Bayesian updating of the reliability of existing RC structures based on the inspection results

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ABSTRACT :

The seismic assessment of existing buildings is subject to uncertainties. These uncertainties can sometimes be comparable to that of the ground motion representation. Seismic codes such as the European code and the Italian Code seem to consider the overall effect of these uncertainties through confidence factors applied to nominal material strength. This work aims to classify and characterize these uncertainties and build a prior distribution of structural lateral load resistance capacity as a function of these uncertainties. The prior probability distribution model for the uncertain parameters is constructed based on the state of knowledge about the building before in situ inspections and tests are conducted. The uncertainties in the parameters are propagated to the structural resistance using an advanced simulation method known as subset simulation. Subset simulation allows for suitable grouping of parameters into groups in order to build a simplified model of correlation across different structural parameters. In the next step, the results of tests are used to update the probability distributions for parameters and to propagate the effects on to the structural resistance. The results of this study are aimed to make recommendations as to how confidence factors can be estimated taking into account all sources of uncertainty and be applied to the structural resistance. Finally, the effect of performing tests in increasing the level of knowledge about the structure are quantified in terms of a decrease in the confidence factor. As a case study, the two-dimensional finite element model of the central frame of an existing school built in the 1960's in Avellino Italy has been used.

KEYWORDS: Structural reliability, modeling uncertainty, Bayesian updating, confidence factors, existing structures, Markov chain Monte Carlo simulation

1. INTRODUCTION

Determining the material properties and structural detailing in existing structures is subject to a significant level of uncertainty. The European and Italian seismic guidelines (e.g., Eurocode C8, OPCM, NTC) seem to synthesize the effect of modeling uncertainties in the so-called confidence factors which are applied to the mean material properties. Evaluation of these confidence factors is rather qualitative and depends on the acquired level of knowledge about the structure. These guidelines define three increasing levels of knowledge, for each of which, they prescribe a certain set of verifying tests and inspections to be performed. The objective of the present study is to take into account the structural modeling uncertainties in a Bayesian framework where the results of tests and inspections can be implemented in order to update both the structural modeling parameters' probability distribution and the structural reliability.

In the presence of structural modeling uncertainty, instead of a unique structural model, a set of plausible structural models can be identified. A *robust* assessment of structural reliability takes into account a whole set of possible structural models that are weighted by their corresponding plausibility. A Bayesian updating framework can be implemented in order to update both the structural modeling properties and the reliability based on test results (Beck and Katafigiotis, 1998). Using this methodology, the probability distributions for a set of specified structural models and the structural reliability are updated.

2. METHODOLOGY

The Bayesian framework used for updating the structural model and its reliability is described in detail in this section.

2.1 Evaluation of robust reliability

Let the vector $\underline{\theta}$ denote the set of uncertain model parameters and let D denote some test data and consider that the set of possible structural models can be defined by M to specify (both the structural and the probabilistic) modeling assumptions used in the analysis. The Bayesian framework used herein provides a rigorous method for updating the plausibility of each of the models described by M in representing the structure. The plausibility of a model is quantified by a probability distribution over the model parameters $\theta = [\theta_1, ..., \theta_n]$ that define a model within the set of possible models. The updated probability distribution can be defined using the Bayes Theorem (Beck and Au 2002):

$$p_D(\theta) = p(\theta | D, M) = \frac{p(D|\theta, M)}{p(D|M)} p(\theta | M) = \frac{p(D|\theta, M)}{p(D|M)} p(\theta | M) \tag{2.1}$$

Where $P(\theta | M)$ is the prior probability distribution for θ specified by M, $P(D|M)$ is the probability distribution for data D specified by M, and $P(D|\theta, M)$ is the probability distribution for observed data D given the vector of parameters θ specified by M.

Updated response predictions can be made implementing data D through $P_D(\theta)$ given by Equation 1. For example, if the probability of a failure event F based on modeling parameters θ is denoted by $P(F | D, M)$, the robust failure probability can be calculated from the following:

$$P(F | D, M) = \int P(F | \theta, M) p(\theta | D, M) d\theta \tag{2.2}$$

Where $P(F | \theta, M)$, the failure probability for the structural model defined by θ. For example, given a specific representation of ground motion, $P(F | \theta, M)$ reduces to a deterministic index function $I_F(\theta, M)$ which is equal to one in the event of failure and equal to zero otherwise:

$$P(F | D, M) = \int I_F(\theta, M) \frac{p(D|\theta, M)}{p(D|M)} p(\theta | M) d\theta \tag{2.3}$$

This paper utilizes a Markov Chain Monte Carlo simulation method to evaluate the robust reliability in Equation 3 (Beck and Au, 2002). This method employs the Metropolis-Hastings (MH) algorithm (Metropolis, 1953 and Hastings, 1970) to generate samples as a Markov chain sequence used to estimate the robust reliability by statistical averaging. The Metropolis-Hastings algorithm is used to generate samples according to an arbitrary PDF when the target PDF is known only up to a scaling constant.

2.3 Generating samples according to target PDF $p(\theta | D, M)$

The MH algorithm can be used to generate samples according to the target PDF $p(\theta | D, M)$. Using Bayes formula one can derive the PDF as:

$$p(\theta \mid D, M) = \frac{p(D \mid \theta, M)\, p(\theta \mid M)}{p(D \mid M)} = c^{-1} p(D \mid \theta, M)\, p(\theta \mid M) \tag{2.4}$$

Where $p(\theta \mid M)$ is *the prior probability distribution* for the parameters θ and $p(D \mid \theta, M)$ known as *the likelihood function* is the probability distribution for the data specified by parameters θ. The MH algorithm can be used to generate samples according to the target updated PDF $f \equiv p(\theta \mid D, M)$ using the product $p^* \equiv p(D \mid \theta, M)\, p(\theta \mid M)$ as the candidate PDF. In order to increase the acceptance rates of the candidate samples during the Markov chain simulation, a sequence of intermediate target PDF's are introduced which vary gradually between the prior PDF $p(\theta \mid M)$ and the updated target PDF $p(\theta_o \mid D, M)$. The target f_i's can be modeled as updated PDF's according to Bayes theorem based on an increasing amount of data : $f_i \equiv p(\theta_o \mid D_i, M)$ were $D_1 \subset D_2 \subset \cdots \subset D_n = D$. That is, at the first level with a target PDF equal to f_1, one could use the prior PDF $p(\theta_o \mid M)$ as the proposal PDF. In order to approximate f_1 a kernel sampling density κ_1 is constructed as a weighted sum of Gaussian PDF's centered about the generated samples. The kernel sampling density generated can be used as the proposal density in the second level: $f_2 \equiv \kappa_1$. In this work, the MH algorithm is used to update the probability distribution $p(\theta \mid M)$ across increasing levels of knowledge.

2.4 Structural Failure

The failure event F can be defined as when structural demand denoted as $D(\underline{\theta})$ exceeds structural capacity $C(\underline{\theta})$: $F = \{\underline{\theta} : D(\underline{\theta}) > C(\underline{\theta})\}$. Assuming scalar demand and capacity, the (scalar) demand to capacity ratio can be defined as $Y(\underline{\theta}) = D(\underline{\theta}) / C(\underline{\theta})$. Therefore, the failure region F can be defined as $F = \{\underline{\theta} : Y(\underline{\theta}) = 1\}$ and the sequence of embedded intermediate failure regions can be generated as $F_i = \{\underline{\theta} : Y(\underline{\theta}) > y_i\}$ where $0 < y_1 < \cdots < y_m = 1$. In this study, the structural capacity is obtained using the pushover analysis as the global displacement at which the first element is in crisis (3/4th of the of ultimate chord rotation in the member). The structural demand is defined as the global displacement corresponding to the intersection of the capacity curve of the equivalent SDOF system and the corresponding code-based seismic response spectra for the seismicity and the soil characteristics at the site of the project (a.k.a, capacity spectrum method, Fajfar, 1999).

3. MODELLING OF UNCERTAINTIES

As it is mentioned in the previous section, the vector of parameters $\underline{\theta}$ contains the uncertain parameters in the problem such as the uncertainty in the seismic action, the uncertainty in the property of the materials and the uncertainty involved in the structural detailing. The present work focuses on the uncertainty in the parameters of structural modeling, which is characterized differently in existing structures and new construction. This type of uncertainty is directly based on the quantity (and the quality) of information that is available on the structure. In this study, two different sources of uncertainty are considered: (1) uncertainty in the mechanical properties of materials used in construction (2) the armature details that affect the component capacity in terms of moment-rotation relation (also known as *structural defects*). As it regards the uncertainties of the second group, those related to the percentage of rebar present in the element, rebar diameter (e.g., different from that specified in the original design notes), and the anchorage quality are considered. The uncertainties in the rebar details are modeled as discrete uncertain variables that can assume a range of possible values with a certain plausibility/weight. In the absence of test results and in situ inspections, the plausibility is assigned qualitatively based on engineering consensus, judgment and experience. Once the test results are available on the quantity in question they can be used in applying the Bayesian methodology described in the previous sections to update its plausibility. As it regards the correlation between different detailing parameters, the grouping of uncertain parameters in subset simulation algorithm is used to group those parameters with possible correlation the same group.

4. NUMERICAL EXAMPLE

The methodology presented in the previous section is applied to an existing structure as a case study.

4.1 Structural Model

As the case-study, an existing school structure located in Avellino, Italy is considered herein. The structure is situated in seismic zone II according to the Italian seismic guidelines (OPCM 3519, 2006). The structure consists of three stories and a semi-embedded story and its foundation lies on soil type B. For the structure in question, the original design notes and graphics have been gathered. The building is constructed in the 1960's and it is designed for gravity loads only, as it is frequently encountered in the post second world war construction. In Figure1a, the tri-dimensional view of the structure is illustrated; it can be observed that the building is highly irregular both in plane and elevation. In order to reduce the computational effort, the main central frame in the structure is extracted and used as the structural model (Figure 1b). The columns have rectangular section with the following dimensions: first storey: 40x55 cm^2, second storey: 40x45 cm^2, third storey: 40x40 cm^2, and forth storey: 30x40 cm^2. The beam, also with rectangular section, have the following dimensions: 40x70 cm^2 at first and second piano, and 30x50 cm^2 for the ultimate two floors. It can be inferred from the original design notes that the steel rebar is of the type Aq40 and the concrete has a minimum resistance equal to 180 kg/cm^2 (R.D.L. 2229, 1939). The finite element model of the frame is constructed assuming that the non-linear behavior in the structure is concentrated in plastic hinges.

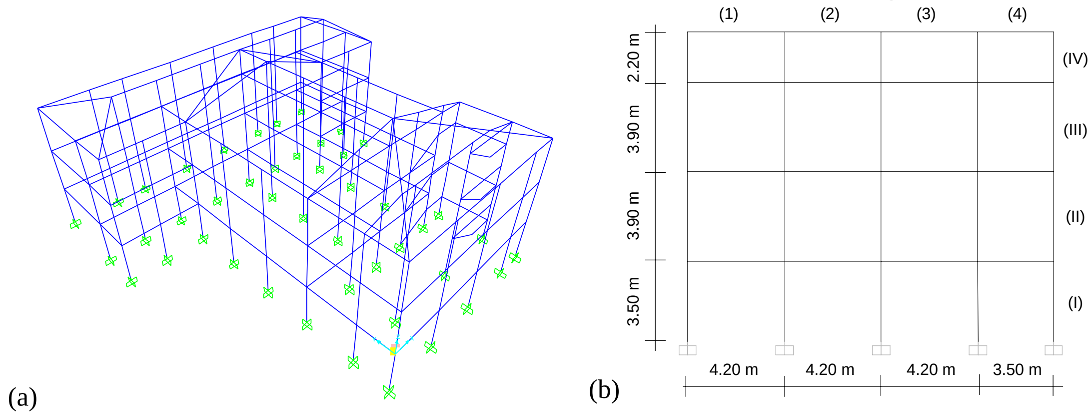

Figure 1: (a) The tri-dimensional view of the scholastic building (b) The central of the case-study building

4.2 Prior to Inspections

The reliability of the case-study frame is calculated based on the state of knowledge about the building before in-situ inspections and tests are conducted. A list of the possible sources of uncertainty in the reinforced concrete section detailing has been constructed by identifying the various possibilities, their relative plausibility, and their correlation with other sources. Table 1 demonstrates a list of possible sources for structural modeling uncertainty represented by discrete probability mass functions and the corresponding correlation structure. Table 2 demonstrates the parameters for constructing a prior probability distribution for the steel yielding strength and concrete strength in compression as material properties. The reliability of the frame is calculated using the Monte Carlo Simulation with 200 simulations. The probability of failure is calculated to be equal to 0.005 with a coefficient of variation equal to 1.0.

Table 1. Probabilistic characterization of the structural detailing parameters

Defects	Possibilities	Prob.	Type
Insufficient anchorage (Beams)	sufficient (100% effective)	0.900	Systematic over floor
	absent (50% effective)	0.100	
Error in diameter (Columns)	ϕ 16	0.950	Systematic over floor and section type
	ϕ 14	0.050	
Superposition (Columns)	100% of the area effective	0.950	Systematic over floor
	75% of the area effective	0.050	
Errors in configuration (columns)	More plausible configuration	0.950	Systematic over floor and section type
	Less plausible configuration	0.050	
Absence of a bar (beams)	Absence of a bar	0.100	Systematic over floor and section type
	Presence of a bar	0.900	
Conceret cover	2 cm	0.125	Systematic over floor
	3 cm	0.750	
	4 cm	0.125	

Table 2. Probabilistic characterization of the mechanical property of RC.

Var	Dist	Mean [kg/cm^2]	COV
f_c	LN	165	0.15
f_y	LN	3200	0.08

3.3 Using test results to update predictions

The test results available for the building consist of (non-destructive) ultrasonic results and (destructive) carrot tests for determining the concrete resistance. The results of the tests are used in two levels in order to update the probability distribution for concrete resistance at different storeys in the structure and to calculate the robust reliability. In the first level half of the destructive test results are implemented and in the second level the other half of the destructive test results the non-destructive test results are used. The test results are implemented using the MH algorithm with 200 simulations at each level in order to update the probability distribution for the concrete strength and also to update the structural reliability. However, in the first level before the data are employed, the same 200 samples generated employing standard Monte Carlo simulation are used.

Table 3.Test results available for the structure

Test	# data	Type	Standard Error
Carrot test Basement	4	Destructive	0.15
Carrot test Ground floor	4	Destructive	0.15
Carote Fisrt floor	4	Destructive	0.15
Ultrasonic test Basement	6	non-destructive	0.335
Ultrasonic test Ground floor	6	non-destructive	0.335
Ultrasonic test First floor	6	non-destructive	0.335
Tension test Reinforcing steel	2	Destructive	0.08

3.4 Results

Figure 2 demonstrates the histograms and the lognormal curves fitted for the demand to capacity ratio for three increasing levels of data. The first level corresponds to the prior lognormal probability distribution for the demand to capacity ratio before taking into consideration the test results. The second level corresponds to the updated distribution after considering half of the carrot test results for concrete and the tension test results for reinforcing steel. The last level illustrates the updated distribution for structural performance variable after considering the rest of the destructive test results and the ultrasonic test results for concrete.

For all three values of confidence level suggested by the code (i.e., FC=1,1.2,1.35) the corresponding demand to capacity ratios for the structure is calculated. The resulting three values for demand to capacity ratio are marked on the curves illustrated in Figure 2. Note that the failure threshold is also marked at the value of 0. The confidence factors can be estimated, for example, as the value of FC that leads to a demand to capacity ratio with say 5% probability of exceedance. In the prior stage, the confidence factor corresponding to a value of demand to capacity ration with 5% probability of exceedance is larger than (but close to) FC=1.35. In the second level, after the distribution for demand to capacity ratio is updated, the demand to capacity ratio with 5% probability of exceedance corresponds to a confidence factor between FC=1.0 and FC=1.20. In the third level, the demand to capacity ratio with 5% probability of exceedance corresponds to a confidence factor slightly greater than 1.0 which corresponds to the code-recommended value for the most complete level of knowledge.

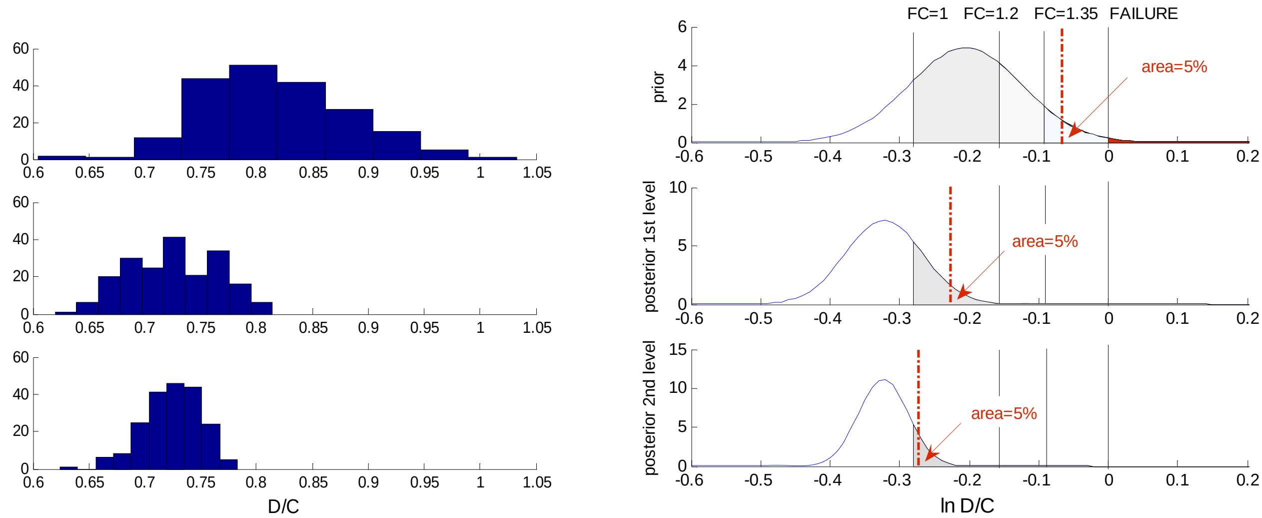

Figure 2: Distribution of the demand to capacity ratio: First Level: The prior lognormal PDF fit to the demand to capacity ratio before test results are being considered, Second Level: The updated lognormal PDF fit to the demand to capacity ratio after implementing the destructive test results, Third Level: The updated lognormal PDF fit to the data after the rest of the test results are also implemented.

3.5 Conclusions

This study aims to characterize, to quantify and to update, based on a probabilistic Bayesian framework, the uncertain modeling parameters (namely mechanical properties of materials and the structural detailing) specific to existing RC buildings as a function of the amount of information available. The motivation behind this research effort is to create a benchmark against which the confidence factors recommended by international codes in the Italian and European seismic guidelines for seismic assessment of existing buildings can be evaluated.

The uncertainties in the structural modeling parameters are related to the mechanical properties of materials and the structural detailing. The structural performance is represented in terms of the ratio of seismic demand to lateral load resisting capacity. Advanced simulation-based reliability methods and Bayesian updating reliability methods are used to estimate and to update the seismic structural reliability. A prior distribution of the structural performance variable is constructed based on available information on the structure and qualitative engineering judgment and experience. A Monte Carlo Markov Chain (MCMC) algorithm is employed in order to both update the structural modeling parameters and reliability after the results of in-situ tests and inspections are being considered. The procedure isapplied to the seismic assessment of an existing school structure in Avellino, Italy, which serves as case-study.

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