APPROXIMATE SEISMIC PERFORMANCE UNCERTAINTY ESTIMATION USING STATIC PUSHOVER METHODS

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ABSTRACT :

An approximate method based on the static pushover is introduced to estimate the seismic performance uncertainty of structures having uncertain parameters. Performance uncertainty is one of the driving forces behind modern seismic guidelines (e.g. FEMA-350) and it is arguably an essential ingredient of Performance-Based Earthquake Engineering (PBEE). We propose a methodology that uses a minimum of static nonlinear analyses and is capable of accurately estimating the demand and capacity epistemic uncertainty. As a testbed, the well-known nine-story LA9 steel frame is employed using beam-hinges with uncertain backbone properties. These range from simple elastic-perfectly plastic backbones with kinematic hardening to full quadrilinear backbones with pinching hysteresis, including an elastic, a hardening, a negative stiffness and a residual plateau branch, terminating with a final drop to zero strength. The properties of the backbone can be fully described by six parameters which are considered uncertain with given mean and standard deviation values. Using latin hypercube sampling with classic Monte Carlo simulation, the pushover curve is shown to be a powerful tool that can accurately estimate the uncertainty in the seismic performance. Coupled with the SPO2IDA tool, such estimates can be applied at the level of the results of nonlinear dynamic analysis, allowing the evaluation of seismic capacity uncertainty even close to global dynamic instability. In summary, the method presented can inexpensively supply the uncertainty in the seismic performance of first-mode dominated buildings, offering for the first time an estimator of the accuracy of typical performance calculations.

KEYWORDS: Performance evaluation, Dispersion, Parameter Uncertainty, Static Pushover Analysis, Incremental Dynamic Analysis, Nonlinear analysis.

1. INTRODUCTION

Structural analysis is plagued by both aleatory randomness, e.g. due to natural ground motion record variability, and epistemic uncertainty, stemming from modeling assumptions or errors. Design codes recognize the importance of uncertainty in the process of seismic design through implicitly including generic safety factors in the model, the material properties and the loads. Unfortunately little data is available on the seismic demand and capacity uncertainty, an issue that is ultimately dealt with tabulated values, which were often meant to act as placeholders. The first code/guideline that treats explicitly uncertainty in seismic design is FEMA-350 (SAC/FEMA 2000). The SAC/FEMA project caused the widespread adoption of the notion of uncertainty in earthquake engineering applications, while novel methodologies on how to include the uncertainty in performance estimation have already appeared (Cornell et al. (2002), Baker and Cornell (2003)). To extend those concepts, in a companion paper, the authors study the influence of plastic-hinge modeling assumptions and provide simplified methods to obtain statistics of the total system response (Vamvatsikos and Fragiadakis 2008).

Although structural reliability analysis methods have considerably evolved during the last few years (e.g., Pinto et al. 2004) the need for new approaches to estimate uncertainty for complex nonlinear structures still exists. Monte-Carlo simulation methods are powerful tools that can handle almost any problem, but this always comes at the expense of a large number of computationally-intensive nonlinear dynamic analyses. For earthquake engineering problems, one of the most comprehensive method to handle the aleatory uncertainty introduced by seismic loading is the Incremental Dynamic Analysis (IDA) method (Vamvatsikos and Cornell 2002), which essentially requires multiple nonlinear dynamic analyses with a suite of ground motion records. To account for other sources of uncertainty, the Monte-Carlo method can be adopted, thus performing a full IDA for each point in the parameter space. This process, although extremely powerful, necessitates the execution of millions of nonlinear dynamic analyses and therefore is beyond the scope of any practical application.

Given the computational obstacle, the usual practice to circumvent this problem is assuming ad hoc values for the dispersions caused by uncertainties, e.g. in the model properties, and either implicitly taking them into account or explicitly including them in the guidelines, as in FEMA-350. A better insight of how the FEMA-350 tabulated values have emerged can be obtained by the study of Yun et al. (2002). Clearly, the proposed dispersion values are either semi-empirical or have been derived from a limited number of benchmark structures. Consequently, these values can be seen as reasonable placeholders that, in the absence of a more rational and proven values, tend to become the de facto standard.

What we are proposing is a new methodology to estimate the dispersion using static pushover methods. A static pushover analysis (SPO) requires considerably less computational resources compared to the hundreds of dynamic analyses necessary for a single IDA run. The link between SPO and IDA is provided by the "Static Pushover to Incremental Dynamic Analysis" tool (Vamvatsikos and Cornell 2005), which for the remainder of the paper will be called "SPO2IDA". SPO2IDA provides rapid estimates of the performance of first-mode-dominated structures, from elasticity and all the way to collapse. Having such a tool at our disposal we can easily perform all the necessary simulations and actually provide an accurate estimate of the effect of uncertainty on the demand and capacity of structures. Using Monte Carlo on top of SPO2IDA we evaluate the dispersion due to the influence of plastic hinges, for each performance level adopting as a reference structure the well known LA9 nine-story steel frame.

2. IDA AND SPO2IDA

Incremental Dynamic Analysis (IDA) is a powerful analysis method that offers thorough seismic demand and capacity prediction capability (Vamvatsikos and Cornell 2002). It involves performing a series of nonlinear dynamic analyses under a multiply scaled suite of ground motion records. By selecting proper Engineering Demand Parameters (EDPs) to characterize the structural response and choosing an Intensity Measure (IM), e.g. the 5%-damped, first-mode spectral acceleration $S_a(T_1,5\%)$, to represent the seismic intensity, we can generate the IDA curves of EDP versus IM for each record and then estimate the 16%, 50% and 84% summarized curves. On such curves the desired limit-states can be defined by setting appropriate limits on the EDPs. Thus the corresponding capacities and their probabilistic distributions are estimated. Such results combined with probabilistic seismic hazard analysis (Vamvatsikos and Cornell 2002) allow the estimation of mean annual frequencies (MAFs) of exceeding the limit-states thus offering a direct characterization of seismic performance. IDA comes at a considerable cost, since it requires the use of multiple nonlinear dynamic analyses that are usually beyond the abilities and the computational resources of the average practicing engineer, even for simple structures. Therefore, a simpler and faster alternative is always desirable.

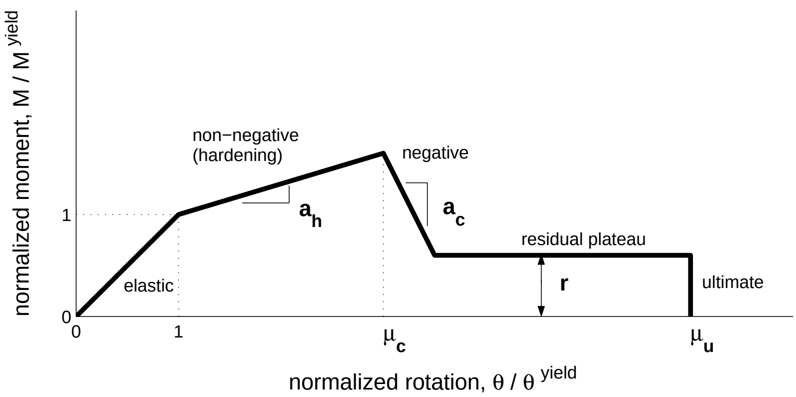
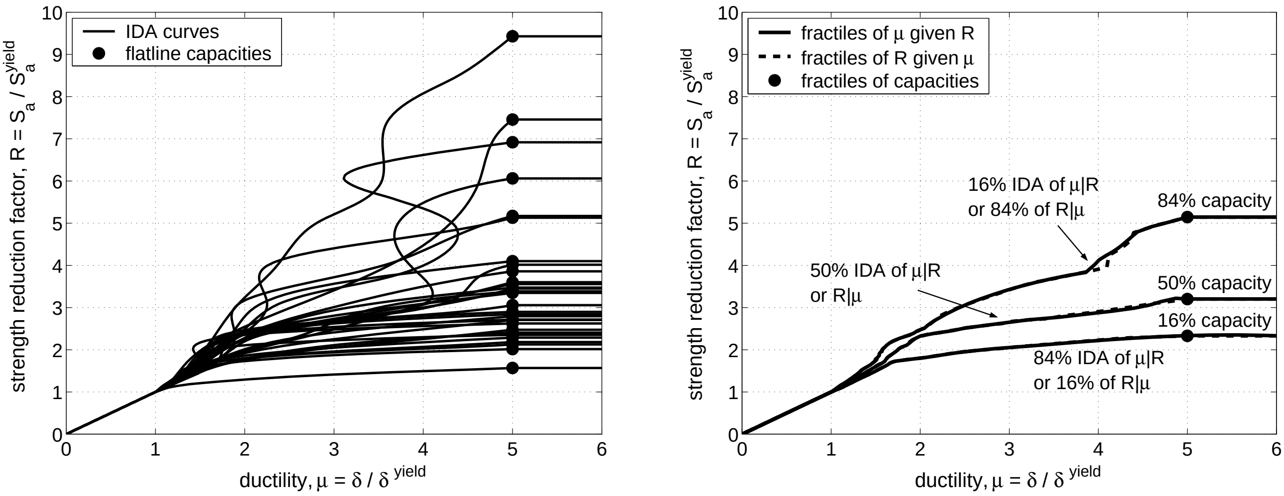


Figure 1 The moment-rotation relationship of the beam point-hinge in normalized coordinates

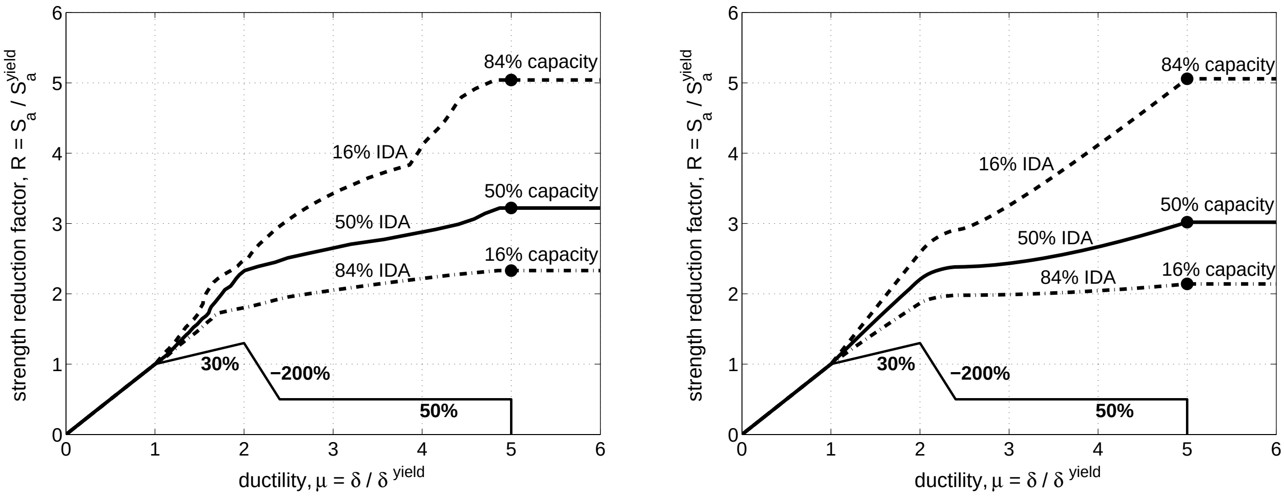
A fast and accurate approximation has been recently proposed for IDA, both for single and multi-degree-of-freedom systems utilizing information from the force-deformation envelope (or backbone) to generate the summarized 16%, 50% and 84% IDA curves (Vamvatsikos and Cornell 2005). The approximation is based on the study of numerous SDOF systems having varied periods, moderately pinching hysteresis and 5% viscous damping, while they feature backbones ranging from simple bilinear to complex quadrilinear with an elastic, a hardening and a negative-stiffness segment plus a final residual plateau that terminated with a drop to zero strength as shown in Figure 1 (Ibarra 2003, Ibarra et al. 2005). Having compiled the results into the SPO2IDA tool, available online (Vamvatsikos 2002), we can get an approximate estimate of the performance of virtually any oscillator without having to perform the costly analyses, almost instantaneously recreating the fractile IDAs in normalized coordinates of R=$S_a(T_1,5\%)/S_a^{\text{yield}}(T_1,5\%)$ (where $S_a^{\text{yield}}(T_1,5\%)$ is the $S_a(T_1,5\%)$ value to cause first yield) versus ductility μ.

An application of SPO2IDA, taken from Vamvatsikos and Cornell (2006), appears in Figures 2 and 3 where the 16%, 50%, and 84% fractile IDA curves of a 5%-damped oscillator with moderately pinching hysteresis and period T = 0.9s, are estimated using both IDA and SPO2IDA. The accuracy achieved by SPO2IDA is remarkable everywhere on the IDA curves, even close to collapse. SPO2IDA is in fact a powerful R–μ–T relationship that will provide not only central values (mean and median) but also the dispersion due to record-to-record aleatory randomness of the strength reduction R factor given μ.



(a) (b)

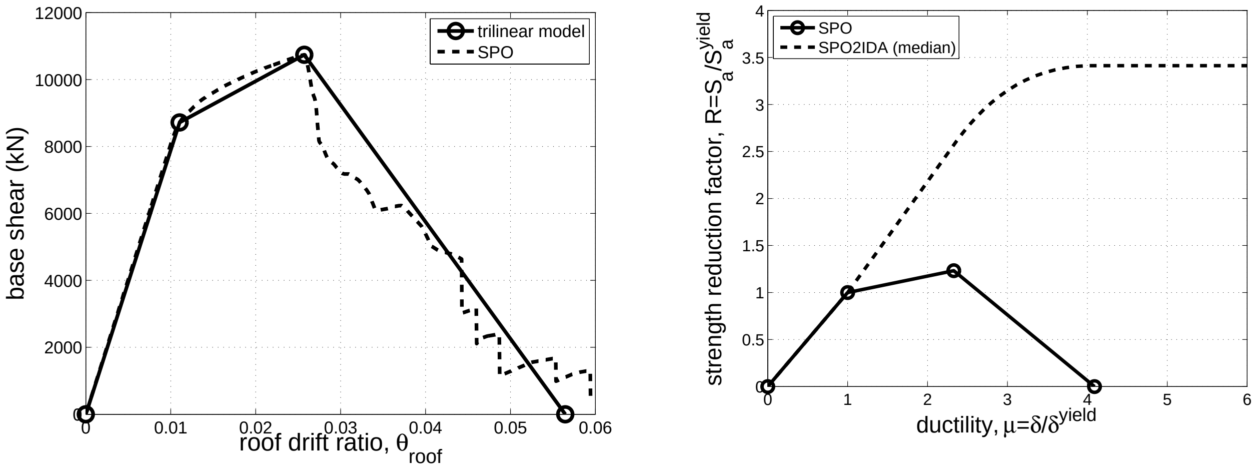
Figure 2 Incremental Dynamic Analysis (IDA) curves for a SDOF system using (a) thirty ground motion records and (b) summarization of the IDA curves into their fractile curves given μ or R.



(a) (b)

Figure 3 (a) The fractile IDA curves of Figure 2b versus the static pushover curve of the SDOF system, (b) the corresponding fractile IDAs estimated by the SPO2IDA tool.

The SPO2IDA tool has been extended to first-mode dominated MDOF structures (Vamvatsikos and Cornell 2005), enabling an accurate estimation of the fractile IDA curves even close to collapse without needing nonlinear dynamic analyses. In addition it has been shown to only slightly increase the error in our estimation, resulting to an accuracy that can be compared to that of the actual IDA using a smaller number, e.g. ten, ordinary ground motion records. Thus it can render performing the multiple IDAs quite effortless, offering an efficient and very simple method for estimating the uncertainty associated with the limit-state capacities of a structure, given the variability in the backbone parameters of the beam plastic hinges.



(a) (b)

Figure 4 (a) The SPO curve of a nine-story steel structure and its approximation with a trilinear model, (b) SPO curve versus the corresponding SPO2IDA capacity curves.

The application of the SPO2IDA tool on a MDOF structure is schematically shown in Figure 4 and is described in detail by Vamvatsikos and Cornell (2005). The process involves approximating the SPO curve with a mutlilinear backbone (Figure 4a) to allow extracting the properties of the backbone curve, as we did for the SDOF structure and provide the data to the SPO2IDA tool to produce the fractile capacities (Figure 4b). While it is advocated to search for the most damaging load patter (Vamvatsikos and Cornell, 2005) it was found that a triangular or first-mode lateral load pattern is sufficient for this model of the LA9 frame.

The fractile capacities provided by the SPO2IDA are in dimensionless R-μ coordinates (Figure 4b) and therefore, the results of SPO2IDA need to be scaled to $S_a(T_1,5\%)$ versus maximum interstory drift ratio, $\theta_{\max}$ coordinates. While SPO2IDA provides all three of the 16%, 50% and 84% fractiles, in the remainder of this paper we only need the median (50% fractile) curve. Thus, extending the approach previously presented for SDOF structures, the scaling can be performed with the following simple calculations:

$$\begin{aligned} \underset{\sim}{S}_a(T_1,5\%) &= \underset{\sim}{R}\, S_a^{yield}(T_1,5\%) \\ \underset{\sim}{\theta}_{max} &= \underset{\sim}{\mu}\, \theta_{max}^{yield} \end{aligned} \tag{1}$$

where the '~' sign denotes a vector. Prior to applying Equation (1) we have to determine $S_a(T_1,5\%)$ and $\theta_{\max}$ at yield. This task is trivial for SDOF systems, but it is not straightforward for MDOF structures, mainly due to the effect of higher modes. Some records will force the structure to yield earlier and others later, thus yielding always occurs at different levels of $S_a(T_1,5\%)$ and $\theta_{\max}$. Driven by our trilinear approximation to the SPO curve, we let the yield roof drift θ_{roof} be defined accordingly as the apparent yield point of the approximation. Therefore, accurate estimation of $S_a^{yield}(T_1,5\%)$ and $\theta_{\max}^{yield}$ comes down to estimating the elastic stiffnesses of the median IDA curves plotted with $\theta_{\max}^{yield}$ and θ_{roof}^{yield} as the EDPs. The stiffnesses, denoted as $k_{\max}$ and k_{roof}, respectively, are the median values obtained using one elastic timehistory run for each record. Finally, $S_a^{yield}(T_1,5\%)$ and $\theta_{\max}^{yield}$ are obtained as follows:

$$S_a^{yield}(T_1,5\%) = k_{roof}\theta_{roof}^{yield}$$
$$\theta_{\max}^{yield} = S_a^{yield}(T_1,5\%)/k_{\max} \tag{2}$$

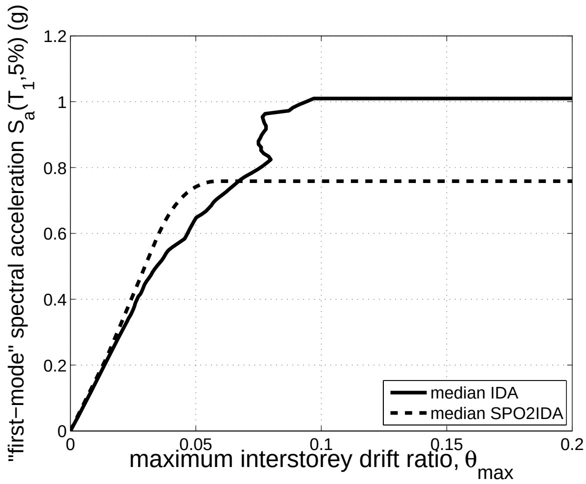


Figure 5 The median SPO2IDA capacity curve of Figure 4 plotted against the actual median IDA.

In summary, the process of deriving an approximate IDA curve involves performing a static pushover which is approximated with an appropriate multilinear model. SPO2IDA will then provide the normalized IDA curves. The final step involves scaling the curves to the $S_a(T_1,5\%)$ and $\theta_{\max}$ coordinates, which necessitates one linear elastic dynamic analysis for each ground motion record (which need only be performed once for all instances of the model) to obtain the elastic slopes of the actual IDAs, k_{roof} and $k_{\max}$, when θ_{roof} and $\theta_{\max}$, respectively, are used for the EDP. With the aid of Equations (2) and (1) we reach the final IDAs. For the SPO curve of Figure 4, the median IDA obtained with the SPO2IDA and the actual IDA curve for thirty ordinary ground motion records (Vamvatsikos and Fragiadakis, 2008) are shown in Figure 5. The error in the procedure is typically 10-20% for our model, while the computing time comes down from 2-3 hours for IDA to just a couple of minutes for SPO2IDA, approximately two orders of magnitude less!

3. MODEL DESCRIPTION AND PARAMETER UNCERTAINTIES

The structure considered is a nine-story steel moment resisting frame with a one-story basement that has been designed according to 1997 NEHRP (National Earthquake Hazard Reduction Program) provisions for a Los Angeles site. We use a centerline model with fracturing connections on OpenSees (McKenna and Fenves 2001). The beams are lumped-plasticity elements allowing the formation of plastic hinges at their ends, while the columns are essentially elastic. Geometric nonlinearities have been incorporated in the form of P-Δ effects, while internal gravity frames have been explicitly modeled. In effect, this is a first-mode dominated structure with a fundamental period of T_1=2.35s and a modal mass of 84%, thus allowing for significant higher modes.

The fracturing connections are modeled as rotational springs with moderately pinching hysteresis and a quadrilinear moment-rotation backbone, shown in normalized coordinates in Figure 1 (see also Ibarra et al. 2005). The random parameters considered refer to the properties of the quadrilinear backbone curve, which initially allows for elastic behavior up to M_y, then hardens at a non-negative slope of a_h that terminates at a normalized rotation μ_c, where the negative stiffness segment starts, having a slope of a_c. The residual plateau appears at a normalized height R, delaying the failure of the connection until the ultimate normalized rotation μ_u. Thus six parameters are necessary to completely describe the backbone: M_y, a_h, μ_u, a_c, r and μ_u, while similar behavior is assumed for both positive and negative moments. This is essentially a complex backbone that is versatile enough to simulate the behavior of numerous moment-connections.

4. METHODOLOGY

In order to estimate the dispersion introduced by the plastic hinge properties, classic Monte-Carlo simulation with Latin Hypercube Sampling (LHS) is performed (McKay et al, 1979). The random variables are the six variables that fully describe the backbone of the plastic hinge moment-rotation relationship (Figure 1). The plastic hinge properties are varied simultaneously for the whole structure, thus being consistent with the scenario where the practicing engineer does not have sufficient data for the moment-rotation backbone and therefore has to rely on his/her judgment and experience. The parameters are considered to be normally distributed with mean and c.o.v as shown in Table 1. To avoid assigning the random parameters with values with no physical meaning, e.g. a_h>1, their distribution is appropriately truncated within ±1.5 standard deviations as shown in the last two columns of Table 1. For the yield moment the c.o.v considered is equal to 20%, while for the remaining parameters it is assumed equal to 40%. Moreover, in Table 1 the M_y values are normalized with the nominal yield moment of the corresponding beam section $M_{y,nom}$. Having determined the distribution of the six random variables we perform latin hypercube sampling to obtain N_{LHS} = 200 Monte-Carlo samples using the Iman and Conover (1982) algorithm to ensure zero correlation among variables. Each realization of the LA9 frame is subjected to an SPO analysis then SPO2IDA to finally obtain N_{LHS} median IDA curves.

Table 1 Random parameters and their statistics

	Mean	c.o.v	Lower bound	Upper bound
M_y	1.0	0.20	0.70	1.30
a_h	0.1	0.40	0.04	0.16
μ_u	3.0	0.40	1.20	4.80
a_c	-0.5	0.40	-0.80	-0.20
r	0.5	0.40	0.20	0.80
μ_u	6.0	0.40	2.40	9.60

5. NUMERICAL RESULTS

Figure 6 shows the N_{LHS}=200 static pushover curves for the LA9 steel frame, while Figure 7 presents the IDA curves obtained through SPO2IDA (Figure 7a) versus the actual IDAs (Figure 7b) computed for thirty ground motion records (Vamvatsikos and Fragiadakis, 2008). Similarly to the median curves of Figure 5, the SPO2IDA curves may appear smoother the actual IDAs but in both cases the final capacities $S_a(T_1,5\%)$ vary similarly between 0.4g and 1.2g, while a consistently constant slope until about θ_{max}=0.07 is observed.

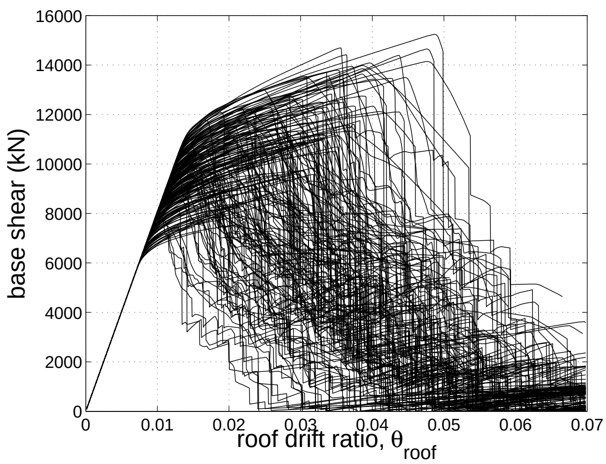
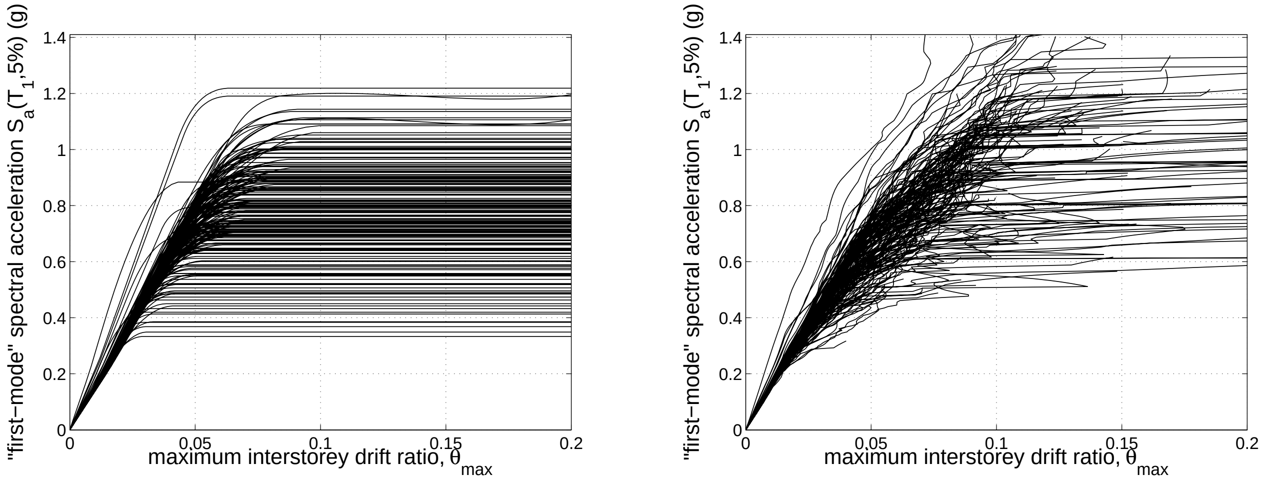


Figure 6 N_{LHS} static pushover curves of the nine-story steel frame

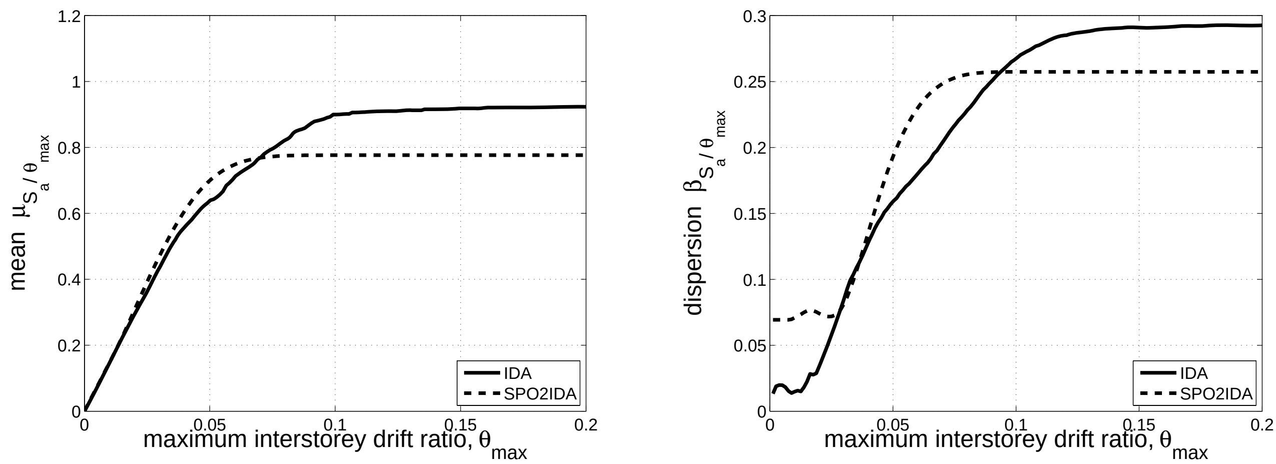


(a) (b)

Figure 7 N_{LHS} IDA curves obtained with (a) SPO2IDA and (b) IDA

The IDA curves can be post-processed to provide mean $S_a(T_1,5\%)$ capacities and dispersion, conditional on the limit-state, θ_{max}. Dispersion is typically represented by β-values, i.e. the standard deviation of the natural logarithms of the data. Figure 8a, shows the conditional mean, $\mu_{Sa|\theta max}$, obtained via SPO2IDA which is very close to that of the actual IDA for the early limit-states, until to 0.04, while for higher limit-states and particularly for the θ_{max} range 0.04÷0.07 the agreement is still remarkable. For high limit-states approaching collapse the error on $\mu_{Sa|\theta max}$ remains constant and approximately equal to 16%. Although this approach is specifically tailored to provide the dispersion $\beta_{Sa|\theta max}$ it is clear that it can also provide very satisfactory estimates of the mean.

Figure 8b shows the dispersion $\beta_{Sa|\theta max}$ of the $S_a(T_1,5\%)$ capacities conditioned on θ_{max}. The agreement is remarkable for the whole range of θ_{max}, even when approaching collapse. More specifically the capacity at the collapse limit-states with the SPO2IDA approach was found equal to 0.26g, and 0.29g with the actual IDA, thus resulting to a 10% error. Contrary to the mean where the SPO2IDA underestimates the capacities, the dispersion is overestimated for lower limit-states, for $\theta_{max}\leq 0.1$, and underestimated beyond this point. As shown both in Figures 5 and 8a, the bias in the prediction of the median IDA is of the order of 10-20% and remains practically constant for every simulation and thus does not affect the estimation of the β-dispersion. This observation provides a partial explanation for the accuracy observed in Figure 8b.



(a) (b)

Figure 8 (a) Conditional mean and (b) conditional β-dispersion values
of the SPO2IDA versus that of the actual IDA using N_{LHS} Monte-Carlo samples.

6. CONCLUSIONS

An innovative approach has been presented to propagate the epistemic uncertainty from the model parameters to the actual seismic performance of a structure, providing inexpensive estimates for the uncertainty in the limit-state capacities. The method has been applied on a nine-story steel moment-resisting frame with quadrilinear fracturing connections that are fully described by six non-deterministic parameters. Monte-Carlo simulation with latin hypercube sampling is performed, while the computationally expensive results of full IDAs are approximated by the SPO2IDA tool. The estimated uncertainty in the limit-state capacities is slightly overestimated for limit-states in the elastic and early inelastic region while it is slightly underestimated close to collapse. The overall accuracy of the proposed method is found to be remarkable and comparable to full IDA results, at only a fraction of the cost. All in all, the proposed tool is an excellent resource for accurate estimation of the seismic performance of structures having uncertain properties, for the first time providing specific results for each limit-state that can be used in the place of the generic, code-prescribed values.

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