

## PERFORMANCE-BASED ASSESSMENT OF THE EFFECTS OF SOIL-STRUCTURE INTERACTION ON THE SEISMIC DEMANDS OF BRIDGE PIERS: A PROPOSED METHODOLOGY

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### ABSTRACT:

A methodology is proposed for simplified estimation of the effects of nonlinear soil-structure interaction (SSI) on the ductility and total displacement demands of bridge piers subjected to earthquake ground motions. The methodology is based on modifying the fixed-base demands by applying SSI Modification Factors that are proposed as functions of the ratio of the flexible-base period to the fixed-base period of the piers ( $T_{sys}/T$ ). The proposed modification factors are estimated by using response databases obtained from nonlinear dynamic analyses of prototype SSI systems with various combinations of soil, foundation, and structure properties and input ground motions. The response data is processed probabilistically to account for the uncertainties involved in the system properties, such as the natural period, and the scatter of the results of the nonlinear dynamic analyses. The results are presented for various performance objectives and reliability levels. Thus, the SSI Modification Factors can be selected with various levels of reliability chosen by the engineer. The methodology is explained through an application example where SSI Modification Factors are presented by using response databases from nonlinear dynamic analyses of prototype pile-supported bridge piers with various natural periods of the fixed-base structure and soil layer subjected to an ensemble of ground motions. The application of the proposed methodology is shown to be suitable for implementation in a design code. Finally, the paper discusses merits and limitations of the methodology and the need for further research.

**KEYWORD:** Nonlinear Soil-Structure Interaction, Inelastic Response, Performance-Based Design, Design Code, Bridge Piers, Pile Foundation

### 1. INTRODUCTION

Estimation of seismic demands of structures involves many challenges, one of which is the evaluation of the effects of seismic soil-structure interaction (SSI) on the inelastic response of structures. Seismic SSI is potentially a highly nonlinear phenomenon, which causes the structural response to differ from that of the ideal structure with rigid base. Rigorous estimation of the effects of SSI requires a system approach to SSI analysis with a fully coupled representation of the main components of the system. It also requires proper consideration of the nonlinearity of soil, structure, and soil-structure interface, as well as the radiation damping of the system. Such SSI analysis provides more reliable predictions of the nonlinear displacements, which are better indicators of damage to structures as opposed to forces obtained from the equivalent linear elastic analysis typically used in practice (Gazetas and Mylonakis, 1998; Martin and Lam, 2000; Crouse and McGuire, 2001; Finn, 2004; Kim and Roesset, 2004).

Rigorous SSI analysis, however, is complex and time consuming, and thus not justifiable in many practical cases. Therefore, simplifications are commonly made, the extent of which depends on the work at hand. Such simplifications typically involve uncoupling of the structure and foundation into separate systems, and/or

approximating the nonlinear behaviour with linear models. It is desirable, however, to explore methods that are convenient for implementation in performance-based design codes or guidelines, which could capture the salient effects of system nonlinearities with less approximation or effort than that introduced by uncoupling or linearization of the system.

In this regard, this paper explores a methodology for predicting the effects of nonlinear SSI on inelastic seismic response of bridge piers by employing response databases of various prototype systems which cover a range of typical structures. If databases of seismic response of prototype systems exist, then they can be used to predict the response of similar systems without performing a complex SSI analysis for each case, by instead modifying the response of the corresponding fixed-base structure. The effects of SSI need to be presented in a format familiar to structural engineers and in accord with the methods of structural analysis and design, e.g. as functions of the structure's natural period. Thus, a link between structural analysis and SSI analysis can be established. The proposed methodology is explained through an application example of prototype pile-supported bridge piers with various natural periods. The prototype systems are described, then response databases are presented, and the implementation of the methodology is explained. Finally, merits and limitations of the proposed methodology are discussed and the need for further research is outlined.

## **2. ANALYSIS OF PROTOTYPE SOIL-FOUNDATION-STRUCTURE SYSTEMS**

A typical pile-supported highway bridge pier with varying properties was analysed to construct seismic demand databases. Bridge structures were chosen since SSI can particularly play a significant role in their seismic response due to their relatively simple structural form and their low degree of redundancy that makes them sensitive to the effects of SSI and SSI-induced displacements. The height of the pier and the thickness of the site soil layer were varied to provide first mode fixed-base structural periods of 0.3, 0.6, 0.8, 1.0, 1.5 and 2.0 s and first mode site periods (at low amplitude of motion) of 0.6, 1.0 and 2.0 s. The soil consisted of saturated soft clay with average shear wave velocity in the upper 30 m of  $V_{s30} = 145$  m/s and average strength (upper 30 m) of  $S_{u30} = 40$  kPa. The foundation consisted of a 6×6 pile group with 0.3 m square piles spaced at 1.25 m and driven to a depth of 15 m below the 7.5×7.5×1.5 m pile cap. The pier had a typical cross section of 1.5 m in diameter with 1% of longitudinal reinforcement. A 3500 kN gravity load was applied at the top of the piers to represent the weight of the bridge's superstructure.

An ensemble of 26 ground motion time-histories was selected from historic ground motions recorded on rock or very stiff soil to represent moderate to severe ground shakings. The selected ground motions are from earthquakes with magnitudes of 5.8 to 7.6 recorded at distances from 8.0 to 48.8 km with peak ground acceleration (PGA) ranging from 0.080g to 0.587g (g is the acceleration of gravity). Peak ground velocities are from 2.9 cm/s to 62.0 cm/s and peak ground displacements are from 0.2 cm to 51.8 cm. To avoid uncertainties introduced by the scaling of ground motions, the records were not scaled or further processed. All records were taken from the Pacific Earthquake Engineering Research (PEER) Center's strong motion database (<http://peer.berkeley.edu/smcat/>). Details of the selection of ground motions can be found in Ghalibafian (2006).

State-of-the-art nonlinear dynamic analyses were carried out by using the commercial finite difference program FLAC (Itasca, 2005). The numerical model included the soil, the foundation, and the bridge pier, and accounted for the soil-foundation interface. The verification of the numerical analysis was performed at both component and system levels. At the component level, the behaviour of each component, modeled individually, was compared against its expected behaviour from the available literature, or against the behaviour predicted by other validated analysis methods. To verify the overall system behaviour, an instrumented pile-supported bridge pier in California that was subjected to an actual earthquake (Shakal et al., 1989) was modeled and the computed response was compared with the recorded response. Results obtained were plausible and verified the analysis procedure. More on the analysis and verification process can be found in Ghalibafian (2006) and Ghalibafian et al. (2006b).

### 3. SEISMIC RESPONSE DATABASES

The effects of SSI are presented here through Ductility Demand Ratio (DDR) and Total Displacement Ratio (TDR), which are ratios that demonstrate how the response without SSI is different than the response when the effects of SSI are accounted for. DDR is the ratio of the bridge pier's ductility demand when SSI is included (flexible-base ductility demand) to that when SSI is not included (fixed-base ductility demand). If  $DDR < 1.0$ , then SSI is reducing the ductility demand, but if  $DDR > 1.0$ , then SSI is amplifying the ductility demand. TDR is the ratio of the total displacement of the flexible-base structure to that of the fixed-base structure. TDR indicates the effects of SSI on the total displacement of the bridge piers. To obtain DDR and TDR, the ductility and displacement demands are obtained from nonlinear dynamic analyses of both flexible-base and fixed-base piers. The input ground motions for the analysis of the fixed-base structures are the free field motions obtained from the site response analysis of the soil layers.

DDR and TDR, obtained through nonlinear dynamic analyses of the prototype bridge piers, were first presented as functions of the fixed-base natural period of the piers (see the companion paper by Ghalibafian et al., 2008). The resulting distributions were examined and the outliers were excluded from the database. Figures 1a and 1b show the cumulative frequency distributions of DDR and TDR, respectively. These figures demonstrate that the data obtained from the dynamic analyses can be represented by lognormal distributions. It is, however, advantageous to relate DDR and TDR to a system parameter rather than merely the fixed-base period of piers. A system parameter is the ratio of the flexible-base natural period to the fixed-base natural period ( $T_{sys}/T$ ), which has been shown to be correlated with the pier-to-foundation stiffness ratio (Finn, 2004). Therefore, the period ratio  $T_{sys}/T$  is employed as a dimensionless parameter to describe the system. It is noted that the system period is commonly referred to as the pier's elongated period. The period elongation is caused by the flexibility of the base due to SSI. Greater  $T_{sys}/T$  indicates greater period elongation and more flexibility at the base of the pier due to a higher pier-to-foundation stiffness ratio. Since piers with shorter periods are stiffer, the period elongation increases with decreasing fixed-base period and it is expected to observe greater effectiveness of SSI with greater stiffness of the piers, i.e., with a greater period elongation (note: foundation stiffness is unchanged here). Table 1 summarizes the means and the standard deviations of DDR and TDR along with the  $T_{sys}/T$  ratios associated with the fixed-base periods  $T$ . An observation is that the distribution of  $T_{sys}/T$  is not linearly correlated with  $T$ . As a result,  $T_{sys}/T$  points are closely spaced between 1.02 and 1.12 while there are no data points between 1.12 and 1.33. This observation suggests that the prototype systems used for studying SSI systems would provide better distribution of data if selected based on  $T_{sys}/T$  rather than the fixed-base period  $T$ . The data in Table 1 is used in the probabilistic analysis explained in the following sections.

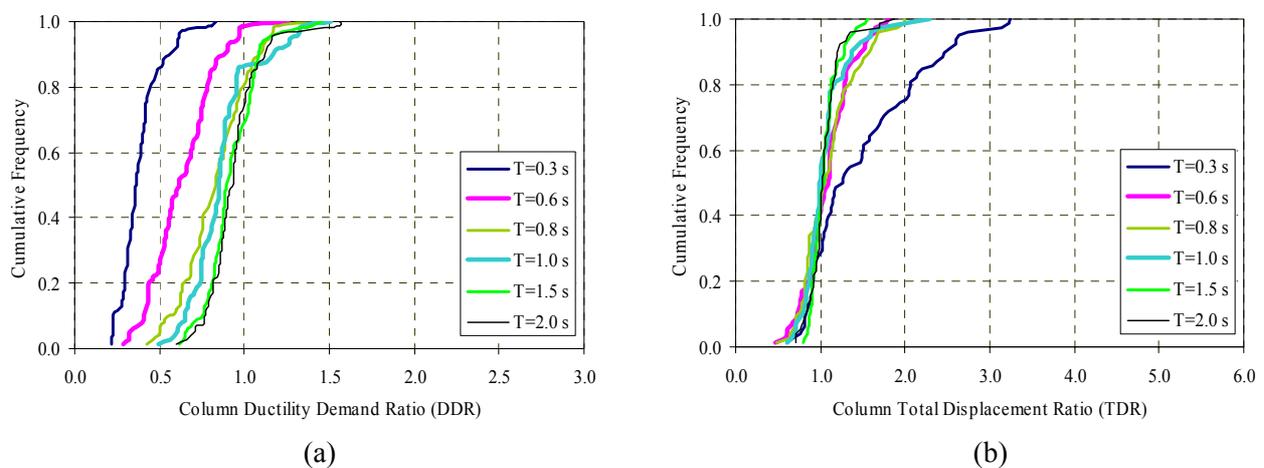


Figure 1: Distribution of (a) Ductility Demand Ratio; (b) Total Displacement Ratio

Table 1: Summary of the statistics of results

Fixed-Base Natural Period T (s)	Period Ratio $T_{sys}/T$	Ductility Demand Ratio (DDR)		Total Displacement Ratio (TDR)	
		Mean	Standard Deviation	Mean	Standard Deviation
0.3	1.33	0.39	0.12	1.50	0.66
0.6	1.12	0.64	0.19	1.08	0.28
0.8	1.10	0.83	0.20	1.10	0.33
1	1.08	0.87	0.19	1.06	0.29
1.5	1.05	0.93	0.16	1.05	0.16
2	1.02	0.94	0.16	1.05	0.20

#### 4. PROPOSED METHODOLOGY

An advantage of the presentation of DDR and TDR as functions of  $T_{sys}/T$  is that the results obtained from the analysis of the prototype systems of this study can be compared to those from studies with different prototype systems but with similar  $T_{sys}/T$  ratios. If it is shown that DDR and TDR thus obtained are comparable for different soil-foundation-structure systems with the same  $T_{sys}/T$  ratio, then they can be used to modify the fixed-base response of SSI systems, identified by their  $T_{sys}/T$  ratio, to estimate their flexible-base response. For instance, the total displacement of a flexible-base bridge pier can be estimated by multiplying the total displacement of its corresponding fixed-base pier by TDR. The mean values of DDR and TDR could be used to modify the response of the fixed-base structure. However, given the scatter of DDR and TDR, the sole consideration of the mean values may obscure some aspects of the system response and may result in wrong predictions of the effects of SSI. For instance, while the mean values of DDR suggest that SSI always reduces the ductility demand, the distributions of Figure 1 demonstrate that this does not necessarily hold true and that SSI could increase the ductility demand of the structure, which depends on the piers' period elongation and the spectral characteristics of the ground motions as passed through the soil layer. Due to the random nature of the problem and the uncertainties involved, the presented statistics cannot be used to conclusively predict the effects of SSI in a deterministic fashion. Therefore, the data should be processed probabilistically so that DDR and TDR can be used to estimate the effects of SSI with known reliability. The probabilistic data processing is carried out by performing reliability analyses to estimate the probability of exceeding selected values of DDR and TDR. In other words, for a selected ratio  $r$ , if  $p$  is found such that the probability of  $TDR < r$  is  $p$ , then the fixed-base total displacement can be multiplied by  $TDR = r$  to obtain the flexible-base total displacement of the pier with the reliability of  $p$ . Note that the reliability analysis is performed by estimating the probability of not meeting the performance objective, i.e., the probability of  $TDR > r$  (in other words, if the probability of  $TDR > r$  is  $P$ , then  $P$  is the probability that the total displacement is greater than that predicted by TDR).

The values of  $r$  can be selected to represent several performance criteria. For instance, the performance function can be formulated to obtain the probability of  $DDR > 1.0$  (i.e.,  $r=1.0$ ) which denotes the probability that SSI is amplifying the ductility demand. Or, it can be formulated to obtain the probability that SSI increases the total displacement by more than 20% (i.e.,  $TDR > 1.2$  for  $r=1.2$ ) or by more than 40% (i.e.,  $TDR > 1.4$  for  $r=1.4$ ). The results of the reliability analyses for various performance criteria can then be plotted together to provide useful information for performance evaluation purposes. An example is demonstrated here by using the databases of DDR and TDR presented in Table 1. The input random variables include the period ratio  $T_{sys}/T$  with a coefficient of variation of 0.05. The reliability analyses were performed using Montecarlo simulation which uses response surfaces of the means and standard deviations as represented by either nonlinear regression models or neural networks (Ghalibafian, 2006; Ghalibafian et al., 2006a). It is noted that the probabilities here are not total probabilities as they are conditional on the occurrence of the ground motions used in this study. Calculation of total probabilities requires an estimation of the probability of occurrence of the input ground motions, which is outside the scope of this work.

Sample results of reliability estimates are shown in Figure 2 for several values of  $r$ . The curves of Figures 2a and 2b show, respectively, the probability of  $DDR > r$  and  $TDR > r$  as functions of  $T_{sys}/T$ . It can be observed, for instance, in Figure 2a, that for a pier with  $T_{sys}/T$  of 1.05, while the probability of  $DDR > 1.0$  (i.e.,  $r=1.0$ ) is about 30%, it is less than 5% for  $DDR > 1.3$  (i.e.,  $r=1.3$ ), which is a much lower probability. Therefore, if the pier has 30% ductility reserve, it could be inferred that ignoring SSI will not pose a significant risk to the pier (less than 5%). As another example, for a system with  $T_{sys}/T$  of 1.20, the probability of  $DDR > 0.9$  is less than 10%, which implies that the ductility demand can be reduced by 10% with more than 90% confidence. Such reduction of demand can become a source of economic savings especially in the retrofit of bridge piers. In another example, Figure 2b shows that there is about 50% probability that piers with  $T_{sys}/T$  of 1.35 experience more than 40% increase (i.e.,  $r=1.4$ ) of total displacement demand due to SSI, but this probability is reduced to about 5% for structures with  $T_{sys}/T$  of 1.02. This information provides additional insight into the performance of the system and could be used for optimizing a design for performances with specified levels of reliability.

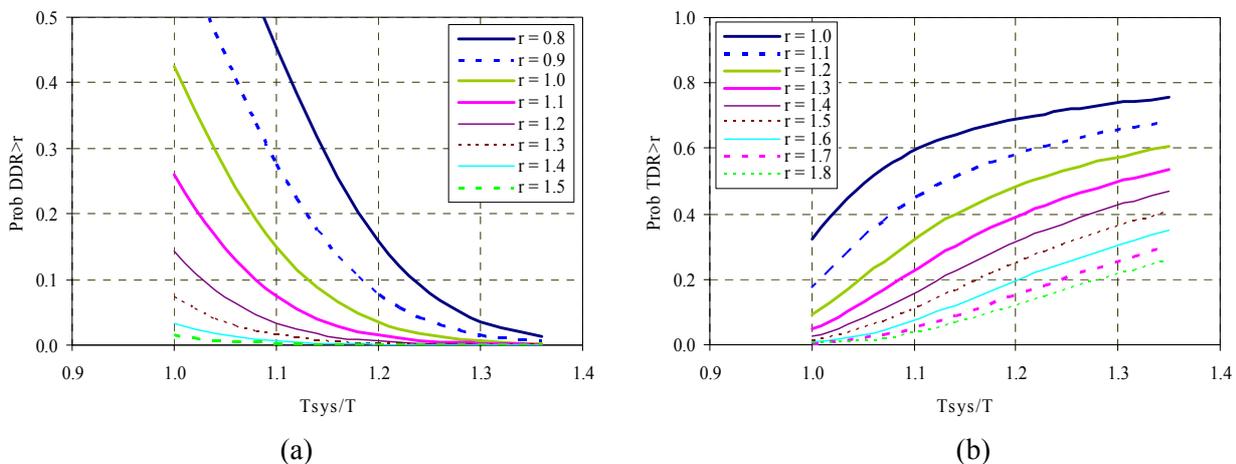


Figure 2: (a) Probability of  $DDR > r$  as a function  $T_{sys}/T$  for  $r$  values from 0.8 to 1.5; (b) Probability of  $TDR > r$  as a function of  $T_{sys}/T$  for  $r$  values from 1.0 to 1.8

The curves of Figure 2 can be rearranged for the purpose of estimating the effects of SSI on the performance of the piers with given target reliabilities. Figure 3 shows the probabilities of  $DDR > r$  and  $TDR > r$  as functions of  $r$  (rather than  $T_{sys}/T$ ) for bridge piers with  $T_{sys}/T$  from 1.02 to 1.30. The advantage of this presentation is that for a specific pier with known  $T_{sys}/T$ , the ratio  $r$ , which represents the response modification due to SSI, can be found with the desired level of reliability. For example, the curves of Figure 3b can be used to obtain the value of  $r$  corresponding to  $TDR < r$  of a bridge pier with  $T_{sys}/T$  of 1.06 with a reliability of 80% (i.e., 20% probability of  $TDR > r$ ). As shown in Figure 3b, the value of  $r$  associated with 20% probability of  $TDR > r$  obtained from the curve for  $T_{sys}/T=1.06$  is 1.23. It denotes that if the total displacement of the bridge pier is obtained from the analysis of the corresponding fixed-base pier, then the effects of SSI can be accounted for by multiplying this fixed-base displacement by  $r=1.23$  to obtain the total displacement of the flexible-base pier with 80% reliability. Figure 3a and Figure 3b are named here, respectively, Performance-Based SSI Assessment Diagrams for ductility and total displacement demands, and the modifying factors obtained from these diagrams (i.e.,  $r$ ) are called SSI Modification Factors.

To better demonstrate the practical application of the proposed SSI assessment diagrams, the bridge example of Figure 4 is considered. This bridge has simply supported spans with soil-foundation-pier systems similar to those of this study. It is assumed that the input motions used in this study describe the seismic hazard at the site of the bridge. To design the supports of the deck, the support length must be estimated which is a parameter sensitive to the relative displacement of the piers with respect to each other. The relative displacement of the piers, on the other hand, is dependent on their total displacements and thus the effects of SSI on the total displacements of the piers must be estimated. For a simple bridge like the bridge of Figure 4, however, performing nonlinear SSI analysis is likely not justifiable due to practical constraints and therefore a simplified

method can be of great help. This task can be quickly performed with the availability of curves such as those of Figure 3, from which SSI Modification Factors can be obtained to modify the demands from the analysis of the fixed-base structure. The level of reliability of the modification factors can be chosen by the designer. For instance, and exceedance probability of 0.2 (or a reliability level of 80%) results in  $r=1.23$  for the pier with  $T_{sys}/T=1.06$  and  $r=1.62$  for the pier with  $T_{sys}/T=1.2$ . Having obtained the values of  $r$  for the two piers, the total displacement of the flexible-base piers can be calculated from the displacements of the fixed-base piers.

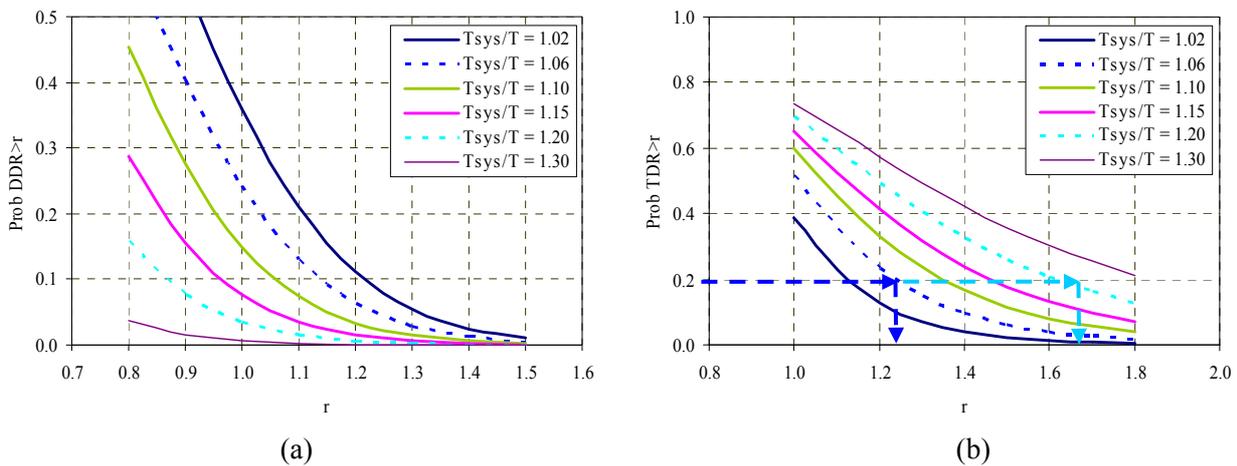


Figure 3: Performance-based SSI Assessment Diagrams for (a) ductility demand; (b) total displacement demand

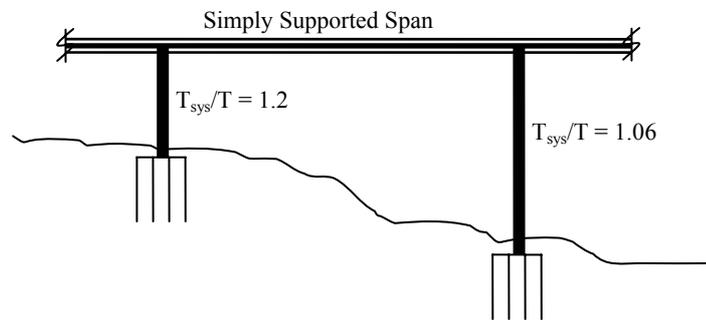


Figure 4: An example bridge with simply supported spans and pile-supported piers on soft soil

## 5. MERITS, LIMITATIONS, AND NEED FOR FURTHER RESEARCH

The proposed SSI Assessment Diagrams have a number of interesting features that make them appealing for the performance-based assessment of the effects of SSI on the seismic response of structures. However, they have some limitations that must be further investigated. Merits, limitations, and the need for further research are outlined as follows:

- One feature is the explicit consideration of the dispersions of the database of demands that was used to generate these curves. Rather than crudely relying on mean or median values with unknown levels of reliability, these curves provide the flexibility of choosing the level of confidence in the estimated modification factors. Therefore, the designer can either use a uniform level of reliability for all performance objectives or use different levels of reliability for different performance objectives tailored for a specific project.

- These curves account for the uncertainties of the system parameters, such as the natural period, that are used to describe the system. The variability of the system parameters must be decided upon prior to generating the curves. Thus, families of curves can be constructed to account for various levels of uncertainty in the system parameters to give designers the flexibility of choosing one that reflects the level of confidence in the estimated system parameter.
- In the examples presented here, only the natural periods were considered to construct the curves. However, this can be extended to other system parameters or a combination of system parameters when constructing the curves. In such cases, complex response surfaces can be represented by neural networks to perform the reliability analyses.
- Constructing the SSI Assessment Diagrams as functions of  $T_{\text{sys}}/T$  can expand their domain of applicability to various combinations of piers, foundations, and soils that are different from those used to generate the curves, but have similar  $T_{\text{sys}}/T$  characteristics. This possibility must be investigated by performing nonlinear dynamic analyses of various pile-supported bridge pier systems on soft soils. If similar behaviour of systems with similar  $T_{\text{sys}}/T$  is observed, then such curves become appealing for implementation in performance-based design codes or guidelines for typical highway bridges or other structures. Simplified estimation of  $T_{\text{sys}}$ , however, remains a challenge which must be addressed if such curves are to be implemented with a design code format. As previously mentioned, the uncertainty of  $T_{\text{sys}}$  can be accounted for in the reliability analysis.
- If the input ground motions used to construct the SSI Assessment Diagrams are all selected to represent a specific level of seismic hazard, then families of curves can be constructed for various levels of seismic hazard. The merits of such selection of ground motions, as opposed to the selection of ground motions with various amplitudes as was done in this study, must be investigated. The preliminary observations of the results of this work, as reported in Ghalibafian (2006), demonstrated that the SSI Modification Factors are to some degree dependent on the intensity of ground motions. Further research is needed to verify this finding and to investigate the effects of ground motion parameters on the SSI Modification Factors.
- For code implementation, guidelines might be provided for the selection of the reliability level to limit the need for individual judgment calls. Although this represents yet another prescriptive approach, it can provide designers with more flexibility to meet specific needs of specific projects. The merits and limitations of such procedures need to be explored by researchers and practicing engineers as well.

## 6. CONCLUDING REMARKS

A method for approximate estimation of the effects of SSI on the ductility and total displacement demands of structures was proposed and its practical code implementation was explored. The idea of the proposed method is to use the statistics of demands obtained from the nonlinear analyses of prototype soil-foundation-structure systems to calculate the effects of SSI on similar systems by modifying their corresponding fixed-base demands. SSI Modification Factors, which are functions of the ratio of the flexible-base initial period to the fixed-base period ( $T_{\text{sys}}/T$ ), can be used to estimate the response of the SSI system from the response of its corresponding fixed-base system. Estimation of the proposed modification factors involves explicit consideration of the scatter observed in the statistics of the response of the prototype systems, and explicit consideration of the uncertainties in the system properties on which they are dependent. The joint consideration of the scatter in the statistics of response and the uncertainties of the system parameters is made by performing reliability analyses. Thus, SSI Modification Factors are estimated for various performance criteria with several levels of reliability as functions of the period ratio  $T_{\text{sys}}/T$  with given uncertainty. Results are presented by Performance-Based SSI Assessment Diagrams from which SSI Modification Factors can be obtained, after deciding on the performance objective and the level of reliability required in the predicted performance. Further research is required to better explore

the merits and limitations of the proposed methodology.

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## REFERENCES

- Crouse, C.B. and McGuire, J. (2001). Energy Dissipation in Soil-Structure Interaction, *Earthquake Spectra*, Vol. 17, No. 2, pp 235-259.
- Finn, W.D.L. (2004). Characterizing Pile Foundations for Evaluation of Performance Based Seismic Design of Critical Lifeline Structures, *13<sup>th</sup> World Conference on Earthquake Engineering*, Vancouver, BC, Canada.
- Foschi, R.O., Li, H., Folz, B., Yao, F., Zhang, J., Baldwin, J. (2000). RELAN: A General Software Package for Reliability Analysis. *Department of Civil Engineering*, University of British Columbia, Vancouver, BC, Canada.
- Gazetas, G. and Mylonakis, G. (1998). Seismic Soil-Structure Interaction: New Evidence and Emerging Issues, *Proceedings of the Specialty Conference on Geotechnical Earthquake Engineering and Soil Dynamics III*, Eds. Dakoulas, P., Yegian, M. and Holtz, R.D. ASCE Geotechnical Special Publication No.75, Vol. 1, pp 1119-1174.
- Ghalibafian, H. (2006). Evaluation of the effects of nonlinear soil-structure interaction on the inelastic seismic response of pile-supported bridge piers, *Ph.D. Dissertation*, University of British Columbia, Vancouver, BC.
- Ghalibafian, H., Foschi, R.O., and Ventura, C.E. (2006a). An Application of Artificial Neural Networks for Reliability Analysis and Performance-Based Design of Pile-Supported Bridge Piers, *Proceedings of the First European Conference on Earthquake Engineering and Seismology*, Geneva, Switzerland.
- Ghalibafian, H., Ventura, C.E., and Byrne, P.M. (2006b). Modeling of Soil-Foundation-Structure Interaction of an Instrumented Bridge Pier Using FLAC, *Proceedings of the 8<sup>th</sup> National Conference on Earthquake Engineering*, San Francisco, California.
- Ghalibafian, H., Ventura, C.E., and Foschi, R.O. (2008). Effects of Nonlinear Soil-Structure Interaction on the Inelastic Seismic Demand of Pile-Supported Bridge Piers, *Proceedings of the 14<sup>th</sup> World Conference on Earthquake Engineering*, Beijing, China.
- Itasca Consulting Group, Inc. (2005). *FLAC – Fast Lagrangian Analysis of Continua, Version 5.0 User's Manual*, Minneapolis, Minnesota.
- Kim, Y.-S. and Roesset, J.M. (2004). Effect of Nonlinear Soil Behavior on Inelastic Seismic Response of a Structure, *International Journal of Geomechanics*, ASCE, Vol. 4, No. 2, pp104-114.
- Martin, G.R. and Lam, I.P. (2000). Earthquake Resistant Design of Foundations – Retrofit of Existing Foundations, *Proceedings of the GeoEng 2000 International Conference on Geological and Geotechnical Engineering*, Melbourne, Australia.
- Shakal, A. et al. (1989). CSMIP strong-motion records from the Santa Cruz Mountains (Loma Prieta), California Earthquake of 17 October 1989, *Report No OSMS 89-06, California Strong Motion Instrumentation Program*, Cal. Dept. of Conservation, Div. of Mines and Geology, Sacramento.