motion for the structure are provided based on the principle of equation between kinetic energy and base shear.

Parameter optimization analysis of the bridge and building complex structural system has been completed, and the parameter influences to the displacement and acceleration response of the system has be analyzed under vehicle dynamic load and earthquake load. The optimal frequency ratio and damping ratio calculation formulas have been proposed when mass ratio is under 0.5-5.

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# REFERENCES

Li, G.Q. and Huo, D. (1993). Structure Control Theory and Application, Wuhan University of Technology Press, Wuhan, China.

Hu, Y. X. (1988). Earthquake Engineering, Earthquake Publishing House, China.

Kelly, J.M. (1997). Earthquake Resistant Design with Rubber. Second Edition. Springer Verlag London Limited.

Liu, W.G. (2003). Mechanics Properties of Rubber Bearings and Earthquake Response Analysis of Isolated Structure. Beijing University of Technology, Beijing, China.

Architectural Institute of Japan. (2001). Recommendation for the design of base isolated buildings. *Maruzen Press*, Tokyo. Japan.

Wen, Y.K. (1976). Method of Random Vibration of Hysteretic System. *Journal of Engineering Mechanics* (ASCE) **102**:249–263.

Li, Z.X. and Zhou, X.Y. (2002). Simplified Analysis Method of Seismically Isolated Regular Building. *Earthquake Engineering and Engineering Vibration* **22:2**, 249–263.

He, W.F. (2008). Earthquake Response Analysis and Experiment Research on Bridge and Building Complex Structural System. Beijing university of technology, Beijing, China.