

# **ASSESSMENT OF A PROBABILISTIC METHOD FOR EVALUATING RESIDUAL RESPONSE PARAMETERS OF EP STRUCTURAL SYSTEMS**

A. D'Ambrisi<sup>1</sup> and M. Mezzi<sup>2</sup>

**1**  *Associate Professor, Dipartimento di Costruzioni, Università di Firenze, Firenze, Italy*  **2**  *Associate Professor, Dipartimento di Ingegneria Civile ed Ambientale, Università di Perugia, Perugia, Italy Email: adam@dicos.unifi.it, m.mezzi@unipg.it* 

### **ABSTRACT:**

Probabilistically controlled design values of the residual response parameters of elasto-plastic structural systems are calculated using a method previously proposed by the authors. Current codes provide for design values of the seismic response of a structure with a nonlinear behaviour computed as the mean value or as the maximum value of the responses to a certain number of accelerograms. The so calculated design values are characterized by a non-exceedance probability that can not be predefined and that depends on the number of accelerograms used in the analysis. The proposed method allows to calculate design values of the residual response parameters characterized by a predefined non-exceedance probability using a limited number of generated accelerograms. The obtained design values can be effectively used in performance based seismic design. The analyses are performed with reference to EPP and ESH SDOF systems representative of nonlinear structural systems.

**KEYWORDS:** residual response parameters, nonlinear structural systems, probabilistic estimates.

#### **1. INTRODUCTION**

Nonlinear dynamic analysis is spreading as a current method for evaluating the seismic response of structural systems since it allows the evaluation of the demand parameters that shall be accounted for in new design strategies, like the performance based seismic design (PBSD) (FEMA 356 2000, FEMA 445 2006, ATC-58 2007). The main difficulty in evaluating results from nonlinear dynamic analysis is that the obtained responses are characterized by a large scattering due to the randomness of the seismic input.

In the ambit of the PBSD reference is made to two different classes of response parameters: the peak response parameters and the residual response parameters. Both are characterized by a large scattering even when spectrum-fitting generated accelerograms are used as seismic input (D'Ambrisi and Mezzi 2005, 2006, 2007). These scatterings can be treated using two different approaches. The first approach consists of considering the actual probability distribution function (PDF) of the seismic responses and carrying out a full probabilistic procedure for the evaluation of the structural system safety. The second approach, discussed in the present paper, requires the definition of an effective and simplified method for calculating design values of the seismic response characterized by a predefined non-exceedance probability, even using a limited number of generated accelerograms.

EC8 (2004) provides for design values of the seismic response of a structure with a nonlinear behaviour computed as the mean value of the responses to seven accelerograms or as the maximum values of the responses to three accelerograms. Both provisions do not allow to define a characteristic design value of the seismic response, that is a value with a predefined probability of not being exceeded by any response. The probabilistic method proposed by D'Ambrisi and Mezzi (2005) overcomes this limitation, since it allows the analyst to calculate a design value of the seismic peak response characterized by a predefined non-exceedance probability using a limited number of generated accelerograms.

Herein the D'Ambrisi and Mezzi (2005) method is extended to the definition of probabilistically controlled design values of residual response parameters using a limited number of generated accelerograms. Estimates of residual response parameters obtained from sample maximums are considered in the analysis.



#### **2. MODEL PARAMETERS AND SEISMIC INPUT**

The analyses are performed with reference to EPP and ESH SDOF systems that are used to represent the global seismic response of nonlinear structural system expressed in terms of base shear vs. top story displacement.

The considered EPP SDOF systems are characterized by an elastic-perfectly plastic cyclic behaviour with no stiffness and strength degradation, while the ESH SDOF systems are characterized by an elastic-strain hardening hysteretic behaviour with no stiffness and strength degradation (modified Clough model). The EPP SDOF models are therefore defined by the initial elastic stiffness and the plastic threshold, while the ESH SDOF models are defined by the initial elastic stiffness, the plastic threshold and the strain hardening ratio. The design parameters corresponding to the initial elastic stiffness and the plastic threshold used in the analysis are the natural period *T* and the behaviour factor *q*. The behaviour factor *q* is defined as the ratio between the maximum response force corresponding to the elastic response spectrum used in the following to generate the accelerograms and the system yielding force. The following values of model parameters for both EPP and ESH SDOF systems are considered: *T* ranging from 0.1 s to 2.5 s in increments of 0.1 s;  $q=2, 3, 4, 5, 6$ . The five assumed values of q cover the range of values currently used in seismic design practice. Moreover for the ESH SDOF systems a strain hardening ratio of 0.02 is assumed.

In the present study the accelerograms are generated following the recommendations of EC8 (2004), that provides for accelerograms generated from response spectra predefined for different subsoil classes. The EC8 (2004) spectrum for subsoil class B-medium stiff soil is assumed as target elastic response spectrum in the accelerogram generation.

250 accelerograms are generated using the SIMQKE code (1976) enhanced with a fitting control procedure. The corresponding spectra are included in the  $\pm 20\%$  target spectrum envelope. The following control parameters are used in the generation of the accelerograms: total duration of motion=30 s; maximum PGA=0.35 g, this value is representative of the PGA corresponding to a reference return period of 475 years in high seismicity zones (EC8 2004); acceleration envelope curve in the time domain: linear in the t=0 to 2 s range, constant in the  $t=2$  to 5 s range (intense phase), hyperbolic in the  $t=5$  to 30 s range (decreasing branch), null in the t=30 to 40 s range; spectrum-fitting rules according to those provided by EC8 (2004). The null post-earthquake tail allows to correctly estimate the residual response parameters.

#### **3. COMPUTATION AND STATISTICAL ANALYSIS OF RESIDUAL RESPONSE PARAMETERS**

Seismic responses of the considered nonlinear SDOF systems are calculated with an improved version of the MCK code (1996). In this study the considered response parameter is the residual displacement, represented with the random variable *X*. The residual displacement is of interest for the damage evaluation within the PBSD. The scatter in the response data is measured through the coefficient of variation  $V_x$ . Diagrams of the coefficient of variation  $V_X$  versus the natural period T for the considered 250 generated accelerograms and for the five behaviour factor values are reported in Figure 1 for both the EPP and the ESH SDOF systems. For the EPP systems diagrams in Figure 1 show that the response scattering is roughly independent on the period for all



Figure 1. Coefficient of variation versus the natural period.



the five considered values of  $q$ , in particular  $V_X$  ranges from 0.7 to 0.8. Similarly for the ESH systems the scattering does not depend on  $q$ , as shown by the diagrams in Figure 1; in this case  $V_X$  ranges from 0.7 to 0.8 in the period range *T*=0.5 to 1.5 s and from 0.6 to more than 0.9 for periods larger than 1.5 s, while it decreases to 0.5 and even less for periods lower than 0.5 s.

The statistical distribution of the response population *X* is represented through the Lognormal PDF (D'Ambrisi and Mezzi 2005, ATC-58 2007).

#### **4. ESTIMATES OF RESIDUAL RESPONSE PARAMETERS**

According to EC8 (2004) provisions the design values of the seismic response of a structure with a nonlinear behavior can be computed as the mean value of the responses to seven accelerograms or as the maximum value of the responses to three accelerograms. In the first case sample means, that are estimators of the mean value of the response population are used, while in the second case sample maximums, that are estimators of characteristic values larger than the mean value, are used.

The two criteria subscribed by EC8 (2004) lead to design values characterized by very different and uncontrolled non-exceedance probability levels (D'Ambrisi and Mezzi 2005, 2006, 2007). According to the EC8 (2004) present formulation these two estimate criteria can be applied to any response parameters, therefore they can be applied not only to the peak response parameters but also to the residual response parameters.

When residual response parameters are studied it is more suitable to refer to maximum values instead that to mean values for two reasons: 1) residual response parameters are currently used in seismic design practice to perform compatibility analysis or damage evaluation, that make reference to their maximum values; 2) residual response parameters are characterized by a very large scattering, but, in spite of this, the sample maximums have a practically constant non-exceedance probability, as shown in the following.

Estimators *Y* are calculated as sample maximums obtained using sets of *s* response values randomly picked from the response population. The estimates *Y* obtained from the population of the 250 responses have a very irregular frequency distribution, that can not be modelled with any PDF. This is due to the fact that 250 responses are not sufficient to ensure an adequate continuity of the values. To overcome this problem the population is artificially enlarged up to 5000 responses, generated using the response PDF, without having actually to perform analyses for a large number of accelerograms. This mathematical process preserves the statistical correlation among the individual responses (ATC-58 2007). Using this larger number of responses the frequency distribution become regular.

To obtain a statistically stable population of the response estimates, a large number  $(10^6)$  of sample maximum values are calculated. Sample sizes *s* of 3, 4, 5, 6, 7, 10 are considered. In particular, according to the EC8 (2004) provisions, *s*=3 corresponds to the minimum number of accelerograms that has to be used in nonlinear dynamic analysis.

The probability, *W*, that the value of the estimates *Y* is not exceeded by the response of any acclerograms can be evaluated through the expression

$$
P(X < Y) = W = \int_0^\infty F_X(y) \cdot f_Y(y) dy \qquad (4.1)
$$

Considering the small dimension of the sample size *s* the *Y* population can be modeled with a Lognormal PDF. Figure 2 shows the *W* versus *T* curves for all considered *q* values and  $s=3$ , for both the EPP and the ESH SDOF systems. It can be observed that *W* is independent of *q* and also independent of *T*: therefore the sample maximums are characterized by a non-exceedance probability that is practically constant. Similar results, characterized by different *W* values, are obtained for the other considered *s* values. Figure 3 shows the *W* versus *T* curves for all considered *s* values and  $q=3$ , for both the EPP and the ESH SDOF systems.

According to these results the non-excedance probability *W* of the design value could be controlled only through the selection of the sample size *s*. Such a procedure does not seem to be adequate in the ambit of current seismic design practice because of two reasons: (1) the safety would depend on the approach followed to estimate the design value; (2) only uncontrolled discrete values of the non-exceedance probability *W,*





Figure 2. *W* versus *T* curves for all considered *q* values and *s*=3.



Figure 3. *W* versus *T* curves for predefined values of the sample size *s* and  $q=4$ .

corresponding to discrete values of the sample size *s*, could be obtained. It is therefore suitable to define a procedure for obtaining predefined values of *W*.

#### **5. PROBABILISTIC CONTROL OF THE RESPONSE ESTIMATES**

The D'Ambrisi and Mezzi (2005) method is based on the definition of an amplification factor of the estimates obtained as mean of the responses to a limited number of spectrum-fitting generated accelerograms. The factor is defined so that the non-exceedance probability of the response estimates can be controlled. The methodology adopted to define the amplification factor  $\zeta$  is described in details in D'Ambrisi and Mezzi (2005).

The ζ factor has been defined as the ratio between a characteristic value of the response population *X* with a non-exceedance probability significantly larger than 0.5 and a characteristic value of the response estimate population *Y* with an exceedance probability significantly larger than 0.5 (D'Ambrisi and Mezzi 2005). If mean values are considered, ζ factors are always larger than 1; in this case, indeed, the estimates *Y* have the same mean value of the response population *X*. If maximum values of the response are considered, ζ factors smaller than 1 can be also obtained.

Following the approach already used by the authors in D'Ambrisi and Mezzi (2005), the amplification factor  $\zeta$ of the estimators *Y* can be applied to calculate the modified estimators *Z*=ζ*·Y*. As already done for the *Y* population, the *Z* population is modeled with a Lognormal PDF. The probability, *W*, that the value of the estimates *Z* is not exceeded by the response of any accelerograms can be evaluated through the expression

$$
P(X < \zeta \cdot Y) = W(\zeta) = \int_0^\infty F_X(z) \cdot f_Z(z) dz \tag{5.1}
$$

Figure 4 show the *W* versus *T* curves for assigned values of  $\zeta$  factor and  $q=4$ , for both the EPP and the ESH SDOF systems.

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Figure 4. *W* versus *T* curves for assigned values of  $\zeta$  factor (*q*=4).

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For both the EPP and the ESH SDOF systems the non-exceedance probability *W* can be considered independent of *T* for all the considered sample sizes *s*, while it depends on the sample size *s*, as it is evident from Figure 4. Moreover, for both considered systems, *W* is also independent of the behaviour factor *q* as it can be noticed from Figure 5 that reports for *s*=5 the *W* versus *T* curves for predefined values of the behaviour factor *q* and for  $\zeta$ =0.7, 1.0, 1.4. Similar results, not reported here for the sake of brevity, are obtained for the other considered values of *s*.

Figure 6 reports for  $q=4$  the *W* versus *T* curves for predefined values of the sample size *s* and for  $\zeta=1.4$ . Considering also Figure 3, reporting the same curves for  $\zeta=1.0$ , it is evident that for all the analyzed cases the variation of the sample size *s* significantly affect the ζ factor to be used for obtaining predefined *W* values.



Figure 5. *W* versus *T* curves for predefined values of the behaviour factor *q* and for ζ=0.7, 1.0, 1.4 (*s*=5).



Figure 6. *W* versus *T* curves for predefined values of the sample size *s* and for  $\zeta = 1.4$  ( $q=4$ ).

Once selected the number of accelerograms to be used in the analysis,  $\zeta$  factor can be usefully applied in evaluating a design value of the seismic response characterized by a predefined non-exceedance probability, due to its practical independence on all considered parameters and to its uniformity over the considered period range.

The minimum number of accelerograms, *s*=3, can be considered for reducing the design analysis efforts without reducing the non-exceedance probability of the computed design value.

The curves reported in Figures 5 and 6 show that once a ζ value is assigned, *W* does not depend on *T* and *q*, therefore these two parameters can be treated as sources of random scattering. With this assumption the frequency distribution of *W* for each value of  $\zeta$  can be approximated by a Normal PDF and an extreme characteristic value, 95% percentile, of *W*(ζ) suitable for structural design applications can be obtained for each  $\zeta$  factor value. These characteristic  $W_k$  values are reported in Table 5.1. From the table it is evident that a high value of  $W_k$  can be obtained even using only three accelerograms, if an adequate value of  $\zeta$  factor is used. A



non-exceedance probability *Wk*=0.8 can be obtained using, for example, an amplification factor ζ≅1.4. If a large non-exceedance probability, i.e.  $W_k=0.9$ , is required, a very high amplification factor, i.e.  $\zeta \geq 2.0$ , should be used due to the large scattering of the residual reponse parameters.

	$\begin{array}{ c c c c c } \hline 0.6 & 0.8 \ \hline \end{array}$		$1.0$ $1.2$ $1.4$ $1.6$ $1.8$		
$W_{\iota}$					

Table 5.1 Characteristic non-exceeding probabilities *W* versus  $\zeta$  ( $s=3$ ).

#### **6. CONCLUSIONS**

A previously proposed method has been extended to the definition of probabilistically controlled design values of residual response parameters of non linear structural systems. The method allows to calculate design values characterized by a predefined non-exceedance probability using a limited number of generated accelerograms. The design values are computed by amplifying the sample maximum of the response estimates through an amplification factor that is independent of the behaviour factor *q* and the natural period *T* of the structural system.

The proposed method is effective in directly controlling the non-exceedance probability of the design value of the residual response parameters even using only three accelerograms. On the contrary, the design values calculated according to the EC8 provisions are characterized by non-exceedance probability that depends on the number of accelerograms used in the analysis.

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#### **REFERENCES**

ATC-58 (2007). *Guidelines for seismic performance assessment of buildings* – 35% draft, Redwood City, California.

D'Ambrisi A, Mezzi M. (2005). A probabilistic approach for estimating the seismic response of elasto-plastic SDOF systems. *Earthquake Engineering & Structural Dynamics*; **34** (14): 1737-1753.

D'Ambrisi A, Mezzi M. (2006). Probabilistic estimate of the non linear seismic response of RC structures. *Proceedings of the 2nd International fib Congress*. Naples, Italy, June 5-8, 2006.

D'Ambrisi A, Mezzi M. (2007). Conservative design values of the nonlinear seismic response of RC frames. *Proocedings of the 3rd International Conference on Structural Engineering, Mechanics and Computation*. Cape Town, South Africa, September 10-12, 2007.

EC8 Eurocode 8 EN 1998-1 (2004). *Design of structures for earthquake resistance*.

FEMA 356 (2000). *Prestandar and Commentary for the seismic rehabilitation of buildings*, Federal Emergency Management Agency and American Society of Civil Engineers.

FEMA 445 (2006). *Next-Generation Performance-Based Seismic Design Guidelines*, Federal Emergency Management Agency.

Gasparini DA, Vanmarcke EH. (1976). Simulated earthquake motions compatible with prescribed response spectra. *R76-4*, Dept. of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Mezzi M. (1996). MCK: A program for the non-linear dynamic analysis of non conventional models. *Bull. Ist. Energetica,* University of Perugia, Perugia, Italy.