SOIL–FOUNDATION–WALL INTERACTION
IN MASONRY COMBINED SYSTEMS

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ABSTRACT:

Soil-Foundation-Structure Interaction (SFSI) has been widely analysed in recent years from theoretical, numerical and experimental viewpoints. Its effects may be largely significant in combined structural systems including masonry or Reinforced Concrete (RC) shear walls. Therefore, such models should be analysed taking into account also the foundation and the soil.

In this paper a matrix algorithm for the seismic analysis of Masonry Combined Systems (MCSs) including Soil-Foundation-Wall Interaction (SFWI) is proposed. The theoretical study is divided into the following stages: (1) modelling of the soil-foundation interaction; (2) analysis of a flexibly-supported solid shear wall under lateral forces. Finally, the results of some parametric analyses on multi-storey Masonry-RC Combined Systems (MRCSs) are discussed showing that SFWI cannot be neglected, since it affects the distribution of lateral forces between different parallel structural elements.

KEYWORDS: Masonry Combined Systems, Soil-Foundation-Wall Interaction.

1. INTRODUCTION

In the force-based seismic analyses the horizontal actions are distributed between parallel structural elements in proportion to their lateral stiffness, which is estimated in fixed-base conditions. This approach may lead to erroneous results in the presence of stiffer elements like Reinforced Concrete (RC) shear walls. In fact, their seismic response is particularly sensitive to both rigid-body rocking and swaying base motions (Goodsir 1985). It follows that the actual horizontal forces applied on each lateral load-resisting element have to be evaluated on flexible-base models. This problem is highlighted by EC8 (CEN 2004) and by national codes as FEMA 451 (BSSC 2006), but it has not yet been studied for Masonry Combined Systems (MCSs) composed by shear walls and/or frame-wall dual systems, with or without coupling beams. Hence, their lateral stiffness should be necessarily estimated by considering also the soil-foundation flexibility.

In this paper Soil-Foundation-Wall Interaction (SFWI) effects on the seismic behaviour of Masonry-RC Combined Systems (MRCSs) are studied and a matrix algorithm is proposed. In particular, the present work deals with the distribution of lateral forces between different parallel elements accounting for foundation rocking. The theoretical analysis is divided in two steps: (1) modelling of the soil-foundation interaction; (2) seismic analysis of a solid shear wall with rotationally-flexible base. Finally, the results of some parametric analyses on MRCSs are discussed.

2. SOIL–FOUNDATION INTERACTION ANALYSIS

2.1. Shallow and embedded foundations on elastic soil

The beam on Winkler elastic medium is obviously the simplest interaction model for a shallow foundation. Alternatively, the soil may be modelled as elastic, homogeneous and isotropic half-space, so the rotational stiffness may be defined in terms of Young’s modulus and Poisson’s ratio of the soil, and of shape factor of the foundation (Bowles 1988), or else through a characteristic dimension of the foundation and an influence factor, which in turn depends on both the foundation shape and the adopted soil model.

Finally, the lateral stiffness of the soil-foundation system can be defined. Recent analytical formulations allow
to evaluate the static foundation stiffness taking into account its actual shape, the embedment depth, and the layering of the soil (Gazetas 1991, ASCE 2000).

2.2. Pile foundations

Even pile foundations do not really ensure fixed-base conditions for shear walls, especially when the soil deformability is high. Therefore, soil-foundation interaction should be included in the structural model. The rigid beam on discrete Winkler medium is a simple interaction model for a shear wall. In this case, group effects (Poulos and Davis 1980, Randolph 1994) may be computed for instance by applying a correcting factor of 0.5 to 2 to the axial stiffness of each pile, as suggested by FEMA 356 (ASCE 2000), or otherwise by defining an equivalent length for every one.

More accurate modelling of the soil-foundation system may be obtained according to Annex C of EC8 (CEN 2004) or to FEMA 450 (BSSC 2003). Such codes provide some formulas for rotational, horizontal and coupled stiffnesses, that are referred to the pile heads for three different soil models. Equivalent horizontal and rotational stiffnesses can then be defined for each pile by means of simple equations, which are different for free- and restrained-head piles. After the global translational and rotational stiffnesses of the whole soil-foundation system are evaluated, one can refer to an uncoupled spring model.

Soil-foundation interaction analysis may also be performed by considering each pile as an equivalent cantilever beam (Tomlinson 1994), in which both the cross-sectional and mechanical properties are assumed to be equal to the actual ones, while its effective length is suitably chosen. The lateral foundation-to-soil stiffness for passive pressure (Wilson 1988) may be also modelled by applying a lateral stiffness at each pile head.

3. MODELLING AND SEISMIC ANALYSIS OF THE SOIL–FOUNDATION–WALL SYSTEM

SFWI may be analysed on a cantilever beam model with both flexural and shear flexibility, having rotationally-deformable base. Assuming that the soil-foundation system does not sway, only its rotational stiffness $k_f$ affects the lateral behaviour of the structural model. In the following treatment $E_{k,p}$ is the generic static or kinematic parameter referred to the $p$-th storey of $k$-th wall; storeys and lateral load-resisting elements are progressively numbered from the top to the base and from the left to the right, respectively; the solid shear wall is labelled as element No. 1.

In a prismatic cantilever wall having height $3H$ subjected to lateral forces at the levels $H$, $2H$ and $3H$, mutual effects in elevation can be considered by applying the principle of effects superposition to three separate schemes (see Figure 1).

![Figure 1. Effects superposition for a cantilever wall of height $3H$](image-url)
Denoting by $k_w = EJ/H$ a flexural stiffness factor of the solid wall, a relative stiffness $c = k_f/k_w$ can be defined for the Soil-Foundation-Wall System (SFWS) to perform seismic analyses on several types of MRCSs. In the $m$-th elementary scheme the rocking rotation of the cantilever wall is given by:

$$\varphi^{(m)} = \frac{M^{(m)}_b}{k_f} = \frac{F_{1,(4-m)} \cdot p \cdot H}{k_f}$$  \hspace{1cm} (3.1)

where $M^{(m)}_b$ is the base bending moment, so the lateral displacement of the SFWS at the $p$-th storey is:

$$\delta^{(m)}_{1,p} = \sum_{m=1}^{s} \delta^{(m)}_{1,p} = \sum_{m=1}^{s} \frac{F_{1,(4-m)} \cdot m \cdot (4-p) \cdot H^2}{k_f}$$  \hspace{1cm} (3.2)

being:

$$\delta^{(m)}_{1,p} = \varphi^{(m)} \cdot (4-p) \cdot H = \frac{F_{1,(4-m)} \cdot m \cdot (4-p) \cdot H^2}{k_f}$$  \hspace{1cm} (3.3)

A set of equations corresponds to Eqn. 3.2 written for all the levels and a flexibility matrix related only to the base rocking may be derived from it, reaching to the assembly of the global flexibility matrix. In order to generalize this formulation to an $s$-storey structural model, one can refer to the analytical expression of the lateral flexibility matrix proposed by Augenti and Parisi (2008) for fixed-base solid shear walls. In the case of a solid shear wall with rotationally-deformable base, a rocking-induced lateral flexibility matrix must be added to the one associated with the fixed-base model. For instance, the former may be obtained once again through the effects superposition of $s$ simple models, each of them subjected to a lateral force $F_{1,(s+1-m)}$ (where $s+1-m = p$), or alternatively via Mohr’s corollaries. In the $m$-th scheme the horizontal displacement of the $p$-th storey can then be written as follows:

$$\delta^{(m)}_{1,p} = \varphi^{(m)} \cdot (s+1-p) \cdot H = \frac{F_{1,(s+1-m)} \cdot m \cdot (s+1-p) \cdot H^2}{k_f}$$  \hspace{1cm} (3.4)

so its total value estimated on all the $s$ schemes is expressed by:

$$\delta_{1,p} = \sum_{m=1}^{s} F_{1,(s+1-m)} \cdot m \cdot (s+1-p) \cdot H^2$$  \hspace{1cm} (3.5)

The rocking-induced lateral flexibility matrix of the SFWS may be derived by the displacements algebraic system corresponding to Eqn. 3.5 written for all the simple shear wall models, so one can get:

$$D^e = \frac{1}{c} \begin{bmatrix}
\frac{EJ}{s^2H^3} & \frac{(s-1)sH^3}{EJ} & \cdots & \frac{sH^3}{EJ} \\
\frac{(s-1)sH^3}{EJ} & \frac{(s-1)^2H^3}{EJ} & \cdots & \frac{(s-1)H^3}{EJ} \\
\cdots & \cdots & \cdots & \cdots \\
\frac{sH^3}{EJ} & \frac{(s-1)H^3}{EJ} & \cdots & \frac{H^3}{EJ}
\end{bmatrix}$$  \hspace{1cm} (3.6)
where the relative stiffness $c$ has been factored out. Therefore, the lateral flexibility matrix of an $s$-storey solid shear wall with rotationally-deformable base can be written in the following form:

$$
D_i = \begin{bmatrix}
D_{1,1} + \frac{s^2 H^3}{c E J} & D_{1,2} + \frac{(s-1)sH^3}{c E J} & \cdots & D_{1,i} + \Phi_{1,i}(s-1)H + \frac{sH^3}{c E J} \\
D_{2,1} + \frac{(s-1)sH^3}{c E J} & \cdots & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
D_{s,1} + \frac{sH^3}{c E J} & \cdots & \cdots & D_{s,s} + \frac{H^3}{c E J}
\end{bmatrix}
$$

(3.7)

being $D_i = D_i^0 + D_i^e$. Such matrix allows to perform seismic analyses on MRCSs accounting for SFSI.

4. PARAMETRIC ANALYSES ON MASONRY–RC COMBINED SYSTEMS

To evaluate preliminarily SFWI effects on MRCSs, some parametric analyses were carried out on the following three-storey models: (1) solid wall – opened masonry wall; (2) solid wall – RC frame. Lateral forces applied on the separate parallel elements were estimated by assuming $c$ equal to 1, 5, 10, and $\infty$ (fixed-base wall). Such analyses were performed under these hypotheses: (1) inverse triangular distribution of external horizontal forces; (2) opened masonry wall modelled through the macro-elements approach by using the RAN method (Augenti 2004); (3) parallel structural elements mutually connected by rigid pendulums; and (4) equal interstorey heights. Every analysis was carried out by assuming the solid shear wall to be made up of masonry (case 1), and of RC (case 2). The following materials were considered: yellow tuff masonry with cementitious mortar, having Young’s modulus $E_m = 3500$ MPa and shear modulus $G_m = 1200$ MPa; concrete of strength class C20/25, with $E_c = 29000$ MPa and $G_c = 12100$ MPa.

4.1. The solid wall – opened masonry wall combined system

A first set of analyses was performed on a three-storey combined model composed by (1) a solid shear wall and (2) an opened masonry wall. The former has interstorey height equal to 350 cm and is 400 cm-long and 40 cm-thick; the latter has two pier panels per storey, which are 130 cm-wide, 260 cm-high and 40 cm-thick. In the case of solid masonry wall (see Figure 2), wall-to-wall interaction induces high variability of lateral forces applied on each element owing to their stiffness-based distribution, either in fixed-base ($c = \infty$) or in flexible-base ($c = 1, 5, 10$) conditions. The lateral stiffness variability along the height of both the lateral load-resisting elements generates inverted forces with respect to the corresponding external ones at the storey 1 of the solid wall and at the storey 3 of the opened masonry wall. That’s why the intensities of the lateral forces applied to these storeys of the opened masonry wall and of the solid wall, respectively, are greater than the ones of the external forces.

Figure 2. Solid masonry wall – opened masonry wall combined model (case 1)
In such combined model, SFSI effects on the distribution of lateral forces are quite significant. As the soil-foundation flexibility increases (which means \( c \) decreasing), the base shear force of the solid wall progressively decreases because for it: (1) the intensity of the lateral force acting on the storey 1 increases, even if it is inverted with respect to the corresponding external one; (2) the intensities of the lateral forces applied to the storeys 2 and 3 decrease, although they are oriented like the external ones. Moreover, the base shear force of the opened masonry wall increases too, because the magnitude of the horizontal forces acting on the storeys 1 and 2 increases; on the other hand, the one of the lateral force applied to the storey 3 reduces, since it is inverted with respect to the corresponding external action.

Figure 3 shows the distribution of lateral forces between the walls as the relative stiffness \( c \) changes: each horizontal action is normalized by the corresponding external one. The sensitivity of the model to the soil-foundation flexibility seems to be more significant at the storeys 1 of the solid wall and 3 of the opened wall. In this case, the magnitude of the horizontal force acting on the storey 1 of the solid wall increases of about 19\% from the fixed-base model to the flexible-base model with \( c = 10 \), while the lateral force applied to the storey 3 of the opened wall reduces of about 15\%. Conversely, the forces acting on other storeys undergo small variations. Therefore, SFSI effects are different along the height with exception of the combined model having \( c = 1 \), at which the minimum base shear force of the solid wall corresponds.

![Figure 3. Lateral forces acting on the solid masonry wall and opened masonry wall](image)

Following to the equilibrium in horizontal direction, as \( c \) decreases the magnitude of the lateral forces acting on each wall changes, but their orientation does not vary. Both the walls are subjected to increasing forces at the storeys 1 and 3, while the solid wall is underloaded and the opened wall is burdened by an increasing force at the storey 2. Soil-foundation flexibility does not affects who is the bracing wall of the structural system. In each examined model the solid wall is braced by the opened wall at the storey 1, whereas the opposite situation happens at the other storeys.

In the case of solid RC wall (see Figure 4), Young’s and shear moduli change: they are 8 and 10 times those ones of the masonry, respectively, so the distributions of lateral forces between the walls are very different from the ones of the case 1.

![Figure 4. Solid RC wall – opened masonry wall combined model (case 2)](image)

In particular, the lateral forces applied to the solid shear wall have the same orientation of the external ones,
while the forces acting on the storeys 2 and 3 are inverted. In the fixed-base model, both the walls get about the same fraction of the horizontal force applied to the storey 1, while the solid wall is the bracing element at the underlying storeys. It is interesting to note that, for the same load pattern, the base shear force of the solid RC wall is 1.67 times the one evaluated on the solid masonry wall. This depends directly on the major uniformity of the vertical distribution of horizontal forces acting on such element. On the contrary, the base shear force of the opened masonry wall is about 28% of the one estimated in the case 1. SFSI effects seem to be not much significant because, as $c$ decreases, the distribution of lateral forces between the walls does not change in relevant manner (see Figure 5). For $c = 1$ the horizontal force acting on the storey 1 of the solid wall largely decreases, so the magnitude of the lateral action applied to the opened masonry wall becomes even 75% of the corresponding external one.

4.2. The solid wall – RC frame combined system
A second set of parametric analyses was performed on a three-storey combined model composed by (1) an opened masonry wall and (2) a RC frame. The former is the same of the section 4.1, while the latter has 350 cm-long columns having cross-section of 40×40 cm and 400 cm-long beams having cross-section of 40×50 cm. In the case of masonry shear wall (see Figure 6), either the wall or the frame provide lateral resistance, according to the definition of dual system given in EC8 (CEN 2004). The horizontal force acting on the storey 1 is resisted almost in the same fraction by the lateral load-resisting elements, whereas the bracing element of the structure at the lower storeys is clearly the shear wall.

Figure 7 shows that the frame is subjected to lateral forces, whose intensities are between 16% and 67% of the corresponding external ones. As $c$ decreases, the forces applied to the storeys 1 and 3 of both the parallel elements change more significantly than those ones acting on the intermediate storey. Note that: (1) the frame is increasingly overburdened at the storey 1; (2) the intensity of the horizontal forces acting at the intermediate storeys of both the load-bearing elements increases; and (3) the magnitude of the lateral forces applied to the storey 3 “rapidly” decreases, until the force acting on the frame reaches the sign of the external loads when $c$ becomes 1. This inversion of the horizontal action is obviously a benefit for the solid wall, since it induces a large reduction of the lateral force acting on it.
In the case of RC solid wall (see Figure 8), the frame can be considered as bracing element at the storey 1 for every value of the relative stiffness $c$: the lateral force acting on it owing to the distribution is between 8% and 13% of the external one. This model is low sensitive to SFSI and force intensity variations up to 10% occur just for the frame (see Figure 9).

5. CONCLUSIONS

The theoretical formulation proposed in this paper allows to perform preliminary seismic analyses on MRCSs taking into account SFSI effects. After a foundation-to-wall relative stiffness and a flexibility matrix of the deformable-base solid shear wall were defined, some parametric analyses on elementary MRCSs were carried out. The distribution of lateral forces between different parallel structural elements was estimated for several values of the relative stiffness, via matrix formulas based on equilibrium and kinematic equations.

A comparison between the base shear forces acting on each lateral load-resisting element, for all the examined
combined models, is showed in Figure 10. SFWI is significant for solid wall – opened masonry wall combined models, while it is negligible for RC wall-frame systems (case 2).

![Figure 10. Comparison between normalized base shear forces](image)

Also these numerical results show that solid wall – opened masonry wall combined systems may be very sensitive to SFSI effects, since as the relative stiffness changes the horizontal forces acting on each wall change too, resulting in a large variability of the base shear forces. Solid wall – RC frame combined models seem to be sensitive to SFSI effects when the shear wall is made up of masonry.

Therefore, given that soil-foundation flexibility may drastically affect the seismic response of MRCSs, the assumption of fixed-base models may result in significant errors and lack of structural safety.

REFERENCES


