Asymmetric Seismic Response of SDOF Systems with Strength Deterioration

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ABSTRACT :

Buildings with soft first stories may incur large deformations due to strength deterioration, which is caused by *P*-*δ* effect. The inelastic deformation of those buildings tends to shift to a single direction. This paper evaluates such biased response caused by strength deterioration and asymmetric property of input ground motion using SDOF systems. We defined a limit strength ratio R_2 wherein the ductility factors for strength degradation systems are obtained by adding 2 to those for elasto-plastic systems. When R is larger than R_2, the SDOF systems exhibit biased response and the response deformations lean heavily towards a single direction in any ground motion.

KEYWORDS: Asymmetric Seismic, SDOF Systems, Strength Deterioration, *P*-*δ* effect

1. Introduction

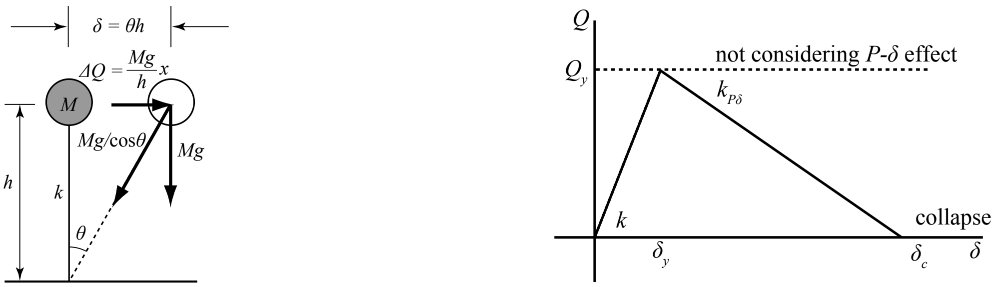
There are buildings with strength deterioration due to compression failure of concrete column or *P*-*δ* effect. Those buildings easily collapse under strong earthquake. Especially buildings with soft first story tend to incur large deformations because of *P*-*δ* effect.
In previous researches, some methods are developed to avoid the building collapse due to strength deterioration. Williamson (2003) evaluated the role of damage accumulation and *P*-*δ* effects on the response of inelastic systems by parametric analysis under various earthquakes. Miranda and Akkar (2003) formulated the limit strength ratio to prevent the collapse of structures with strength deterioration as parameters with natural period and postyield stiffness. However it is difficult to determine limit strength to avoid collapse of those structures because of variation of inelastic response. The inelastic deformation of those buildings tends to shift to a single direction, although there are few investigations on such a bias in the response. Moreover, such response bias may be caused by the bias of input ground motion itself, for it is known that some near field ground motion records have asymmetrical acceleration. This paper evaluates the response bias caused by strength deterioration or by asymmetric properties of input ground motions using SDOF systems with various stiffnesses and strengths under 68 real ground motions and some simplified artificial ground motions.

2. *P*-*δ* effect

The additional shear force caused by the *P*-*δ* effect is proportional to the system displacement *δ*. Therefore, the *P*-*δ* effect can be considered by an additional spring with negative stiffness $k_{p\delta}$ as shown in Figure 1a. The constant of the additional spring is given by the following equation:

$$k_{P\delta} = \frac{Mg}{h} \tag{1.1}$$

Figure 1b shows the static-force-deflection relations considering *P*-*δ* effect. In this paper, collapse of the SDOF system with *P*-*δ* effect is defined as case that shear force of the system is less than $Q_y/100$.

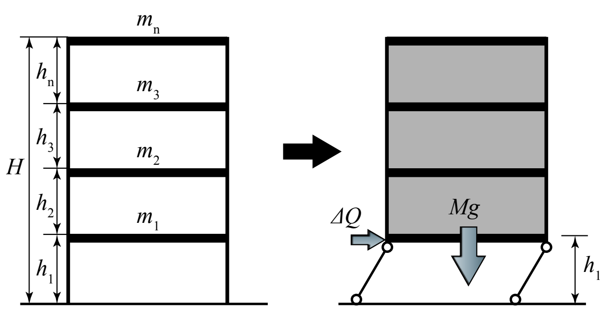


(a) Single Degree of Freedom System (b) Relation between Force and Deformation

Figure 1 *P-δ* Effect

3. Analysis models and Input Ground Motion

Figure 2a shows a multi-degree-of-freedom (MDOF) system, assuming real reinforced concrete structure with soft first story. Under the situation where only the soft first story deforms, the stiffness of the 1st story deteriorates by additional shear force ΔQ considering *P-δ* effects as shown in Figure 2b. Therefore we replace the MDOF system by a SDOF system considering negative stiffness $k_{p\delta}$, where we substitute the entire mass of the system and the height h_1 of a story for mass M and height H in the formula 1, respectively.
Three cases of SDOF systems are defined as shown in Table 1, considering standard 5, 10 and 15 story buildings. The damping factor of each model is 0.05. Two types of hysteresis models are used for each building model, the elasto-plastic model and the strength degrading model (*P-δ* system) as depicted in Figure 3a and 3b.As shown in Table 1, the post-yield stiffness ratio $k_{p\delta}/k$ becomes larger with an increase in natural period T.

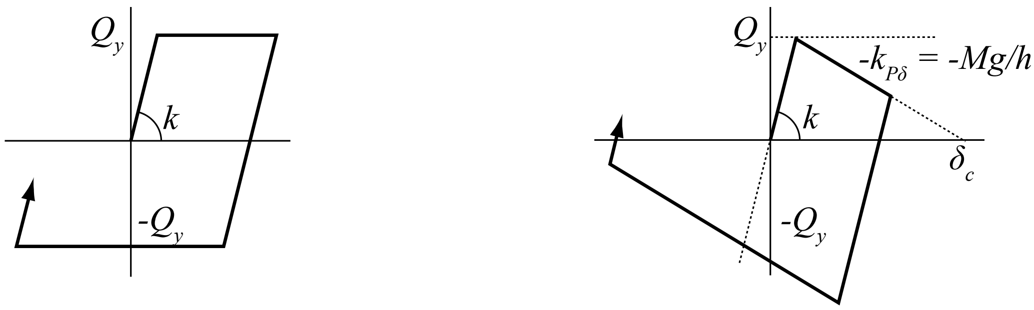


(a) (b)

Figure 2 *P-δ* Effect of Multi Degree of Freedom System

Table 1 Analysis Models

	Model 5		Model 10		Model 15	
	EP	*Pδ*	*EP*	*Pδ*	*EP*	*Pδ*
story	5		10		15	
mass M (kg)	225000		450000		675000	
natural period T(sec)	0.35		0.70		1.05	
stiffness k (kN/m)	73×10^3		36×10^3		24×10^3	
post-yield stiffness $k_{p\Delta}$ (kN/m)	0	0.6×30^3	0	1.26×30^3	0	1.89×30^3
post-yield stiffness ratio ($k_{p\delta}/k$)	0	0.0087	0	0.035	0	0.078
hysteresis characteristic (Figure 3)	(a)	(b)	(a)	(b)	(a)	(b)



(a) Elasto-Plastic Model

(b) Strength-Degrading Model considering P-δ Effect (P-δ system)

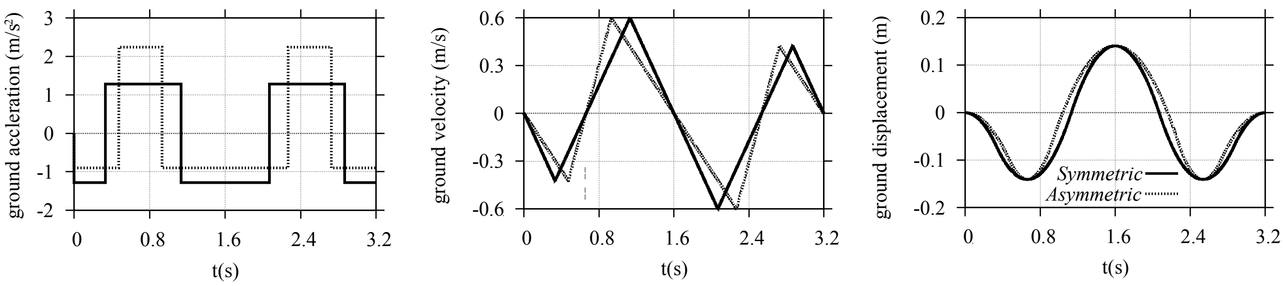
Figure 3 Hysteresis Models

4. Simple Motion Response

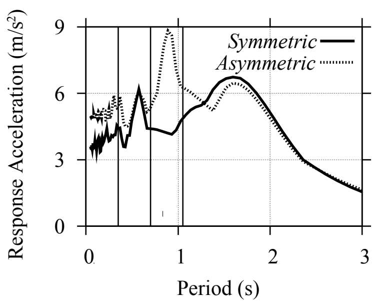
In this section, effect of hysteresis model and ground motion characteristics on the bias of response is evaluated. Marubashi (2006) pointed out that the bias of the input ground motion acceleration strongly affects on the response bias of SDOF systems with short period and small strength by using simplified periodic motions. First, the bias of response is estimated in this paper by using simplified periodic motions.

4.1. Input Ground Motion

Figures 4a, 4b, and 4c show the time-histories of the ground acceleration and velocity and displacement, respectively. In this study, two simplified periodic ground motions are used, namely '*Symmetric*' and '*Asymmetric*.' The maximum velocity is the for each ground motion, while the acceleration is biased to one direction for '*Asymmetric*.' Figure 4d shows the elastic acceleration spectrum, where the solid lines represent natural periods of SDOF systems used in this study.



(a) Ground Acceleration (b) Ground Velocity (c) Ground Displacement



(d) Elastic Acceleration Spectrum

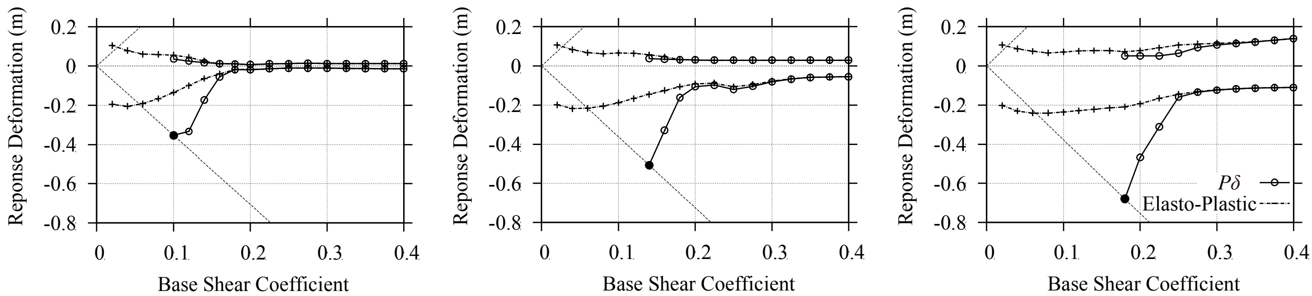
Figure 4 Simplified Periodic Ground Motions

4.2. Maximum Response Displacement

Figures 5 and 6 show the relation between base shear coefficient and the maximum response deformation in each direction computed under *Symmetric* and *Asymmetric* motions, where a, b and c are results of Model 5, Model 10 and Model15, respectively. The broken lines represent collapse displacement δ_c of $P\delta$ systems (shown in Figure 3b). In cases where the response deformation of $P\delta$ system exceeds δ_c, the results are not plotted in Figures 5 and 6. The filled circle represents the limit of base shear coefficient to avoid collapse.

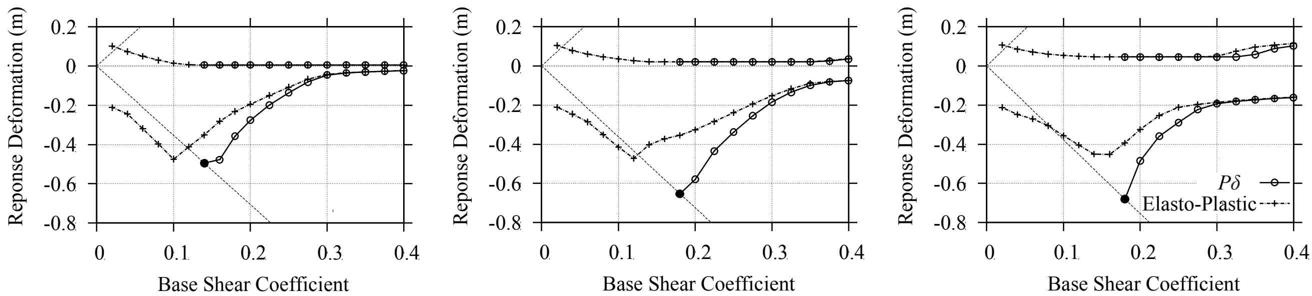
The responses for the *Symmetric* motion are shown in Figure 5. The difference of the maximum response to positive and negative directions are small, in other words, the bias of response is small for the elasto-plastic systems compared to the $P\delta$ systems. On the other hand, the response of $P\delta$ system tends to lean toward negative direction. The required of base shear coefficient to avoid collapse for $P\delta$ system (filled circle in Figures 5) are larger in order of 5a, 5b and 5c because the stiffness degradation $k_{p\delta}$ became larger in the order of Model5, Model10 and Model15 as shown in Table 1. The inelastic response deformation of the $P\delta$ system is very sensitive to the input ground motion acceleration.

The responses for the *Asymmetric* motion are shown in Figure 6. The bias of the response is large even for the elasto-plastic systems. As for the difference of the input motion characteristics, the limit of base shear coefficient to avoid collapse of $P\delta$ system subjected to *Asymmetric* motion (Figures 6a and 6b) is larger than that subjected to *Symmetric* motion (Figures 5a and 5b). On the other hand the values of base shear coefficient to avoid collapse of $P\delta$ system in Figures 5c and Figure 6c were almost the same. It seems that the systems with short period like Model 5 and Model 10 are very sensitive to the input ground acceleration compared to the systems with large period like Model 15.



(a) Model 5 (b) Model 10 (c) Model 15

Figure 5 Relation between Base Shear Coefficient and Positive and Negative Response Deformation subjected to *Symmetric* Motion



(a) Model 5 (b) Model 10 (c) Model 15

Figure 6 Relation between Base Shear Coefficient and Positive and Negative Response Deformation subjected to *Asymmetric* Motion

4.3 Relation between Strength Ration R and Response Bias

In this paper, the strength ratio R (Miranda (2003)) is used as the index which represents the relation between the ground motion intensity and the strength of SDOF systems. R is defined by the following equation:

$$R = \frac{MS_a(T)}{Q_y} \tag{4.1}$$

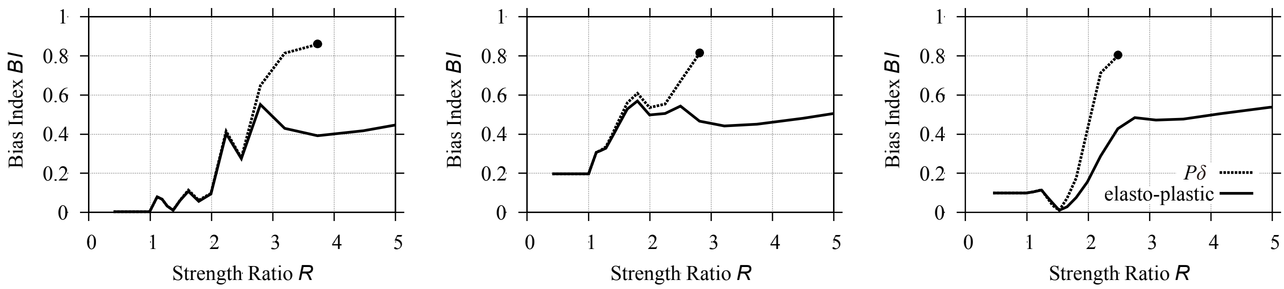
where $S_a(T)$ is the elastic response acceleration of the ground motions corresponding to the fundamental period T of the SDOF systems and Q_y is the yield strength of the hysteresis model. The system stays in elastic range when $R \leq 1$. Hear, an index to represents the response bias is defined by following equation:

$$BI = \frac{\mu}{\left(\mu_n + \mu_p\right)/2} - 1 \tag{4.1}$$

where μ is the maximum ductility factor whichever the positive or the negative direction and μ_{n} and μ_{p} are the ductility factor to the negative and the positive direction, respectively. The bias index *BI* takes a value between 0 and 1. The system shifts to a single direction when $BI = 1$.

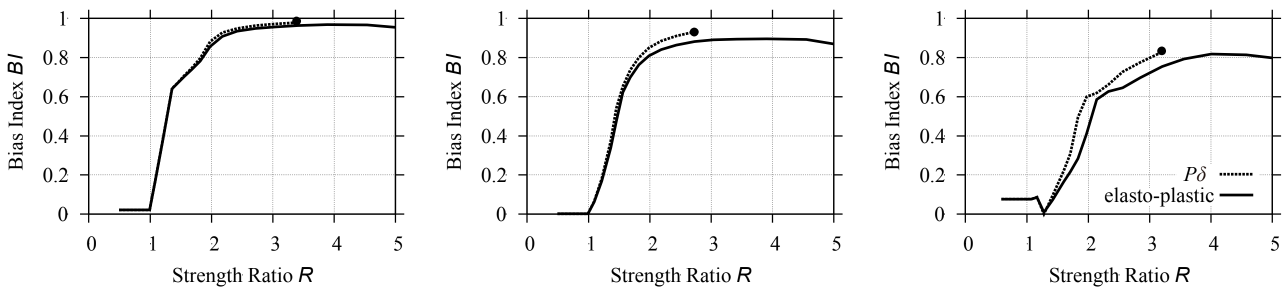
Figures 7 and 8 show the relation between the strength ratio R and the bias index *BI*. When R of the $P\delta$ system is larger than the filled circle in Figures 7 and 8, the response displacement exceeds the collapse displacement δ_c (See Figure 3b). Therefore the strength ratio R at the filled circle is the limit strength in order to avoid collapse of the analysis model, which is called as the collapse strength ratio R_c (Miranda (2003)).

In Figure 7, the bias index *BI* of the elasto-plastic system was smaller than 0.6. The collapse strength ratio R_c of the $P\delta$ system was larger in order of 7a, 7b and 7c. The *BI* of the $P\delta$ system greatly exceeded that of the elasto-plastic system when R is R_c. If R is smaller than R_c, the *BI* of the $P\delta$ system decreased rapidly with a decrease of R and it became close to the *BI* of the elasto-plastic system. In both elasto-plastic and $P\delta$ system, the *BI* subjected to *Asymmetric* motion was large as shown in Figure 8. The bias index *BI* for *Asymmetric* motion was larger than that for *Symmetric* motion. It seems that Model 5 which is short period vibration (Table 1) is the most sensitive to asymmetric property of ground acceleration because the value of *BI* for Model 5 is the largest among the three models.



(a) Model 5 (b) Model 10 (c) Model 15

Figure 7 Strength Ratio R – Bias Index *BI* subjected to *Symmetric* Motion



(a) Model 5 (b) Model 10 (c) Model 15

Figure 8 Strength Ratio R – Bias Index *BI* subjected to *Asymmetric* Motion

5. Real Motion Response

5.1 Variation of Collapse Strength Ratio

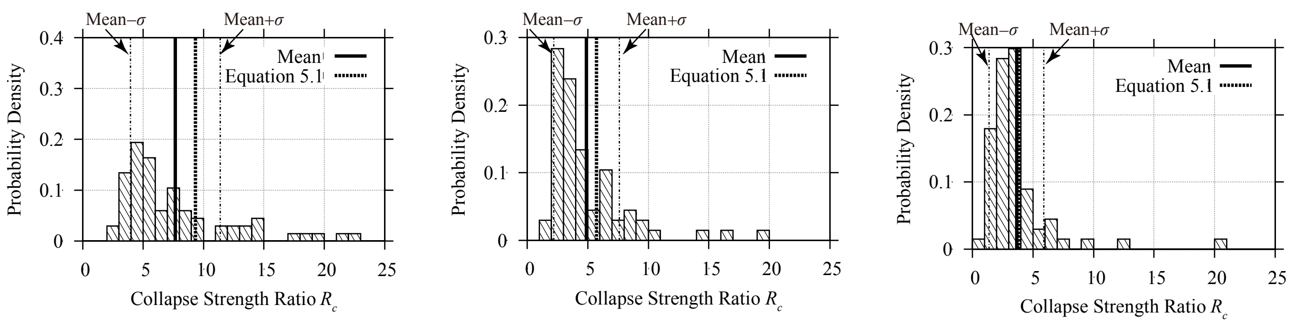
In this section, collapse strength ratio R_c is evaluated using real ground motion records. Figures 9a-9c show histograms of the collapse strength ratio R_c computed with 68 ground motion records set for the $P\delta$ systems. Those records were selected from earthquakes with magnitude range from 6.5 to 7.9 and with maximum velocity range from 70 cm/s to 180 cm/s. Figures 9a, 9b and 9c are the results for Model 5, Model 10 and Model 15, respectively. The solid line represents mean of those results and σ represents standard deviation. The broken line was obtained using the formulation proposed by Miranda (2003):

$$R_c = 1 + a(\alpha)^{-b} \tag{5.1}$$

$$a = 0.26\left(1 - e^{-7.5T}\right) \tag{5.2}$$

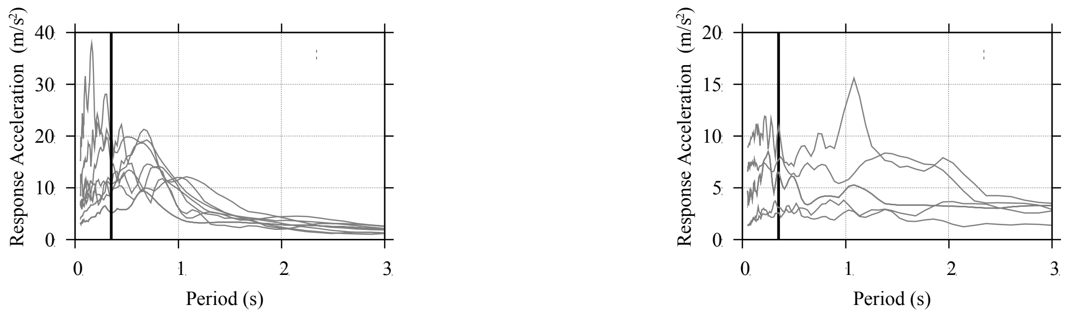
$$b = 0.89 + 0.04T + 0.15\ln(T) \tag{5.3}$$

where α is the post-yield stiffness ratio and T is the natural period of the SDOF systems.
As shown in Figure 9, most results computed with the real ground motion records were less than 10. The mean of the distribution was almost the same as R_c predicted by Equation 5.1 in Figure 9c, although the peak was less than R_c predicted by Equation 5.1 in Figures 5a and 5b. In each case, results were less than R_c predicted by Equation 5.1 due to distribution proportional with peak to the left side like lognormal distribution. The standard deviation is larger in order of Model 15, Model 10 and Model 5. This is attributed to the difference of the natural period. It seems that dispersion of the results of Model 5 is caused by response bias due to asymmetry of the ground acceleration. In case where $P\delta$ systems are subjected to *Symmetric* motion, R_c of Model 5 was the largest, and R_c of Model 15 was the smallest as shown in Figures 7. The same relations can be seen in Figure 9. In contrast, R_c of Model 10 subjected to *Asymmetric* motion was the smallest as shown in Figures 8. Therefore characteristic of the real ground motions like *Asymmetric* motion may affect on variation of the values R_c.
The shape of the elastic spectrum of ground motion can also affect on the variation of R_c. We divided input ground motion records into two groups: (a) the results computed with the motions exceeding mean + σ in Figure 9a and (b) the results less than mean – σ in Figure 9a. Figures 10a and 10b show respectively elastic response spectrum of the group (a) and that of the group (b), where the solid line represents the natural period of Model 5 (T = 0.35s). The ground motions under which R_c values were overestimated (Figure 10a) have relatively short predominant periods. In the systems with small stiffness degradation, the apparent fundamental periods are elongated for the large collapse displacement and the response for the ground motions with short predominant period tends to be smaller than estimated by this method. Thus, it is difficult to estimate the intensity of ground motion records using only elastic response computed from the natural period of the SDOF systems.



(a) Model 5 (b) Model 10 (c) Model 15

Figure 9 Histogram of Collapse Strength Ratio R_c computed with 68 Real Ground Motions

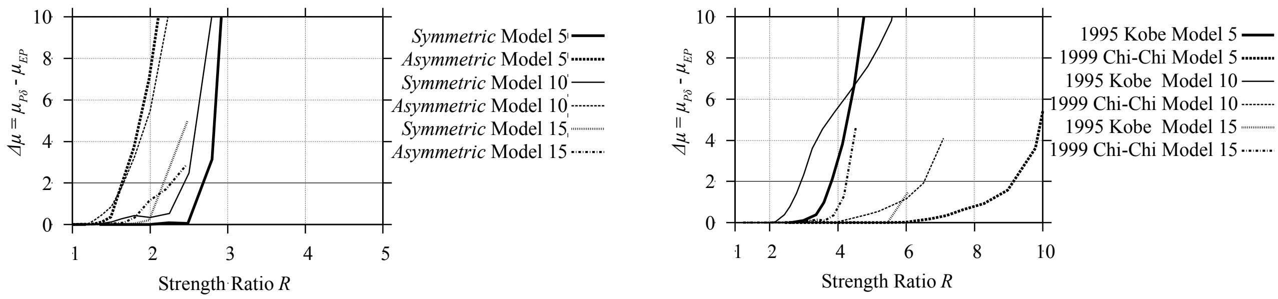


(a) Ground Motions with R_c Overestimated (b) Ground Motions with R_c Underestimated

Figure 10 Elastic Response Spectrum

5.2 Relation between Strength and Bias response of the $P\delta$ system

We evaluated relations between strength and bias response by comparing the $P\delta$ system with the elasto-plastic system. As shown in Figures 7 and 8, the bias index *BI* of $P\delta$ system is almost the same as that of the elasto-plastic system when the strength ratio *R* is small. The *BI* of the $P\delta$ system rapidly increases with an increase of *R* when *R* exceeds certain value. In order to evaluate that *R* value, Figures 11 show relation between strength ratio *R* and difference of ductility factor $\Delta\mu$, where $\Delta\mu$ represents the difference between ductility factor of the $P\delta$ system $\mu_{P\delta}$ and that of the elasto-plastic system μ_{EP}. Figures 11a and 11b were respectively computed with two simple motions and with two real ground motions (1995 Kobe earthquake at the JMS station and 1999 Chi-Chi, Taiwan at the TCU068 station). The values of strength ratio *R* where $\Delta\mu$ started to increase varied by each motion and by each model. When $\Delta\mu$ was equal to about 2, the increasing rate of $\Delta\mu$ tended to became large with an increase with *R*. We defined a limit strength ratio R_2 wherein the ductility factors for $P\delta$ system are obtained by adding 2 to those for elasto-plastic system.



(a) Simple Motions (b) Real Ground Motions

Figure 11 Relation between Strength Ratio *R* and Difference of Ductility Factor $\Delta\mu$

Figure 12 shows relation between the collapse strength ratio R_c and the limit strength ratio R_2, where these results were computed with 68 real ground motion records. The solid line represent the regression line as following equation:

$$R_2 = 0.37R_c + 1.56 \tag{5.4}$$

In Figure 12, the coefficient of correlation between them is 0.72. The limit strength ratio R_2 is computed from Equations 5.1, 5.2 and 5.3 formulated by Miranda (2003)

$$R_2 = 1.93 + 0.37a(\alpha)^{-b} \tag{5.5}$$

Figure 13 shows histograms of the limit strength ratio R_2, where a, b and c are the results of the $P\delta$ system with

hysteresis of Model 5, Model 10 and Model 15, respectively. The solid straight line and σ represent mean and standard deviation. The broken line is obtained from Equation 5.5.
In Figures 13, the difference between mean of the distribution and the value predicted by Equation 5.5 is smaller than that in Figures 9. The dispersion of R_2 values in Figures 13 is considerably smaller than that of R_c values in Figures 9.

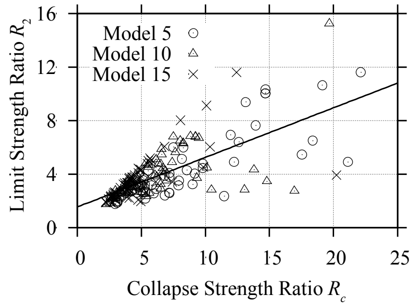
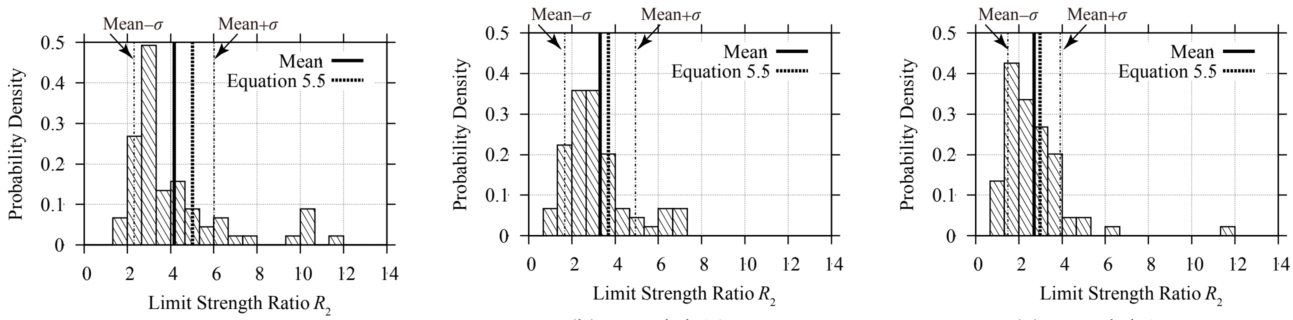


Figure 12 Relation between Collapse Strength Ratio R_c and Limit Strength Ratio R_2



(a) Model 5 (b) Model 10 (c) Model 15

Figure 13 Histogram of Limit Strength Ratio R_2 computed with 68 Real Ground Motions

6. CONCLUSION

The inelastic response deformation is extremely sensitive to the strength reduction factor R, especially when R is larger than a certain value. We defined the limit strength reduction factor R_2 where the ductility factor for the degradation systems are adding 2 to those for the elasto-plastic systems, and formulated R_2 as a parameter of the stiffness degradation factor and the fundamental period. When R is smaller than R_2, response deformations are moderate and almost symmetric except under eccentric ground motions. Otherwise, the SDOF systems exhibit biased response and the response deformations lean heavily towards a single direction in any ground motion.

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