Research on energy spectrum of elastic SDOF

TENG Jun DONG Zhijun
Harbin Institute University of Technology Shenzhen School, Shenzhen 518055,China
Email:tengj@hit.edu.cn

ABSTRACT:
According to the spectrum analysis theory the energy spectrum including input energy spectrum, damping energy spectrum, hysteric energy spectrum and kinetic energy spectrum of elastic SDOF subjected to earthquake had been obtained. 160 strong earthquake records were used to get the input energy spectrum in four site classifications. The results have shown that the input energy spectrum curve can be divided to three parts: the ascending part for short periods, the plateau part for moderate period and the descending part for long period. The site classifications have some influence to the plateau part. Simplified formulations for the three parts of the spectrum curve had been regressed. The rationality of the simplified formulations had been certified.
KEYWORDS: elastic SDOF, energy spectrum, four site classifications, simplified formulations

1 Preview

The energy based seismic design methodology not only takes into account the acceleration, velocity and displacement response that subjected to earthquake of the structure, but also of the duration, the damping energy and the hysteric energy. In a sense it can consider the damage accumulation and the performance degradation of the structure under strong earthquake. It can more accurately describe the seismic problem than the methodology based on capability or strength.
In 1950s, G.W.Housner proposed the energy concept to analysis the seismic response[1]. Used the energy based seismic analysis method on the limit design of water tower which can be considered as a single degree of freedom. He thought if the dissipation capacity is not less than the input energy and the energy can be distributed rationally in the structures it will can be avoided the destroy and collapse. After that, the researchers did some work on the energy calculation, structural energy distribution and the energy dissipation. The results have shown that the energy based seismic design method can make a rational explanation for the structural response under the near earthquakes[2-7].
To research the energy response and energy spectrum for SDOF is helpful to know the regularity and characteristics of structural energy response. It is the essential problem for the energy based on seismic design. In this paper, according to the Chinese seismic code (GB50011-2001)[8], 160 strong earthquake records in 4 site classifications have been used to obtain the input energy spectrum. The characteristics and the influence of the site classification had been studied. A simplified formulation for input energy spectrum has been regressed for practice use.

2 Basic concept of energy method

2.1 The energy calculation for elastic SDOF

The balance function for the elastic SDOF under earthquake show as Eq.(1)

$$m\ddot{x} + c\dot{x} + f(x) = -m\ddot{x}_g \qquad (1)$$

where m denotes the mass for system; c denotes the damp coefficient; $f(x)$ denotes the elastic restore force, $f(x)=k$, k denotes the stiffness of system; x , $\dot{x}$ and $\ddot{x}$ denote the relative displacement, velocity and acceleration respectively; $\ddot{x}_g$ denotes the ground motion.

Multiply at both sides with relative speed $\dot{x}$ and integrate from 0 to t which is the duration time of earthquake, the energy response balance equation can be obtained as follow:

$$\int_0^t m\ddot{x}\dot{x}dt + \int_0^t c\dot{x}\dot{x}dt + \int_0^t f(x)\dot{x}dt = -\int_0^t m\ddot{x}_g\dot{x}dt \tag{2}$$

or be expressed as

$$E_K + E_D + E_H = E_I \tag{3}$$

where $E_K = \int_0^t m\ddot{x}\dot{x}dt$ denotes the kinetic energy of the system; $E_D = \int_0^t c\dot{x}\dot{x}dt$ denotes the damping energy of the system; $E_H = \int_0^t f(x)\dot{x}dt$ denotes the hysteric energy of the system, including elastic deform energy E_E and plastic deform energy E_P. In the elastic system, there is only elastic deform energy E_E; $E_I = -\int_0^t m\ddot{x}_g\dot{x}dt$ denotes the input energy; t denotes the duration of earthquake record.

Eq.(3) tells that the earthquake action can be considered as a procedure that the energy input, translation, storage and dissipation. The earthquake input the energy into the structures. And than some of the energy is translated by movement and elastic deformation while some of the energy is stored by plastic deformation and some of them is dissipated by structural damping. The kinetic energy and elastic energy come to zero after the earthquake and they could not cause damage to the structures. But the damping energy and plastic energy cause the plastic deformation and the performance degradation. Under the certain input, the aftershock performance of the structure is up to the plastic deform ability and the damping dissipation ability. If the structure has a strong ability of deform and energy dissipation, it will be serviceable with some damage after the earthquake. Contrarily the structure will be destroyed or even be collapse.

2.2 The solution of energy spectrum

By the record of discrete earthquakes, from Eq. (1) the relative velocity can be got by doing Duhamel integration as follows:

$$\dot{x}(t) = -\int_0^t \ddot{x}_g(\tau) e^{-\zeta\omega(t-\tau)} \bullet \sin\omega_D(t-\tau)d\tau \tag{4}$$

where ζ is the structural damping ratio, ω_D is damped natural frequency.

The relative input energy, the kinetic energy, the damping energy and the hysteric energy can be obtained as follows:

$$E_I = m\int_0^t \ddot{x}_g \left\{\int_0^t \ddot{x}_g(\tau) e^{-\zeta\omega(t-\tau)} \bullet \sin\omega_D(t-\tau)d\tau\right\}dt \tag{5}$$

$$E_K = -m\int_0^t \ddot{x} \left\{\int_0^t \ddot{x}_g(\tau) e^{-\zeta\omega(t-\tau)} \bullet \sin\omega_D(t-\tau)d\tau\right\}dt \tag{6}$$

$$E_D = \int_0^t c\left\{\int_0^t \ddot{x}_g(\tau) e^{-\zeta\omega(t-\tau)} \bullet \sin\omega_D(t-\tau)d\tau\right\}^2 dt \tag{7}$$

$$E_H = \int_0^t f(x)\left\{\int_0^t \ddot{x}_g(\tau) e^{-\zeta\omega(t-\tau)} \bullet \sin\omega_D(t-\tau)d\tau\right\}dt \tag{8}$$

Suppose there are a series of N single degree of freedom systems with natural periods T_i ($i=1,2,\ldots,N$) and the same damping ratios. Under a given earthquake record $\ddot{x}_g$, the maximum input energy response, the maximum kinetic energy response, the maximum damping energy response and the maximum hysteric energy response can be got as follows respectively $S_{E_I}(T_i,\zeta) = |E_I|_{\max}$, $S_{E_K}(T_i,\zeta) = |E_K|_{\max}$, $S_{E_D}(T_i,\zeta) = |E_D|_{\max}$ and $S_{E_H}(T_i,\zeta) = |E_H|_{\max}$,from the numerical calculation the structural input energy spectrum, the kinetic

energy spectrum, the damping energy spectrum and the hysteric energy spectrum can be obtained
The energy response and energy spectra of the mass of one single degree of freedom is 1kg, period 1s, and damping ratio 5% under an El Centro earthquake wave, the duration of which is 54s .The energy time history of the structure under El Centro record is shown in Fig.1 and the energy spectrum are shown in Fig.2. From the Fig.2 we know that the input energy spectrum of the elastic structure envelops the kinetic energy spectrum, the damping energy spectrum and the hysteric energy spectrum. It can be considered as the representative value of the energy response. So it is valuable to research the characteristics and the regularity of the input energy spectrum. The following research is based on the input energy spectrum of unit massed single degree of freedom systems.

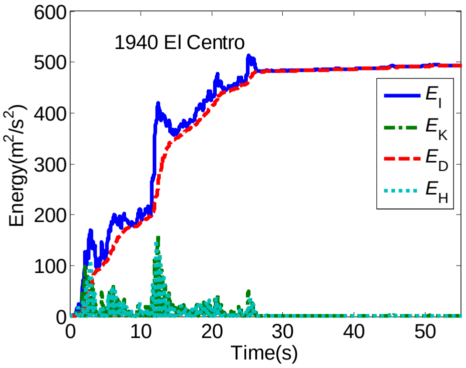


Fig.1 The energy time history of the structure under El Centro record

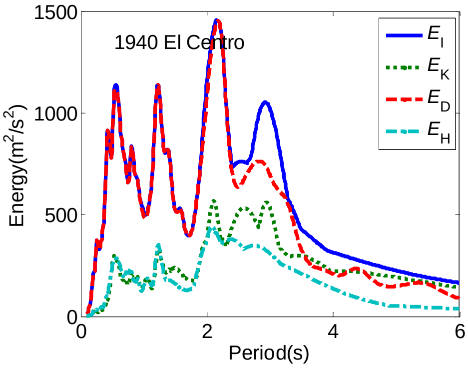


Fig.2 The energy spectrum under El Centro record

3 The selection of earthquake records

There are four site classifications in Chinese seismic design code which considered the equivalent velocity of shear wave and the thickness of the covered soil. It is similar to the GMX's C3. To assure the reliability of the spectrum analysis enough strong earthquake records should be selected. In this paper 40 records were selected for each site from the PEER strong earthquake records database. The records can take into account the most important influence factors of earthquake, such as the intensity, the duration, the frequency components, the magnitude and the hypocentral distance. The statistics of the records are shown in table 1. The rang of PGA and the duration is shown in figure 3. The rang of magnitude and hypocentral distance is shown in figure 4.

Table 1 The statistics of the earthquake records

Site classification	Records number	Magnitude(M)	Hypocentral distance(km)	PGA (g)	Duration (s)
Rock &shallow soil	40	4.7-7.3	3-162.6	0.008-0.608	10.085-149.04
Deep narrow soil	40	4.7-7.3	3.4-172.5	0.009-0.617	6.85-152.98
Deep broad soil	40	5.4-7.4	2-195	0.01-0.728	11.14-138.55
Soft deep soil	40	5.2-7.6	1.2-193.3	0.025-0.694	11.02-150.4

4 The establishment of the spectrum shape

The unit massed structure systems with 5 percent damp had been used to calculate the input energy spectrum for each site classification. The PGA of earthquake records is 35cm/s^2 and the period domain of structure system is 0-6s. The input spectrum curves and its statistic curves are shown in Fig.5. The statistic curves include the mean value curves, the mean value plus standard deviation curves, the mean value plus double deviation curves and the mean plus three times deviation curves.

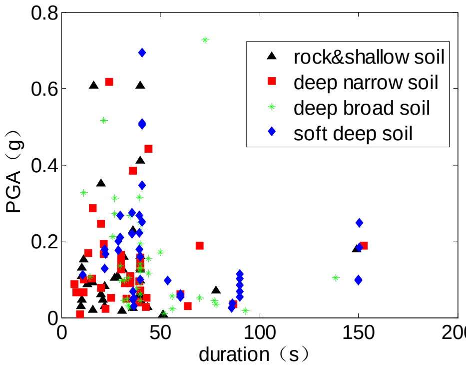


Fig.3 The range of PGA and duration

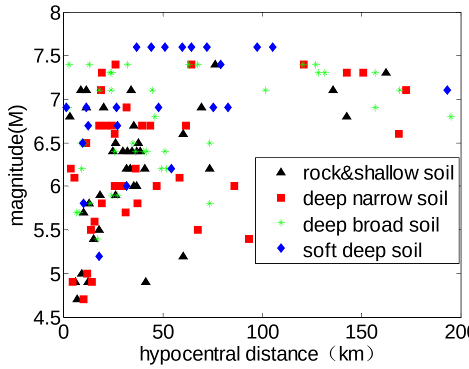
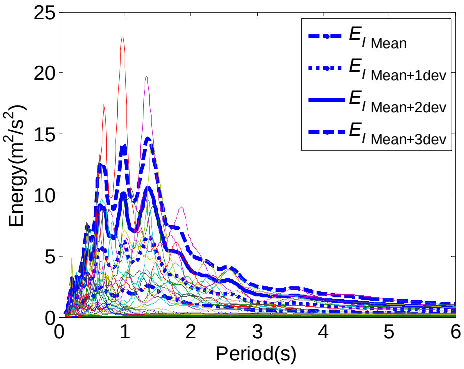
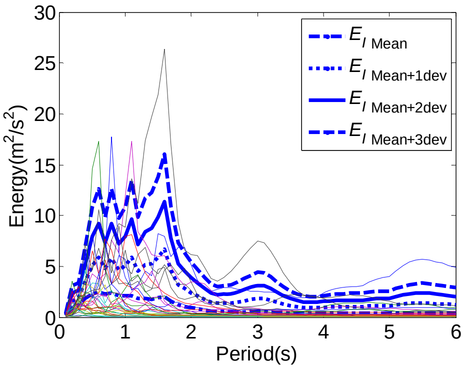


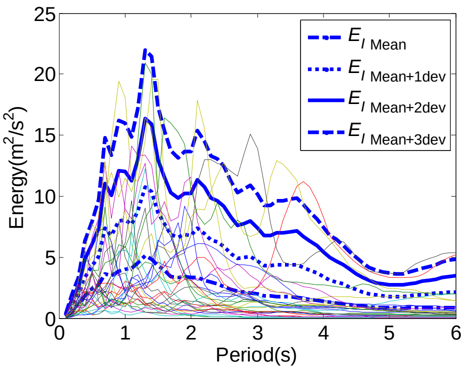
Fig. 4 The range of magnitude and hypocentral distance



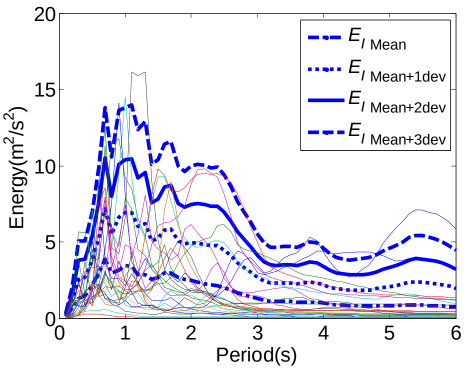
(a) Input energy spectra in rock & shallow soil



(b) Input energy spectra in deep narrow soil



(c) Input energy spectra in deep broad soil



(d) Input energy spectra in soft deep soil

Fig.5 The input energy spectra and the statistics

From Fig.5 we know that there is a big difference of each record. The mean plus double standard deviation is proposed as the representative value of input energy spectrum for enough reliability and not to be excessively conservative. The representative curves for each site classification are shown in Fig.6 and the mean curve is shown in Fig.7.

From Fig.6 and Fig.7 we know that the energy spectrum curves can be divided into three parts. 1. Ascending part: in the short periods, the input energy increase as the period. The four curves are almost superposition. 2. The plateau part: in the moderate periods, the curves surge at some value. The length of this part goes to long as the soil becomes soft. 3. The descending part: in the long periods, the curves decline as the period becomes long and the descending velocity is similarly.

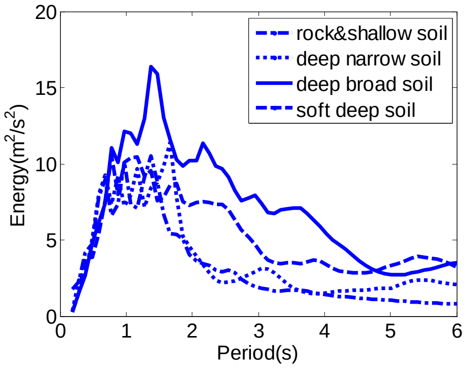


Fig.6 The representative curves for each site

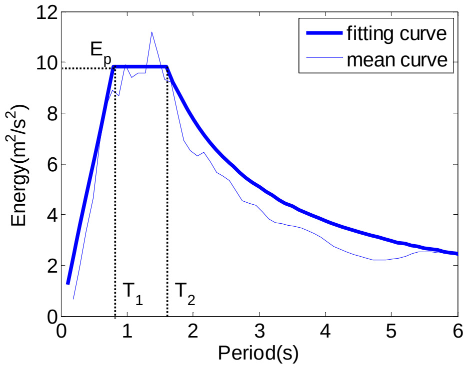
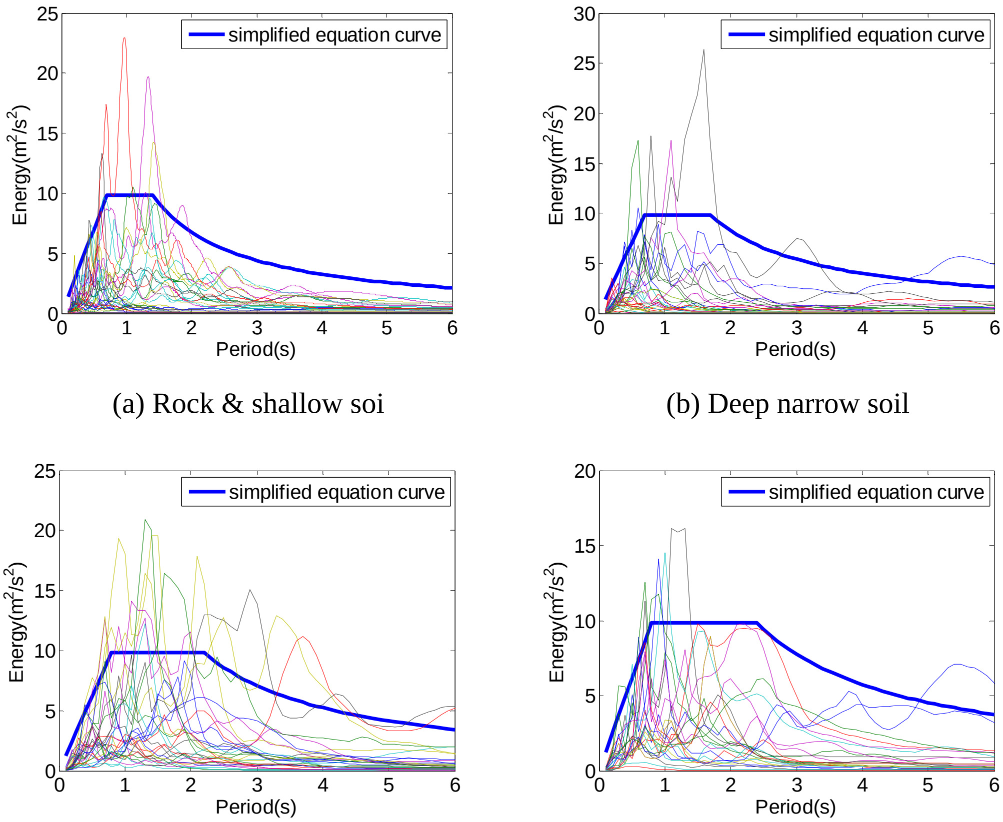


Fig.7 The mean curve and fitting curve for energy spectra

5 The regression of the simplified calculation formulation for input energy spectrum

To observe the representative curves in Fig.6, we know that the shape of the energy spectrum curve is



(a) Rock & shallow soi

(b) Deep narrow soil

(c) Deep broad soil

(d) Soft deep soil

Fig.8 The verification for the simplified equation in each site

almost the same except the length of the plateau part. Namely there is some difference of T_1 and T_2 for each site classification. The statistic of the T_1 and T_2 of four site classifications is shown in table 2. The simplified calculation formulation of input energy spectrum has been obtained by regressed for the three parts separately (Eq.9). The regression curve is shown in Fig.7.

Table 2 T_1 and T_2 of input energy spectra curves for each site

Site classification	Rock & shallow soil	Deep narrow soil	Deep broad soil	Soft deep soil
T_1（s）	0.7	0.7	0.8	0.8
T_2（s）	1.4	1.7	2.2	2.4

$$E_I(T)=E_p\frac{T}{T_1} \qquad 0\le T\le T_1$$
$$E_I(T)=E_p \qquad T_1<T<T_2 \qquad (9)$$
$$E_I(T)=E_p\left(\frac{T_2}{T}\right)^k \qquad T\ge T_2$$

where E_p is proposed to use the mean of plateau part of the representative curves. k can be regressed according to the descending part of the representative curves. In this paper E_p=9.82m^2/s^2，k=1.05. The comparison of the curve calculated by the regressed simplified equations and the curve of each record is shown in Fig.8. The simplified equation curve can envelop most of the curves and not to be excessively conservative. In a way it is reasonable.

6 Conclusions

The input energy spectrum of 160 records in 4 site classifications had been got by numerical calculation. The characteristics of input energy spectrum had been studied and a simplified formulation was proposed by regression. The main conclusions were got as follows:

1 The energy spectrum of SDOF includes input energy spectrum, kinetic energy spectrum, damp energy spectrum and hysteric energy spectrum. The input energy spectrum can envelop the other spectrum can be considered as the main representative value.

2 The input energy spectrum can be divided into three parts: the ascending part, the plateau part and the descending part. The site classifications have some influence on the length of the plateau part. The length goes to be longer as the soil becomes soft.

3 A simplified formulation to calculate input energy spectrum had been proposed by regression. By comparison the input energy spectrum got by simplified formulation and the spectrum of the earthquake records on each site classification, we know that the simplified formulation can envelop most of the spectra. In a way the formulation is reasonable.

REFERENCES

[1] Housner G. Limit design of structures to resist earthquakes [A]. *Proceedings of the first world conference on earthquake engineering*[C], 1956

[2] Erberik A and Sucuoglu H. Seismic energy dissipation in deteriorating systems through low-cycle fatigue [J]. *Earthquake Engineering & Structural Dynamics*, 2004: vol.33, 49~67

[3] Luis D. Decanini, Fabrizio Mollaioli. Formulation of elastic earthquake input energy spectrum [J]. *Earthquake Engineering & Structural Dynamics*, 1998:vol. 27, 1503~1522

[4] Riddell R and Garcia J E. Hysteretic energy spectrum and damage control [J]. *Earthquake Engineering & Structural Dynamics*, 2001:vol.30, 1791~1816,

[5] Manfredi G. Evaluation of seismic energy demand [J]. *Earthquake Engineering & Structural Dynamics*, 1995:vol.24, 1195~1213

[6] LIU Zhefeng. SHEN Pusheng. Research on the input energy spectrum subjected to earthquakes. *Earthquake Resistant Engineering and Retrofitting*. 2006;（4）1-5 (in Chinese)

[7] CHENG Guangyu and YE Lieping. The input energy spectrum of SDOF under earthquake. *Earthquake Resistant Engineering and Retrofitting*. 2006;（5）1-8 (in Chinese)

[8] Code for seismic design of buildings (GB50011－2001). Ministry of Housing and Urban-Rural Construction. Beijing, 2001