ANALYTICAL SIMULATION OF PROGRESSIVE COLLAPSE OF PERIMETER FRAMES DUE TO OUT-OF-PLANE BEHAVIOR

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ABSTRACT :

Typically steel moment-resisting frame structures in the United States, similar to many super-high-rise framed-tube structures being constructed in Japan, have stiff beams and columns only around the perimeter of the building and often have no or few seismic beams inside the building. The lack of strong, stiff connections between the perimeter frames and the floor system may result in minimal resistance to out-of-plane motion by the perimeter frames and may lead to unstable behavior when perimeter frames are separated from the floor systems as a result of accidental or earthquake loading. To investigate this behavior, simulations of the progressive collapse of these systems were carried out using simple structural models. Structural stability was evaluated by conducting eigenvalue analysis at each time-step of the analysis. The results of this study show that when frame-floor connections have insufficient tensile strength, progressive collapse of perimeter frames occurs due to out-of-plane motion of the frames and that on-set of collapse may be predicted by a negative instantaneous eigenvalue.

KEYWORDS : High-rise Framed-tube Structure, Perimeter Frame, Dynamic Stability, Progressive Collapse

1. INTRODUCTION

Steel moment-resisting frame structures in the United States typically consist of a few seismic frames located around the perimeter of the structure and many gravity frames inside structure, as illustrated in Figure 1(a). This framing system has a similar plan to the framed-tube structures such as the World Trade Center Building in New York and many base-isolated, super-high-rise buildings being constructed in Japan, as illustrated in Figure 1(b). These framing systems are attractive to designers because they provide flexibility for floor layout. However, with these systems, progressive collapse of the perimeter frames, due to instability resulting from failure of the floor-frame connections is a concern.

The connections between the perimeter frame and floor system prevent buckling of the perimeter frame columns by limiting the column buckling length to the story height. If a perimeter column deforms towards the inside of the structure, as illustrated in Figure 1(c), the floor system resists this motion and prevents buckling. However, if a perimeter column deforms towards the outside of the structure, as illustrated in Figure 1(d), large tensile forces may develop in the connection between the perimeter frame and floor-system. Since simple connections between the floor system and perimeter frame typically have relatively small strength, tensile failure may occur at these connections. Tensile failure of the frame-floor connection results in a longer column buckling length and, possibly, progressive collapse, as illustrated in Figures 1(e) and (f). Relatively low tensile capacity of the simple connections between the perimeter frames and floor system is considered to be one of many factors that contributed to the progressive collapse of the World Trade Center Building[1)] in New York.

The objectives of this study are to (1) numerically simulate the progressive collapse of perimeter frames in typical steel moment-resisting frame structure in the United States and (2) evaluate how the structure loses its stability and exhibits large deformations as tensile failure of frame-floor connections progress under earthquake loading. To achieve these objectives, simple 1D structural models proposed by the authors[2)] are used to simulate progressive collapse and eigenvalue analysis is carried out at each time-step of the nonlinear time-history analysis to evaluate the dynamic stability of the structure.

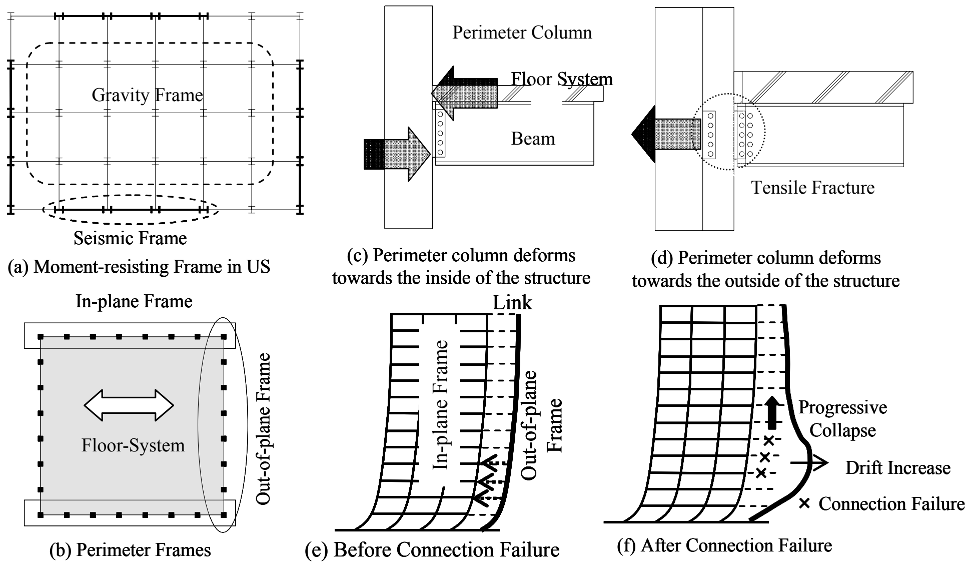


Figure 1. Progressive Collapse of Perimeter Frames due to Connection Failure in Tensile Direction

2. STRUCTURAL MODELS

The building used for this study is a 9-story steel moment-resisting frame structure consisting of seismic frame (Ⓐ), gravity frames (Ⓑ), and out-of-plane frame (Ⓒ) as illustrated in Figure 2(a). This building is representative of special moment-resisting steel frame structures in the United States, and was designed and analyzed as part of the SAC steel research project[3)]. The dimensions of fame members are listed in Table 1. As part of this study, the 2D frame model of the structure, shown in Figure 2(b), was simplified to the 1D shear-flexural-beam model[2)] with links, shown in Figure 3(c). Here, the springs in the shear-beam simulate the lateral resistance of each story-subassembly, assuming inflection points develop at mid-height of the columns, and the flexural-beam simulates flexural resistance associated with column continuity for the seismic and gravity frames as well as the perimeter frame in the out-of-plane direction. The link-element between the shear-beam and flexural-beam (Ⓐ+Ⓑ) is perfectly-rigid to provide identical drifts in the seismic and gravity frames (Ⓐ and Ⓑ). The link-element between the shear- and flexural-beams (Ⓐ+Ⓑ) and flexural-beam (Ⓒ) simulates the connection between the out-of-plane and in-plane frames. Thus, this link is defined to have large stiffness and strength in compression and relatively small strength in tension, as illustrated in Figure 4. When tensile force exceeds the capacity of the connection, strength and stiffness decrease to zero to simulate connection failure.

The formulation of the 1D shear-flexural-beam model with the links is summarized in Table 1. The incremental equation of motion of this system is given by Equation (1). Here, $[M]$, $[C]$, and $[K]$ are the mass, viscous damping, and stiffness matrices, $d\{\Delta_s\}$ and $d\{\Delta_f\}$ are the incremental relative drift vectors of the shear-beam and flexural-beam, $d\Delta_{\mathrm{ground}}$ is the incremental ground displacement, and $\{I\}$ is the eigenvector. $[K]$ is the sum of the shear-beam, flexural-beam, and link-element stiffness matrices, $[K_{shear}]$, $[K_{flexure}]$, and $[K_{link}]$. Each spring of the shear-beam has bilinear hysteresis with initial and post-elastic stiffness calibrated for the SAC 9-story building[2)]. $[K_{flexure}]$ is obtained by the boundary condition of $\{M_{total}\}=\{0\}$ as illustrated in Figures 3. Here, $\{M_{total}\}$ is the moment resultants at the nodes of the flexural-beam. The flexural-beam is assumed to be elastic and connected to the ground with rotational pin. The flexural stiffness ratios ($\alpha_i=\Sigma EI^{fc}_i/k^s_i$) of in-plane seismic frame, gravity frame, and out-of-plane seismic frame are 0.2, 0.2, and 0.07, respectively[2)]. The geometric matrices given by Equation (9) for the shear-beam and Equation (10) for the flexural-beam are included in the formulation of $[K]$.

The Newmark-Beta method (α=1/2, β=1/4)[4)] is used to solve the incremental equation of motion. Rayleigh damping is used for $[C]$ so that the structure has 2% viscous damping ratio at the 1st natural period of the system, T=2.23s, and at T=0.2s. The modified Newton-Raphson iteration scheme[4)] is used to compute nonlinear incremental drifts corresponding to incremental external forces. Assuming that the in-plane seismic frame, the gravity frame, and the out-of-plane seismic frame carry, respectively, 20%, 60%, and 20% of the total weight of the structure, the perimeter frame column in the first story experiences gravity force equal to 37% of the axial yield strength of perimeter column, P_y. Tensile strength of the link-element is expressed by the ratio of it to P_y in this study. The AIJ Recommendation for Limit State Design for Steel Structures[5)] specifies lateral-support strength as 3% of P_y.

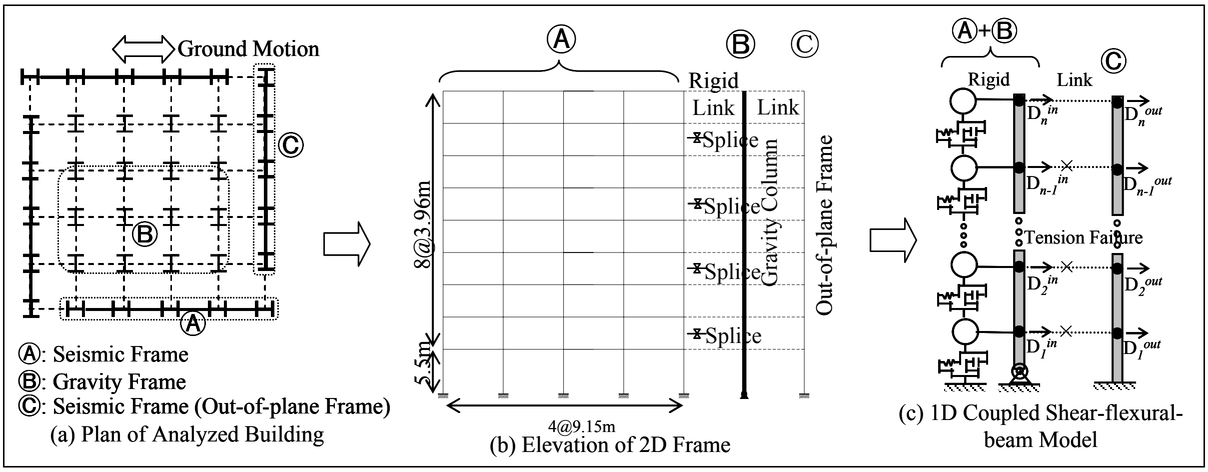
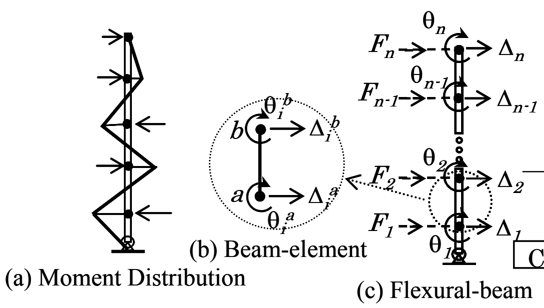


Figure 2. Building and Structural Model used for Analysis

Table 1: Section Size

	Beam	Int. Column	Out. Column
9F	W610×101	W360×382	W360×347
8F	W690×125	W360×382	W360×347
7F	W760×147	W360×421	W360×382
6F	W920×201	W360×421	W360×382
5F	W920×201	W360×551	W360×421
4F	W920×201	W360×551	W360×421
3F	W920×201	W360×677	W360×551
2F	W920×238	W360×677	W360×551
1F	W920×238	W360×744	W360×551



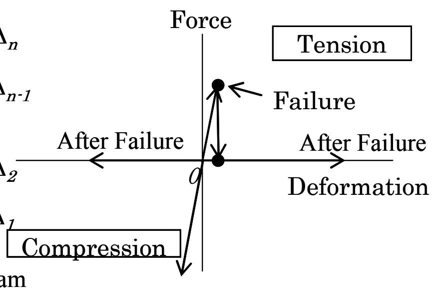


Figure 3. Mechanism of Flexural-beam

Figure 4. Hysteresis of Links

Table 1. Formulation of 1D Shear-flexural-beam Model with Links

Equation of Motion

$$[M]\begin{Bmatrix} d\ddot{\Delta}_s \\ d\ddot{\Delta}_f \end{Bmatrix} + [C]\begin{Bmatrix} d\ddot{\Delta}_s \\ d\ddot{\Delta}_f \end{Bmatrix} + [K]\begin{Bmatrix} d\ddot{\Delta}_s \\ d\ddot{\Delta}_f \end{Bmatrix} = -[M]\begin{Bmatrix} I \\ I \end{Bmatrix} d\ddot{\Delta}_{ground} \quad (1)$$

Mass Matrix

$$[M] = \begin{bmatrix} [M_{shear}] & [O] \\ [O] & [M_{flexure}] \end{bmatrix}_{2n\times 2n} \quad (2)$$

Stiffness Matrix

$$[K] = \begin{bmatrix} [K_{shear}]+[K_l] & -[K_l] \\ -[K_l] & [K_{flexure}]+[K_l] \end{bmatrix}_{2n\times 2n} \quad (3)$$

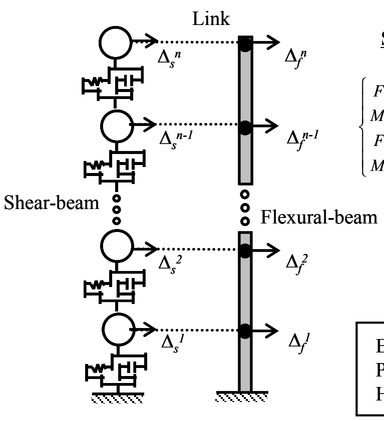
Stiffness Matrix of Shear-beam $[K_{shear}]$

$$[K_{shear}] = \begin{bmatrix} k_1^s + k_2^s & -k_2^s & 0 & \cdots & 0 \\ & k_2^s + k_3^s & 0 & \ddots & \vdots \\ & & \ddots & 0 & 0 \\ & sym. & & k_{n-1}^s + k_n^s & -k_n^s \\ & & & & k_n^s \end{bmatrix}_{n\times n} \quad (4)$$

Stiffness Matrix of Link $[K_{link}]$

$$[K_{link}] = \begin{bmatrix} [K_l] & -[K_l] \\ -[K_l] & [K_l] \end{bmatrix} \quad (8a)$$

$$[K_l] = \begin{bmatrix} k_1^l & 0 & \cdots & \cdots & 0 \\ & k_2^l & \ddots & \ddots & \vdots \\ & & & \ddots & \vdots \\ & sym. & & k_{n-1}^l & 0 \\ & & & & k_n^l \end{bmatrix}_{n\times n} \quad (8b)$$



Coupled Shear-flexural-beam Model with Link

Stiffness Matrix of Flexural Beam $[K_{flexure}]$

$$\begin{Bmatrix} F_i^a \\ M_i^a \\ F_i^b \\ M_i^b \end{Bmatrix} = \frac{E\left(\sum I_i^{fc}\right)}{H_i^3} \begin{bmatrix} 12 & 6H_i & -12 & 6H_i \\ & 4H_i^2 & -6H_i & 2H_i^2 \\ & & 12 & -6H_i \\ sym. & & & 4H_i^2 \end{bmatrix} \begin{Bmatrix} \Delta_i^a \\ \theta_i^a \\ \Delta_i^b \\ \theta_i^b \end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix} \{F_{total}\} \\ \{M_{total}\} \end{Bmatrix} = \begin{bmatrix} [K_{\Delta\Delta}] & [K_{\Delta\theta}] \\ [K_{\theta\Delta}] & [K_{\theta\theta}] \end{bmatrix} \begin{Bmatrix} \{\Delta_{total}\} \\ \{\theta_{total}\} \end{Bmatrix} \quad (6)$$

$$[K_{flexure}]_{n\times n} = [K_{\Delta\Delta}] - [K_{\Delta\theta}][K_{\theta\theta}]^{-1}[K_{\theta\Delta}] \quad (7)$$

Geometric Matrix $[k_g]$

$$[k_g] = \frac{P_i}{H_i} \begin{bmatrix} 1 & 0 & -1 & 0 \\ & 1 & 0 & -1 \\ & & 1 & 0 \\ sym. & & & 1 \end{bmatrix} \quad (9)$$

$$[k_g] = \frac{P_i}{H_i} \begin{bmatrix} \frac{6}{5} & \frac{H_i}{10} & -\frac{6}{5} & \frac{H_i}{10} \\ & \frac{2H_i^2}{15} & -\frac{H_i}{10} & -\frac{H_i^2}{30} \\ & & \frac{6}{5} & -\frac{H_i}{10} \\ sym. & & & \frac{2H_i^2}{15} \end{bmatrix} \quad (10)$$

$E(\Sigma I_i^{fc})$: Flexural Stiffness of i-story Flexural-beam
P_i : Weight on i-story
H_i : Height of i-story

3. STABILITY OF STRUCTURES

The dynamic stability of structures during an earthquake, which may determine seismic response, is closely related to the instantaneous eigenvalue of the tangent stiffness matrix normalized by the mass matrix, $\Omega(t)$. The relationship between the dynamic stability of structures and their eigenvalues is discussed below for the Single-Degree-of-Freedom (SDOF) and Multi-Degree-of-Freedom (MDOF) systems.

For the SDOF system, the incremental equation of motion is given by Equation (11). The incremental drift, Δu, that solves this equation is the sum of the particular solution, Δu_P, and the homogeneous solution, Δu_H, as given by Equation (12).

$$m(\Delta\ddot{u}) + c(\Delta\dot{u}) + k(t)\cdot(\Delta u) = -m(\Delta\ddot{u}_g) \tag{11}$$

$$\Delta u = \Delta u_P + \Delta u_H \tag{12}$$

The particular solution, Δu_P, does not become extreme large except in the case of resonance[6]. The homogenous solution, Δu_H, is given by Equations (13) through (15). For simplification, we consider the case of c equal to zero and drift response from the stationary state (i.e. Δu_H (t=0)=0, $\Delta\dot{u}_H$ (t=0)=0). Three types of response may be observed for this system. In Case (a), where $k(t)$ is positive, the eigenvalue, $\Omega(t)$ (=$k(t)/m$), is positive and Δu_H (t) is oscillatory motion around the equilibrium point, as illustrated in Figure 5. In Case (b), where $k(t)$ is zero, Δu_H (t) is linearly increasing. In Case (c), where $k(t)$ is negative, $\Omega(t)$ is negative and Δu_H (t) increases exponentially.

(a) When $4\cdot k(t)m - c^2 > 0$

$$\Delta u_H = C_1\exp\left(-c/(2m)\cdot\Delta t\right)\cos\left(\sqrt{4km-c^2}/(2m)\cdot\Delta t\right) + C_2\exp\left(-c/(2m)\cdot\Delta t\right)\sin\left(\sqrt{4km-c^2}/(2m)\cdot\Delta t\right) \tag{13}$$

(b) When $4\cdot k(t)m - c^2 = 0$

$$\Delta u_H = C_1\exp\left(-c/(2m)\cdot\Delta t\right) + C_2\exp\left(-c/(2m)\cdot\Delta t\right)\Delta t \tag{14}$$

(c) When $4\cdot k(t)m - c^2 < 0$

$$\Delta u_H = C_1\exp\left(\left(-c+\sqrt{c^2-4km}\right)/(2m)\cdot\Delta t\right) + C_2\exp\left(\left(-c-\sqrt{c^2-4km}\right)/(2m)\cdot\Delta t\right) \tag{15}$$

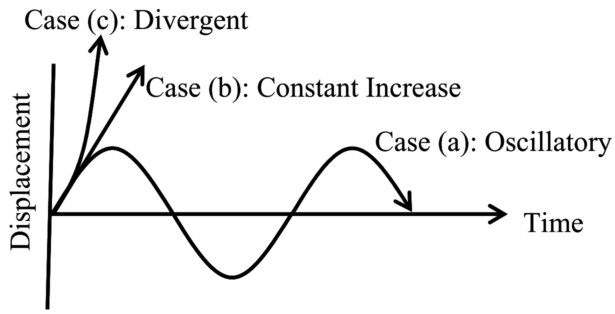


Figure 5. Movement from Stationary State

For the MDOF system, the incremental equation of motion is given by Equation (16). Here, the response of the structure is the summation of the response of each mode. The instantaneous eigenvalue of the i^{th}-mode, $\Omega_i(t)$, which is the square of the instantaneous frequencies, ω_i^2(t), is given by Equation (17). This corresponds to $\Omega(t)$ (=$k(t)/m$) in the SDOF system. If the structure exhibits negative $\Omega_i(t)$ during earthquake excitation, the instantaneous frequency, ω_i^2(t), becomes imaginary and the structure exhibits unstable response during the time-step[7].

$$[M]\{d\ddot{\Delta}\} + [C]\{d\dot{\Delta}\} + [K(t)]\{d\Delta\} = -[M][I]\{d\ddot{\Delta}_g\} \tag{16}$$

$$\Omega_i(t) = \frac{\{\Phi_i(t)\}^T[K(t)]\{\Phi_i(t)\}}{\{\Phi_i(t)\}^T[M]\{\Phi_i(t)\}} = \omega_i(t)^2 \tag{17}$$

However, this does not necessarily mean that the structure collapses when $\Omega_i(t)$ becomes negative. This is because the inertial force term, $[M]\{d\ddot{\Delta}\}$, and viscous damping term, $[C]\{d\dot{\Delta}\}$, may stabilize the structural response[8]. Also, while the structure has a negative eigenvalue, $\Omega_i(t)$, the direction of ground motion may change, resulting in unloading of the structural elements and negative eigenvalues changing to positive eigenvalues[6]. However, negative $\Omega(t)$ can be an important index to express the onset of unstable behavior, possibly resulting in complete collapse.

4. STABILITY OF STRUCTURE ASSUMING PLASTIC AND FRACTURE MECHANISM

Yield and fracture mechanisms have a large impact on structural stability, and hence the eigenvalues of the stiffness matrix. To understand how the structure loses stability as frame-floor connection failure progresses up the height of a structure, eigenvalue analyses were conducted using for the 1D shear-flexural-beam model with links. Here it was assumed that the connection failure progressed (a) from the top to the bottom of the structure and (b) from the bottom to the top, and two modes of structural response were considered: (1) the shear-beam is elastic and (2) the shear-beam is inelastic. The minimum eigenvalue, Ω_{min}, is plotted against the number of connection failures in Figure 6 for the case of an elastic shear-beam and in Figure 7 for the case of an inelastic shear-beam.

<u>Case (1): Shear-beam is Elastic in All Stories</u>

The data in Figure 6 show that the minimum eigenvalue, Ω_{min}, decreases rapidly when connection failure progresses from the top to the bottom of the structure. This is because the separation between the out-of-plane and in-plane can be large in the upper stories of the structure, where significantly decreases out-of-plane stiffness. When connections fail at 6 of 9 stories, Ω_{min} becomes negative, indicating that the structure is unstable. If connection failure progresses from the bottom to the top of the structure, Ω_{min} remains almost constant until connections fail at 6 floors, and then begins to decrease rapidly. This is because the separation between the out-of-plane and in-plane frames stays relatively small until a large number of connections have failed, as by the mode-shapes in Figure 6.

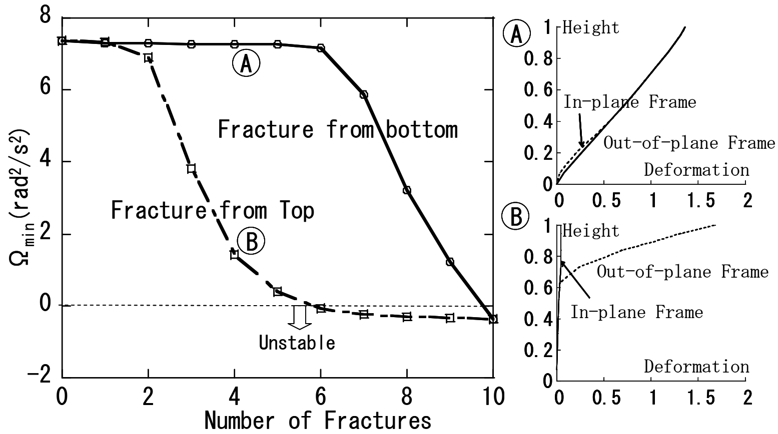


Figure 6. Minimum Eigenvalue (Elastic Shear-beam)

<u>Case (2): Shear-beam is Inelastic in All Stories</u>

For the case of an inelastic shear beam, the minimum eigenvalue, Ω_{min}, becomes negative before the first connection fails (Figure 7). This is due to the reduced tangent stiffness of shear-beam and large *P*-Δ effects. For the case of fracture starting at the top, the Ω_{min} decreases gradually until 4 connections fail and then decreases for rapidly with connection failure. This initial, gradual decrease in Ω_{min} is because the separation of the in- and out-of plane frames in the upper stories is not large, as indicated by the mode-shape in Figure 7. When connection failure progresses from the bottom to the top, large *P*-Δ effects result in more rapid initial decrease in Ω_{min}.

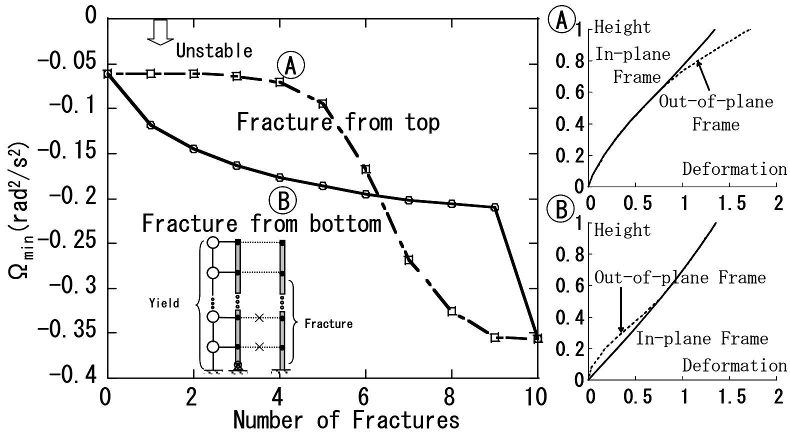
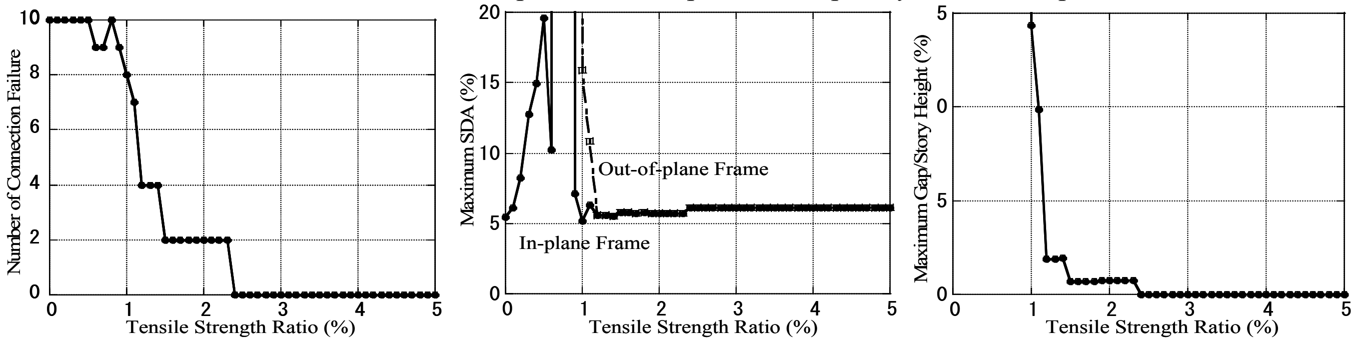


Figure 7. Minimum Eigenvalue (Inelastic Shear-beam)

5. STABILITY OF STRUCTURE DURING EARTHQUAKE LOADING

In order to understand dynamic stability and seismic response of the steel moment-resisting frame structures, nonlinear time-history analyses were performed using the 1D shear-flexural-beam model with links subjected to the NF17 record[9)]. The tensile strength of the links was varied from 0% to 5% of the yield strength of the perimeter column. The number of connection failures, the maximum inter-story drift angle of in-plane and out-plane frames, and the maximum gap between the in-plane and out-of-plane frames are plotted versus link tensile strength in Figure 8(a), (b), and (c), respectively.

When the link strength was greater than 2.4% of the axial yield strength of perimeter column, P_y, the links did not fail and in-plane and out-of-plane frames behaved as a single unit. When the link strength was between 1.5% and 2.3%, two links broke, resulting in separation of the in-plane and out-of-plane frames. However, the gap was less than 1% of the story height. When the link strength was less than 1.2%, the out-of-plane frame experienced large drifts and collapsed, and the number of connection failures increased. When link strength was between 0.7% and 0.8%, in-plane and out-of-plane frames exhibited story drift angles (SDA) of more than 20% and collapsed. When the link strength was less than 0.7%, all of the links failed and the out-of-plane frame separated completely from the in-plane frame.



(a) Number of Connection Failures (b) Maximum Story Drift Angle (%) (c) Maximum Gap

Figure 8. Response for Various Tensile Strength of Connections

<u>Case (1): No failure of links (Link strength: 2.5%)</u>

When the tensile strength of the links was 2.5%, the links did not fail and in-plane and out-of-plane frames behaved as a single unit and exhibited identical drifts as shown in Figure 9(a-1). The maximum SDA was 5.82% at 11.06s. Between 9.98s and 10.06s, the minimum eigenvalue decreased to 0.257(rad^2/s^2) as shown in Figure 9(b-1), but maintaining the positive value. Therefore, even if the structure exhibited more than 5% SDA, it behaved in stable manner.

<u>Case (2): Few links fail (Link strength: 2.0%)</u>

When the tensile strength of the links was 2.0%, the links in the 1st and 3rd stories failed, and after the failure, in-plane and out-of-plane frames behaved differently. As shown in Figure 9(a-2), the maximum SDAs of in-plane and out-of-plane frames were 3.87% and 4.56%. Even though connection failures occur, as shown in Figure 9(b-2), the structure maintained positive eigenvalues during the earthquake, resulting in stable behavior.

<u>Case (3): Many links fail (Link strength: 0.8%)</u>

When the tensile strength of the links was 0.8%, the instantaneous eigenvalue decreased rapidly to a negative value as connection failure progressed from the 1st-story to the 3rd, 4th, 5th, 8th, 10th, 9th, 7th, and 2nd-story as shown in Figure 10. After 8.80s, the minimum eigenvalue did not recover to a positive value as shown in Figure 9(a-3). As a result, the structure continued to response in an unstable manner, and exhibited very large drifts and finally collapsed.

<u>Case (4): All links fail (Link strength: 0.0%)</u>

When the tensile strength of the links was 0.0%, as shown in Figure 9(b-4), the eigenvalue decreased to negative right after the ground motion started. As a result, the structure exhibited unstable behavior as a whole and out-of-plane frame accumulated drift in one-direction. Even though the behavior of the entire structure was unstable, the in-plane frame behaved in a stable manner. This was because if in-plane frame alone were analyzed for the same ground motion, in-plane frame behavior was stable.

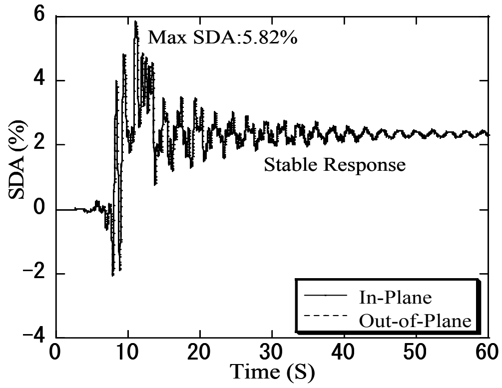


Figure 9(a-1) 9th-story Drift History

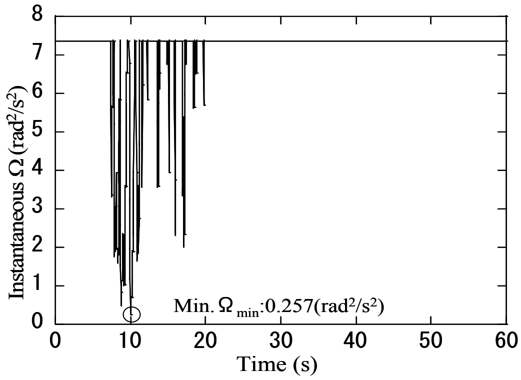


Figure 9(b-1) History of Instantaneous Eigenvalue

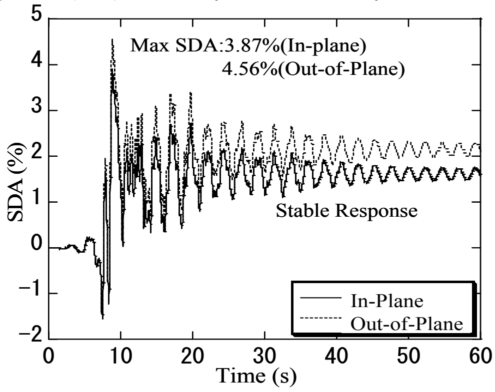


Figure 9(a-2) 2nd-story Drift History

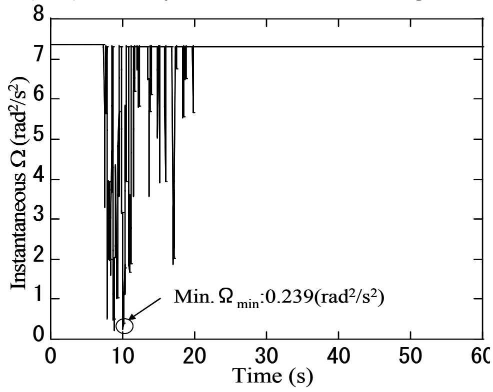


Figure 9(b-2) History of Instantaneous Eigenvalue

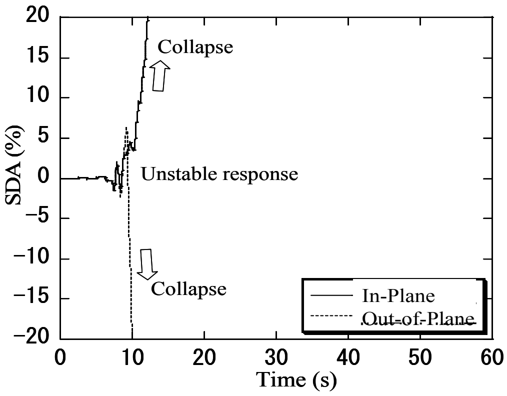


Figure 9(a-3) 2nd-story Drift History

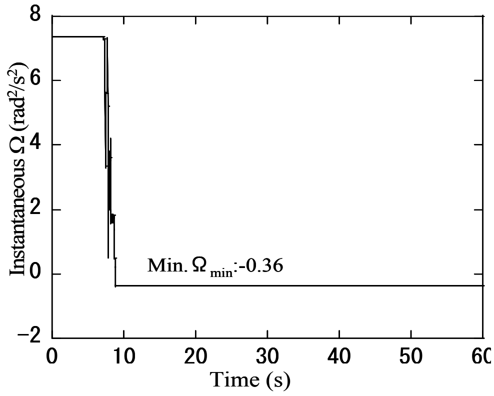


Figure 9(b-3) History of Instantaneous Eigenvalue

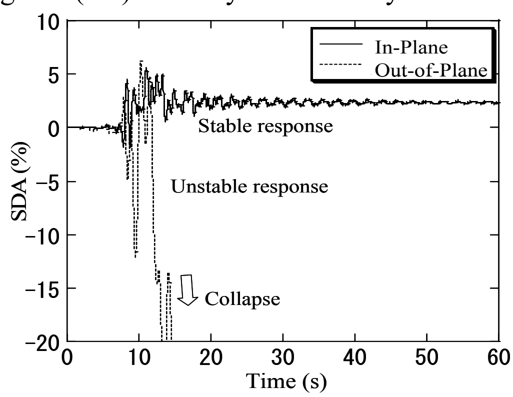


Figure 9(a-4) 9th-story Drift History

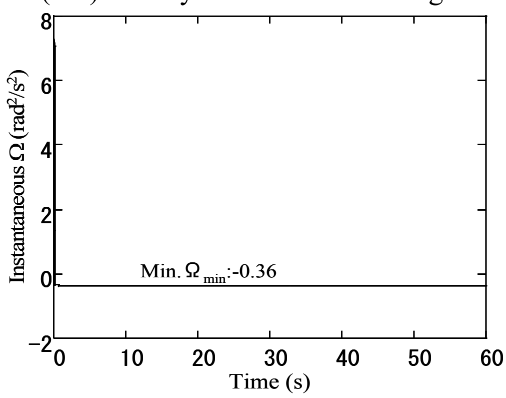


Figure 9(b-4) History of Instantaneous Eigenvalue

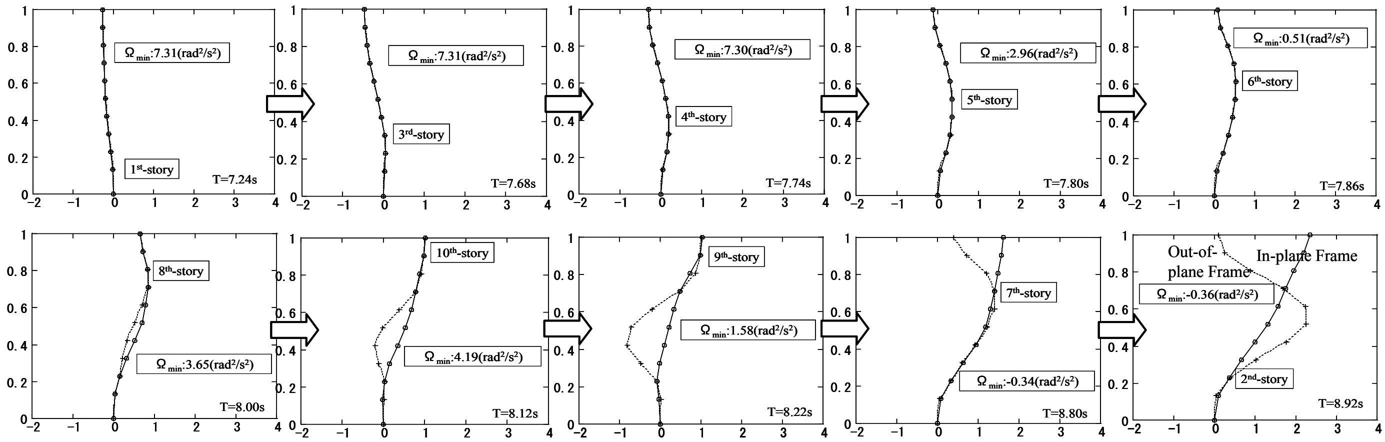


Figure 10. Transition of Drifts and Eigenvalues during Progressive Collapse, 0.8% Tensile Strength (x: Floor Drift/Building Height (%), y: Height)

6. CONCLUSIONS

In this study, the progressive collapse of steel moment-resisting structures typical of construction in the United States is simulated using the 1D shear-flexural-beam model with link-elements. In this model, horizontal springs in the shear-beam simulate the lateral resistance of story-subassemblages from the seismic frame and the flexural-beam simulates the additional resistance provided by column continuity in the in-plane and out-of-plane seismic frames. The link-element simulates the restoring force of the simple connection between the perimeter, seismic frame and the floor-system. This link is modeled as possessing large stiffness and strength in compression and relatively small strength in tension. Major findings of the study include:

1. Through eigenvalue analysis of system with assumed plastic frame mechanism and assumed link fracture mechanisms, the relationship between structural stability and connection failure progresses is evaluated. If the in-plane frame is elastic, the structure loses its stability more rapidly when connection failure progresses from the top of the structure to the bottom than when connection failure progresses from the bottom to top.
2. When the tensile strength of the connection is very small, the out-of-plane frame is separated from the in-plane frame and collapses as a result of earthquake loading. However, the in-plane frame behaves in stable manner. In this case, the entire structure has the negative minimum eigenvalue. As the tensile strength of the links increases, the number of connection failures decreases and the in-plane and out-of-plane frames behave as a single unit and in more stable manner.

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