EFFECTIVE RIGIDITY OF REINFORCED CONCRETE ELEMENTS IN SEISMIC ANALYSIS AND DESIGN

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ABSTRACT :

Seismic analysis and design of reinforced concrete structures are performed based on linear response, however it is universally accepted that under severe earthquakes inelastic response and cracking is accepted. Therefore element properties should reflect this condition and inertias of beams and columns should be reduced accordingly. Several procedures are suggested to considered effective rigidity: Priestley (2003), ACI 318S-05, FEMA 356, Paulay and Priestley (1992). In this work the convenience to consider the effective stiffness of elements is demonstrated. However since there is not a uniform criteria, a comparison of these procedures was performed. Most world seismic standards do not establish effective stiffness for seismic analysis, although all of them accept inelastic incursions. Therefore it is useful to find a common or reasonable criterion to reduce inertias. Priestley (2003) procedure is applied to find the effective rigidity of elements, which are dependent on element strength. A direct consequence of reduced inertias is larger elastic displacements. On the other hand seismic standards specify displacements are computed by factoring elastic displacements from analysis, therefore using reduced inertias a substantial increase of estimated displacements would occur produced turning analysis into over conservative.

KEYWORDS: Effective Stiffness, seismic analysis, member properties, lateral displacements.

1.0. INTRODUCTION

Most Seismic Design Codes do not precise effective stiffness to be used in seismic analysis for structures of reinforced concrete elements, therefore uncracked section properties are usually considered in computing structural stiffness. This is actually not the case because cracking occurs not only under gravity loads in beams but in seismic events of intensity lower than that of design earthquake. Uncracked stiffness will never be fully recovered during or after seismic response and it can be said it is not a useful estimation of effective stiffness, Priestley (2003).

Priestley (2003) points out that using modal analysis with uncracked sections stiffness for different elements makes it impossible to obtain precise seismic forces, even response within elastic range. Computed elastic periods are probably wrong, and moreover, force distribution throughout the structure, which depends on relative stuffiness of the elements may be excessively mistaken.

It is also known flexural cracking varies along the element length, therefore moment of inertia (second moment area), I, also varies along element length. In each cross section, moment of inertia, I, depends on the magnitude and sign of the bending moment, the amount of reinforcement, cross section geometry and axial load. There are other factors that cause more variations of element stiffness, such as: effects of tension in the concrete sections in between cracks, diagonal cracking due to shear stresses, intensity and direction of axial load, etc (Paulay & Priestley 1992). It is not of practical use to evaluate geometrical properties in various cross sections along each element, therefore, a reasonable average value must be adopted.

Some design codes recognize the influence of cracking. They consider stiffness of the cracked section EI_e proportional to the stiffness of the gross uncracked section EI_g, specifying reduction factors to be applied to the stiffness of the uncracked cross section. However, Priestley in his 2003 study indicates the reductions factors proposed by these codes are yet inadequate to represent stiffness to a precision degree appropriate to justify a modal analysis, since the influence of bending steel ratio and that of the axial load are not being considered, therefore, stiffness of the elements is assumed independent of the bending strength, which in his opinion, is not valid. Priestley (2003) sustains that experimental evidence and detailed analytical results have demonstrated that yield curvature is independent of strength and therefore stiffness is directly proportional to yield strength with a constant yield curvature

This research has followed Priestley´s proposal with the purpose to compare results obtained with his proposal with results obtained with reduction factors proposed by codes and then try to balance precision with simplicity.

2. REDUCTION FACTORS IN CODES AND PROPOSALS

Among the codes to propose reduction factors of gross moments of inertia are listed as allowed values to be used in a second order analysis, but are also used when a first order general analysis of frames is being made to compute lateral relative story displacements.

Table 1. Reduction Factors

Element	*New Zealand Code*	*ACI 318S-05 Design Code*
Beams	$0.35I_g$	$0.35\ I_g$
Columns	$0.40 – 0.70I_g$	$0.70\ I_g$
Walls uncracked	----------------	$0.70\ I_g$
Walls cracked	----------------	$0.35\ I_g$

FEMA (2000) proposes similar reductions, although for bending shear and axial stiffness for prestressed, nonprestressed beams, columns and walls. These range from 0.5 to 0.8 EI_g .

2.2. Reduction Factors proposed by Paulay and Priestley (1992)

Table 2. Element Effective Moment of Inertia (Paulay y Priestley, 1992)

Element	**Range of I_e**	**I_e recommended**
Rectangular Beams	$0.30\text{-}0.50\ I_g$	$0.40\ I_g$
T and L Beams	$0.25\text{-}0.45\ I_g$	$0.35\ I_g$
Columns		
$P > 0.5f'cA_g$	$0.70\text{-}0.90\ I_g$	$0.80\ I_g$
$P = 0.2f'cA_g$	$0.50\text{-}0.70\ I_g$	$0.60\ I_g$
$P = -0.05f'cA_g$	$0.30\text{-}0.50\ I_g$	$0.40\ I_g$

3. EVALUATION OF SECTION STIFFNESS BASED ON MOMENT CURVATURE

Section stiffness can be evaluated based on moment curvature according to beam equation. Eqn. 3.1.

$$EI = \frac{M_N}{\phi_y} \tag{3.1}$$

M_N is nominal moment capacity of the section; ϕ_y is yield curvature of equivalent bilinear representation of the moment curvature diagram. It is accepted that linearization of moment curvature relation is given by an initial straight line (elastic) through “first yield”, up to the nominal bending strength, M_N, and a second line post yield connecting to the ultimate strength and curvature. “First Yield” of a section is defined as the Moment M_y and curvature ϕ'_y when the section first reaches yield strain in tension, ε_y, or the extreme compression fiber reaches a strain of 0.002, whatever occurs first.

Nominal bending strength, M_N develops when extreme compression fiber reaches 0.004, or when strain in tension reaches 0.015 whatever comes first. (Figure 1). Thus yield curvature is given by

$$\phi_y = \frac{M_N}{M_y}\phi_y^{'} \qquad (3.2)$$

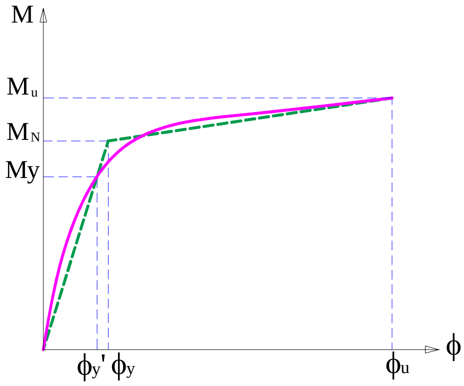

Figure 1. Moment-curvature relationship and bilinear approximation

Element stiffness depends on variation of curvature along the element, not only in the critical section where yielding occurs.

4.0. HYPOTHESIS OF STIFFNESS INDEPENDENT OF STRENGTH

This implies, yield curvature is directly proportional to bending strength, M_N. (See Figure 2(a))

$$\phi_y = \frac{M_N}{EI} \qquad (4.1)$$

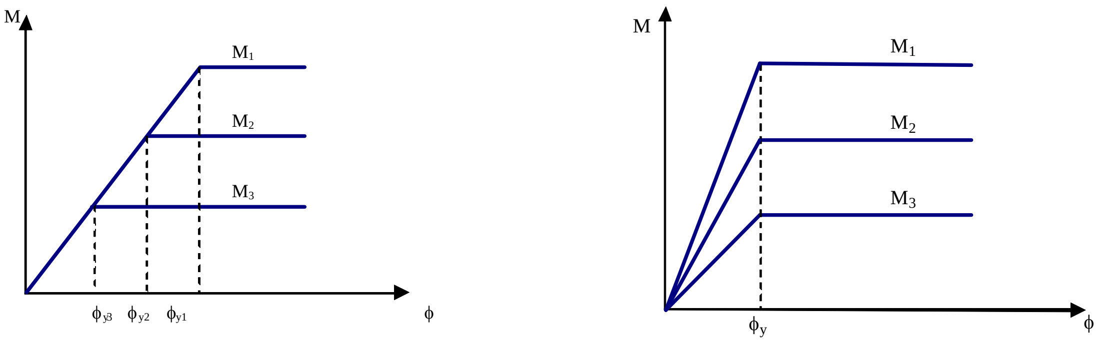

Figure 2. (a) Stiffness independent of strength. (b) Stiffness depending on strength. Constant yield curvature

The consequence of this in conventional seismic design is that stiffness of the structure can be predicted al the beginning of design process considering section properties of uncracked sections or properties of the effective section and thus compute period of vibration.

Later, this period is used to obtain design spectral acceleration and give the structure appropriate strength required by this associated lateral forces. This greatly simplifies the design process, but as Priestley (2003) shows, the initial hypothesis of stiffness independent of strength is in fact wrong

5.0. HYPOTHESIS OF STIFFNESS DEPENDING ON STRENGTH

Research from Priestley y Kowalsky (2000) and Priestley, (2003) have demonstrated through experimental testing and detailed analytical computations, that stiffness actually cannot be supposed independently of strength. On the contrary, it has been found elements yield curvature is effectively independent of strength and can be taken as a constant for the known section dimensions. Therefore, it can be concluded that stiffness is directly proportional to bending strength as shown in Eqn. 3.1. This relationship is illustrated in Figure 2(b).

This way it is not possible to perform a precise analysis of the elastic structural periods or elastic distributions of required strength through the structure until final strength of the elements has been determined. This means conventional seismic design based on elastic stiffness of the elements and considerations based in force must be an iterative process where stiffness of the elements are changed in each iteration.

Priestley (2003) indicates that yield curvature can be found as a function of geometric properties of the elements:

-	Circular Walls	$\phi_y = 2.25\ \varepsilon_y/D$	(5.1a)
-	Rectangular Columns	$\phi_y = 2.10\ \varepsilon_y/h_c$	(5.1b)
-	Rectangular Walls in cantilever	$\phi_y = 2.00\ \varepsilon_y/l_w$	(5.1c)
-	T Beams	$\phi_y = 1.70\ \varepsilon_y/h_b$	(5.1d)

6.0 PERIOD COMPUTATION, MAXIMUM DISTORTION AND DUCTILITY DEMAND

Periods obtained from seismic analysis taken into account Priestley (2003) proposal are larger than periods obtained considering stiffness of the uncracked section of the elements. A relation follows:

$$T_2 = \frac{T_1}{\sqrt{K_e/K_g}} \tag{6.1}$$

where T_1 is period without cracking, T_2, period with Priestley's assumption, K_e, effective stiffness considering Priestley's assumption, K_g, stiffness without cracking

Therefore, maximum distortion computed using real effective stiffness is larger and much more than indicated by codes. If a building is designed based on stiffness of uncracked sections, a short period building with an apparent large base shear will be obtained, in consequence is not a conservative design, because the building which will really have larger distortions mostly unacceptable. In addition, displacement ductility demand will be lower since it was designed for a larger shear than it should, yield displacement will be larger than that obtained with a lower base shear and therefore ductility will tend to be reduced (Figure 3). Yield displacement will be equal to:

$$\delta_{y_2} = \frac{\delta_{y1}}{K_e/K_g} \tag{6.2}$$

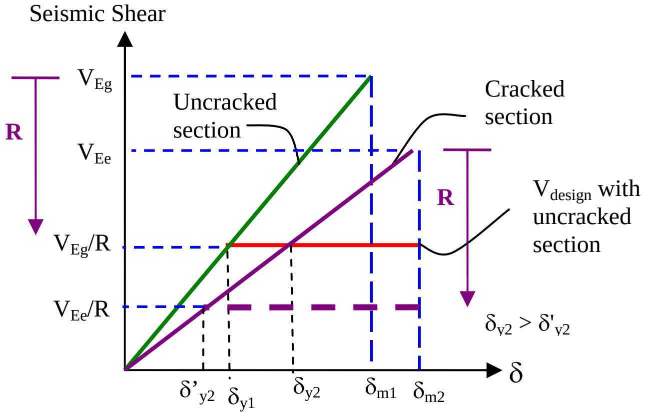

Figure 3. Relation between shear and lateral displacement

The actual maximum displacement can be obtained considering the displacement spectrum is directly proportional to period which is a consequence of considering the acceleration spectrum based on the hypothesis of a constant velocity spectrum. Therefore the relation of both maximum displacements, considering both cracked and uncracked sections is:

$$\delta_{m2} = \frac{T_2}{T_1}\delta_{m1} \tag{6.3}$$

7.0. APPLICATION OF PRIESTLEY CRITERION TO SAMPLE BUILDING

A four story existing building in Lima was chosen to conduct this research, Burgos (2007). The structure is a space frame plus a single shear wall in the staircase. (Figure 4).

Moment curvature diagrams were computed for the main structural elements with different steel ratios. Effective stiffness was computed based on the bilinear approximation for moment curvature. Eqn. 5.1 (a-d) are rather useful to compute effective stiffness without having to define moment curvature curves, particularly for beams because knowing moment capacity from analysis $M_N = M_u/\phi$, effective stiffness can be found after few iterations with Eqn (3.1)

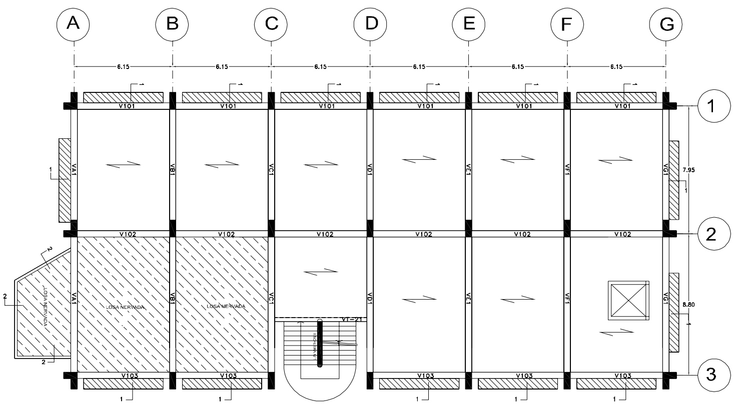

Figure 4. Typical plan of studied building

7.1. L Shaped, Rectangular (.40 x 1.05m) Column and Beam (.40 x .75m)

Moment curvature for X, and Y directions were computed. UCFyber (Chadwell, 1998) was used and compared with other programs for moment curvature. In Figure 5(a) influence of axial load on strength of L column can be appreciated. There is almost no variation of factors in either X or Y direction or moment direction. Therefore a unique reduction factor can be taken for X or Y direction properties. Effective stiffness ratios to uncracked increases with axial load and reinforcement, ranging from 0.23 to 0.43.

In contrast to L shaped columns, rectangular columns show an increment in effective stiffness ratios with increased axial load and steel ratio varying between 10 to 20% in X and Y directions. This suggest a different factor should be used in each direction for seismic analysis.

For rectangular beams EI / EI_g effective stiffness factors are presented in Table 3. These results indicate:

- Effective stiffness increases proportional to reinforcement. (Figure 5(b)
- For sections with different positive and negative reinforcement an average relation is used in seismic analysis.
- Reduction factor corresponding to minimum reinforcement is smaller than required by codes.

Table 3. Effective stiffness reduction factors as a function of steel ratio, nominal bending strength of beam.

Steel ratio	MN(-)	MN(+)	EI/EIg (-)	EI/EIg (+)	EI/EIg $_{average}$
0.35%-0.35%	28.33	28.33	0.21	0.21	0.21
0.45%-0.45%	37.22	37.22	0.27	0.27	0.27
0.45%-0.33%	36.90	28.49	0.27	0.21	0.24
0.52%-0.33%	41.88	28.85	0.30	0.21	0.26
0.63%-0.35%	49.71	30.37	0.36	0.23	0.29
0.94%-0.94%	67.97	67.97	0.47	0.47	0.47
1.39%-1.39%	95.29	95.29	0.60	0.60	0.60

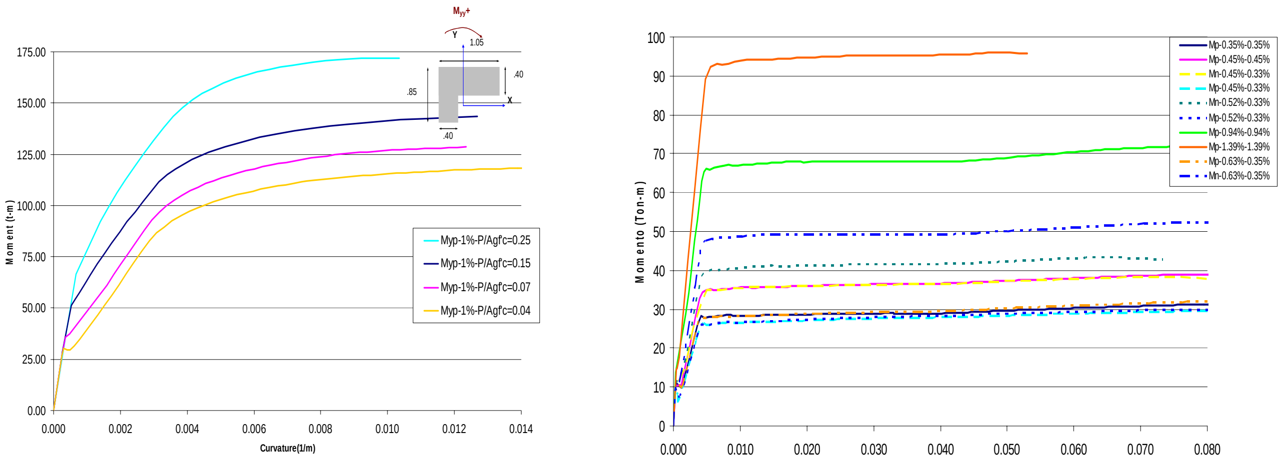

Figure 5. (a) L column (p= 1%, Y axis) Variable axial load. (b) Rectangular beam. Variable steel ratio

8.0. EFFECTIVE STIFFNESS RESULTS COMPARISON: PRIESTLEY (2003); PAULAY AND PRIESTLEY (1992); UNCRACKED SECTIONS

Priestley's procedure is rather complex and requires more effort for it is an iterative process where element stiffness is updated in each iteration until strength in all elements (bending moments and shears) do not change significantly. In other words, for each iteration step needed element strength equals a steel ratio, and this in time corresponds to an effective stiffness, with this analysis is done again and again until a final strength of the elements is reached.

To compare precision with simplicity and analysis of the building was made using effective stiffness obtained with Priestley's criterion (2003) and using Paulay and Priestley's reduction factors, which are similar to those of New Zealand standard

8.1. Reduction Factors

Figure 6 shows reduction factors in beams and columns after Priestley (2003) iterative procedure using reduction factors computed after Secc. 7. One frame in each direction of the plan. All beams in the Y direction have a reduction factor of 0.22, and correspond to minimum reinforcement. However in the X direction the factor is larger for beams in floors 1, 2 and 3.

8.2 Periods

Periods double using Priestley's criteria(2003) with respect to uncracked sections. Paulay and Priestley's (1992) range somehow in between.

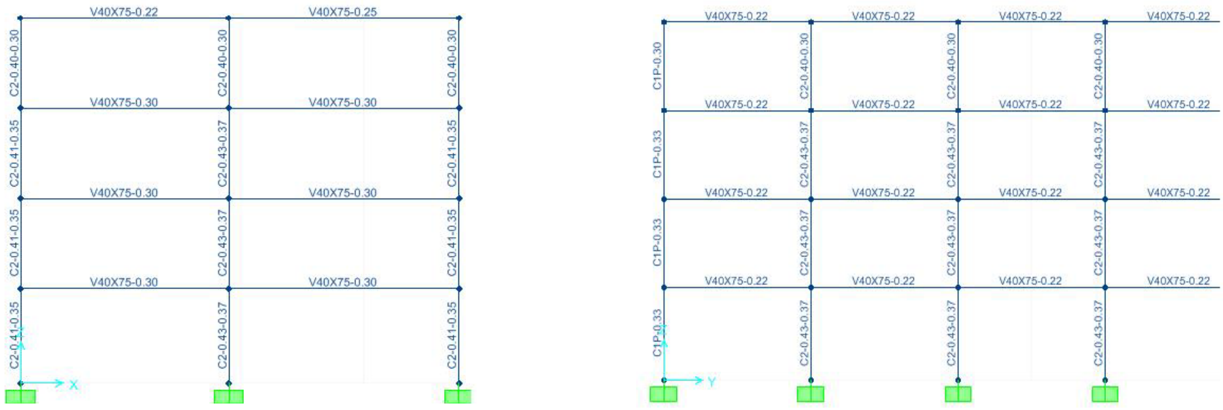

Figure 6. Frame B (X direction) and Frame 2 (Y direction). Reduction factors in beams and columns.

Reduction factor using Paulay and Priestley (1992) criteria (Table 2) are: for rectangular beams = 0.4; for columns with $P/A_g f'_c \leq 0.20 = 0.40$. These modifying factors were used for following seismic analysis

8.3 Negative Moments in beams (1.2D+L+E)

Change observed in negative moments, using reduction factors by Priestley (2003) and Paulay y Priestley (1992), is small for beams in direction X. On the other hand for beams in direction Y, there is almost no variation because minimum reinforcement controls design.

8.4. Maximum Displacements and Distortions

Table 4. Drifts or distortions in % for each reduction criteria for effective.

Story	Priestley (2003)		Paulay & Priestley (1992)		Uncracked section	
	Drift X%	**Drift Y%**	**Drift X%**	**Drift Y%**	**Drift X%**	**Drift Y%**
1	1.16%	1.50%	0.92%	0.78%	0.50%	0.45%
2	1.08%	1.34%	1.07%	1.10%	0.65%	0.69%
3	0.98%	1.63%	0.99%	1.47%	0.68%	0.92%
4	0.51%	1.21%	0.52%	1.13%	0.39%	0.71%

A significant increment in computed displacements can be observed from distortions in Table 4. Between double and triple as those obtained with uncracked sections. This has strong design implications because buildings may not comply with allowable displacements. All depends on effective stiffness used. Peruvian Seismic Code has proven successful after stringent displacements requirements were introduced in 1997 (Pique, 2004) but using reduced stiffness may be excessive. This needs further research to determine which is appropriate or changes to be introduced in seismic design standards

CONCLUSIONS

1. Cracking must be considered in seismic analysis of building structures and thus to get realistic distortions, in nonlinear range, since these are computed from an elastic analysis. It has been shown that a seismic analysis with uncracked sections, design moments are larger than with other two evaluated procedures and therefore it is possible to end up with a conservative design in strength but with larger distortions.

2. Results using effective stiffness with factors given by Paulay and Priestley (1992) have a small variation as compared to results obtained by Priestley's iterative procedure (2003). It can be concluded there exists a balance between precision and simplicity when Paulay and Priestley's (1992) reduction factors are used. If the seismic standard does not include requirements respect to the effective stiffness to be consider for reinforced concrete structures a simple approach would be to use reduction factors proposed by Paulay and Priestley's (1992) which take into account the influence of axial load.

3. For a more precise analysis of structural response or for retrofitting and evaluation of existing structures criterion proposed by Priestley's (2003) could be convenient and even necessary.

4. When design is controlled by minimum reinforcement, particularly in beams, special attention should be given to computation of real periods and maximum distortions. Using only Priestley's (2003) methodology it has been observed that effective stiffness of a beam with minimum steel ratio is much lower than that obtained with the proposed reduction factors. As a consequence real periods and actual maximum distortions can be even larger.

5. It has been found in computed moment curvature diagrams the hypothesis that yield curvature is independent from element strength is correct and effective stiffness EI is dependent of strength, which in turn is a function of axial load and steel ratio.

6. Seismic design standards must be calibrated together with appropriate reduction factors to estimate and control lateral deflections. Since professional practice usually uses uncracked stiffness, distortions are lower and have proven adequate, but if reduced stiffness is used then current limits might be larger, to accommodate larger computed distortions.

REFERENCES

- American Concrete Institute. (2005) "Building Code Requirements for Structural Concrete". (ACI 318S – 05). Michigan. USA
- Burgos, M. (2007) Study of Capacity Design for Reinforced Concrete Frames as Design Alternative in Peruvian Standards. Master's thesis supervised by Dr. Javier Pique. Faculty of Civil Engineering. National University of Engineering. Lima. Peru
- Crowley, H. (2003), "Periods of vibration for displacement-based assessment of RC buildings", Thesis for Master degree in Earthquake Engineering supervised by Dr. Rui Pinho, European School of Advanced Studies in Reduction of Seismic Risk, Rose School, Pavia, Italy, pp. 41-56.
- Chadwell, Ch. (1998) "UCFyber cross section analysis software for structural engineers", University of California at Berkeley. USA.
- Elwood, K. J. & Eberhard, M. O. (2006). Effective Stiffness of Reinforced Concrete Columns, Research Digest Nº 2006-1. Pacific Earthquake Engineering Research Center PEER. University of California at Berkeley. USA.
- Federal Emergency Management Agency FEMA 356 (2000) "Seismic Rehabilitation Guidelines".
- Ministry of Housing Peru (2003) "Earthquake Resistant Design" Peruvian Standard E-030. (in Spanish)
- New Zealand Standard NZS 3101: Part 1 (1995), "Code of practice for the design of concrete structures", New Zealand Standards Association, Wellington, New Zealand
- Park, R., Paulay, T. (1975), Reinforced Concrete Structures, Wiley Insterscience.
- Paulay, T , Priestley M. J. N. (1992). Seismic Design of Reinforced Concrete and Masonry Buildings. Wiley Interscience
- Pique, J. Martel, P. (2004) "How Peruvian Seismic Code greatly improved building response to real earthquakes". 13 WCEE. Vancouver. Canada. Paper 1825.
- Priestley, M. J. N. (2003), "Myths and fallacies in earthquake engineering, revisited" (In the Ninth Mallet Milne Lecture). Rose School, Pavia, Italy., 9-31.