

# ON THE DESIGN AND EVALUATION OF SEISMIC RESPONSE OF RC BUILDINGS ACCORDING TO DIRECT DISPLACEMENT-BASED DESIGN APPROACH

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### **ABSTRACT:**

This paper describes a research work on the evaluation of seismic response of reinforced concrete frames designed according to Direct Displacement-Based Design (DDBD) approach. A group of plane RC frames, characterized by a variable number of storeys, was designed by means of this methodology. Then, seismic performance of designed frames was studied by carrying out pushover and non-linear dynamic analyses. Results of analyses were compared with the seismic behavior expected from design. Some evaluations are also made on the differences between DDBD and more traditional force-based design procedures.

**KEYWORDS:** Direct-Displacement Based Design, Pushover analysis, Non-linear dynamic analysis

# **1. INTRODUCTION**

Actual seismic codes are generally based on force-based design procedures, which are characterized by check that strength of structural members is larger than seismic induced force determined by applying a force reduction factor. This factor depends on ductility of the structure, which for new buildings is implicitly assured by design rules. Within force-based methods, the application of capacity design criteria is aimed to control the inelastic response, by providing the building a proper distribution of strength to avoid plastic hinges in columns and shear failure. Also the addition of drift limits allows to improve force-based design by recognizing the importance of displacement as a parameter related to seismic damage. Anyway force-based design is characterized by several problems (Priestley, 2003). First, seismic damage is mainly correlated to strain or drift while its correlation with strength is not clear. Force-based design, then, is based on the assumption of unique force reduction factor, depending on ductility capacity, for a given structural type and material, while various studies showed that ductility capacity is related to a wide range of other factors, as for example axial load ratio, reinforcement ratio or structural geometry. Different structures designed considering the same value of force reduction factor may experience different levels of damage, thus implying that the requirement of uniform risk is not satisfied. Some problems arises also in the definition of stiffness. Force-based design uses initial stiffness of members for determining the period and the distribution of forces between structural elements. The initial stiffness is assigned independently from strength, while various studies showed that stiffness is essentially directly proportional to strength. Moreover, the distribution of seismic forces on the basis of initial stiffness can not provide in general adequate estimates of force distribution in the inelastic range. In the last years a lot of research has been aimed to mitigate the problems of current force-based design. In particular, several design approaches have been proposed for designing structures in order to achieve a specified deformation state under the design earthquake (Calvi, 2003). These approaches are characterized by the assumption of displacement as fundamental design parameter, and they are known as displacement-based design procedures. When the design method does not require iterations, it is called direct displacement-based design. The approach proposed by Priestley (2000, 2003) is a direct displacement-based procedure characterized by the use of secant stiffness to

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maximum displacement. In this way the problems related to initial stiffness can be avoided. The method is based on the characterization of a substitute structure and on the use of highly damped displacement spectra. This approach has been developed in a rather complete form, and has been applied to a wide category of structures. Recently a draft displacement-based code (Priestley et al., 2007) with criteria for application of the method to various structural typologies according to more recent research results (Pettinga and Priestley, 2003; Sullivan et al., 2006) has been proposed. The purpose of research presented here was to give a further contribution to the investigation and validation of the direct displacement-based procedure in the light of the mentioned draft code. The study was carried out with regard in particular to reinforced concrete frame structures. A set of RC frames with variable number of storeys was designed according to the direct displacement-based design method. Seismic performance of designed frames was evaluated through pushover and non-linear dynamic analyses. Results were compared with expected behaviour from design.

#### 2. DESIGN PROCEDURE

The direct displacement-based design of the structures under study was performed according to the criteria and rules proposed by Priestley et al. (2007) with reference to reinforced concrete frame structures. In a first phase, for a given design displacement, the design base shear is derived through the definition of the substitute structure. From strain or drift limits, defined depending on design seismic intensity, it is possible to obtain design displacement of each storey *i* using the following proposed relationships:

$$\Delta_{i} = \omega_{\theta} \delta_{i} \left( \frac{\Delta_{c}}{\delta_{c}} \right); \quad \text{for n>4:} \quad \delta_{i} = \frac{4}{3} \left( \frac{H_{i}}{H_{n}} \right) \left( 1 - \frac{H_{i}}{4H_{n}} \right)$$
(2.1)

where  $\Delta_c$  is the displacement limit of the critical storey,  $\omega_{\theta}$  is a reduction factor which accounts for higher mode amplification, *n* is the number of storeys and  $\delta_i$  is a normalized inelastic mode shape dependent on height  $H_i$ . The design displacement of the substitute structure can then be determined from displacements of the storeys:

$$\Delta_d = \sum_{i=1}^n \left( m_i \Delta_i^2 \right) / \sum_{i=1}^n \left( m_i \Delta_i \right)$$
(2.2)

where  $m_i$  is the mass of the storey *i*. Equivalent mass and effective height of substitute structure are given by:

$$m_{e} = \sum_{i=1}^{n} (m_{i}\Delta_{i}) / \Delta_{d}; \quad H_{e} = \sum_{i=1}^{n} (m_{i}\Delta_{i}H_{i}) / \sum_{i=1}^{n} (m_{i}\Delta_{i})$$
(2.3)

The ductility demand of the substitute structure is  $\mu = \Delta_d / \Delta_y$ . The yield displacement  $\Delta_y$  can be calculated once it is known the yield drift  $\theta_y$  as  $\Delta_y = \theta_y H_e$ . It has been shown (Priestley, 1997) that yield drift depends mainly on geometry and not on strength. Various expressions have been provided for  $\theta_y$ . In particular, for reinforced concrete frames  $\theta_y = 0.5 \varepsilon_y L_b / h_b$ , where  $L_b$  and  $h_b$  are beam length and depth respectively, and  $\varepsilon_y$  is the yield strength of reinforcement steel. For RC frames the equivalent viscous damping may be evaluated as follows:

$$\xi_{eq} = 0.05 + 0.565 \left(\frac{\mu - 1}{\pi \mu}\right) \tag{2.4}$$

The effective period  $T_e$  at maximum displacement is obtained as a function of design displacement from the displacement spectrum associated to the equivalent viscous damping calculated with Equation 2.4. The effective stiffness of the substitute structure and the design base shear of the structure are given by:

$$K_e = 4\pi^2 m_e / T_e^2; \ V_{base} = K_e \Delta_d$$
 (2.5)



P- $\Delta$  effects may be included by adding to base shear the contribution  $0.5P\Delta_d/H$  when stability index exceeds 0.1. The base shear force is distributed to the floor levels in proportion to the product of mass and displacement. For high-rise frame buildings 10% of base shear is additionally applied at roof level to reduce higher mode effect:

$$F_{i} = F_{t} + 0.9V_{base}\left(m_{i}\Delta_{i}\right) / \sum_{i=1}^{n} \left(m_{i}\Delta_{i}\right)$$

$$(2.6)$$

where  $F_t=0.1V_{base}$  at the roof and  $F_t=0$  at all other levels. The building under these forces is then analyzed in order to determine required flexural strength of structural elements, in particular at location of plastic hinges. The analysis shall be based on effective stiffness at maximum displacement of structural members which are expected to undergo inelastic deformations. Alternatively, required flexural strength may be obtained using a simplified method based on equilibrium conditions. Shear forces in beams are derived from seismic axial force in the exterior columns. Considering that the difference between total overturning moment induced by lateral forces at the base and the sum of the column base moments  $M_{Cj}$  is equal to the axial force T in the exterior columns times the distance  $L_{base}$  between these columns, it is possible to obtain:

$$\sum_{i=1}^{n} V_{Bi} = T = \left(\sum_{i=1}^{n} F_{i}H_{i} - \sum_{j=1}^{m} M_{Cj}\right) / L_{base}$$
(2.7)

In this equation the sum of the seismic beam shears  $V_{Bi}$  of all storeys is given by the axial forces in the exterior columns at the base. The values of the column base moments depends on design choice. Assuming that the point of contraflexure in the base columns occurs approximately at 60% of the storey height, the sum of the column base moments are evaluated as  $0.6V_{base}H_1$ . The single values of beam shear  $V_{Bi}$  of each storey may be defined considering a vertical distribution of total beam shear along the building in proportion to the storey shears  $V_{Si}$ :

$$V_{Bi} = T\left(V_{Si} / \sum_{i=1}^{n} V_{Si}\right); \quad V_{Si} = \sum_{k=i}^{n} F_{k}$$
(2.8)

From equilibrium of each beam span it is possible to find beam design moments at the column centrelines at the left and right end of the beam. From these moments, which in general are not equal, the corresponding values at the column faces are then obtained. Also column moments, as beam moments, are calculated directly. The total storey shear may be distributed between the columns of each storey assuming that shear absorbed by internal columns is double than that of external columns. Column moments may be determined from the column shears, by making, for example, the approximate assumption of central contraflexure point. They can be found alternatively from beam moments at the column centrelines, by distributing the sum of the moments of beams framing into a joint between the columns framing into the same joint. The described procedure allows to obtain beam and column moments induced by seismic action. In addition to these moments, also those due to gravity loads should be considered. It has been observed (Priestley et al., 2007) that this operation is too conservative. It would lead to an increase of the cost of the structure and to a reduction of displacement demand below the design value. Therefore it has been recommended to design beams for the larger of the seismic moments and the factored gravity-load moments. It has been recommended also to calculate flexural capacity of beams considering design values of material properties larger than characteristic values according to  $f'_{ce}=1.3f'_{c}$  and  $f'_{ve}=1.1f'_{v}$ . The columns are designed in order to not exceed the elastic limit under seismic action, and hence capacity design criteria have to be applied. The flexural strength of columns  $M_N$  has to satisfy the requirement  $M_N \ge \phi^0 \omega_f M_E$ , where  $M_E$  is the moment due to seismic action,  $\phi^0$  is the ratio of overstrength moment capacity to required capacity and  $\omega_f$  is a dynamic amplification factor due to higher mode effects. Factor  $\phi^0$  may be set equal to 1.35 while  $\omega_f$  can be calculated with following relationship:

$$\omega_f = 1.15 + 0.13 \left( \mu / \phi_0 - 1 \right) \tag{2.9}$$



#### **3. STRUCTURES UNDER STUDY**

The procedure described above was applied to design the structures under study. They are three RC plane frames characterized by three spans and by a number of storeys equal to six, nine and twelve (Fig. 1). These frames are referred respectively as frame 3-6, 3-9 and 3-12. Length of all bays is equal to 6.0 m and height of all storeys is equal to 3.2 m. Assumed mechanical properties of materials are: concrete cylinder strength equal to 25 Mpa and steel yield strength equal to 430 Mpa. Considered non-factored loads on beams are the same for all frames and storeys: dead load equal to 35 kN/m and live load equal to 12 kN/m. This assumption provided uniform mass distribution along the height. Adopted dimensions of beams are: width equal to 300 mm and depth equal to 600 mm for the beams of the first three storeys of frame 3-12 and to 500 mm for all other beams. The dimensions of columns cross sections were defined in order to limit the normalized axial force. Variable dimensions along the height were adopted in order that consequent variations of elastic stiffness of gross section remain in percentage below 30%. The elastic response spectrum provided by Eurocode 8 (CEN, 2003) was considered in the design. Peak ground acceleration (PGA) equal to 0.35g and soil type C, associated to an amplification of PGA equal to 1.15, were adopted. For this intensity level a drift limit equal to 0.025 was assumed as starting value for the design procedure. The displacement response spectrum was obtained from the acceleration spectrum considering a corner period equal to 4 sec. Table 3.1 shows principal design parameters of the three frames related to the first phase of the procedure, which ends with definition of design base shear.

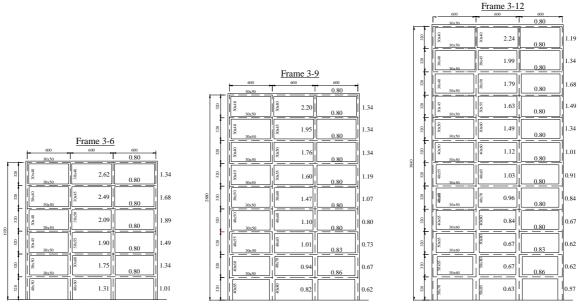


Figure 1 Structures under study (dimensions in cm, reinforcement ratios in percentage)

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Frame	Design Drift	$H_n[\mathbf{m}]$	$m_{tot}$ [kNs <sup>2</sup> /m]	$\Delta_{ytop}$ [mm]	$\Delta_{dtop}$ [mm]	$H_e[\mathbf{m}]$	$m_e$ [kNs <sup>2</sup> /m]	$\Delta_y$ [mm]	$\Delta_d$ [mm]	μ	ξ <sub>eq</sub> [%]	$T_e[s]$	V <sub>base</sub> [kN]
3-6	0.025	19.2	425	265	376	13.5	360	187	283	1.51	11.1	2.57	606
3-9	0.025	28.8	638	398	555	19.7	529	273	409	1.50	11.0	3.71	619
3-12	0.025	38.4	850	442	608	26.0	696	299	443	1.48	10.8	4.00	761

Table 3.1 Design parameters of the structures under study

With regard to design of beams, equal areas were adopted for the top and bottom reinforcements. Within a single storey the same reinforcement areas were considered for all plastic hinge regions of the beam. As recommended, design strength of beams was based on the larger of gravity and seismic moments. The gravity moments resulted dominant for all beams of frame 3-6 and for the beams of the last six storeys of the other two frames. Adopted reinforcement ratios for beams and columns are indicated in Figure 1.



# 4. NON-LINEAR MODEL AND PERFORMED ANALYSES

Non-linear analyses were carried out in order to asses the performance of structures under study. The OpenSees software (McKenna et al., 2003) was used for the non-linear analyses. This computer program allows to study the structure with distributed plasticity finite elements characterized by a fibre modelling of the control sections. Each structural member, column or beam, was modelled with a single finite element. Five control sections were adopted, two located at the ends and the other along the element. A bilinear stress-strain relationship with hardening ratio equal to 0.005 was adopted for the steel fibres. A constitutive law which includes the effect of confinement due to stirrup and the stiffness degradation due to cyclic loading was considered for the concrete. Different types of behaviour were adopted for the cover concrete and the concrete core: in the first case the effect of confinement was neglected, in the second case it was included according to the model proposed by Mander et al. (1988). Assumed steel and unconfined concrete strength in the non-linear analyses are the same as the ones used in the design procedure. Gravity loads were applied as uniformly distributed loads on beams. The geometrical non-linearity was considered in terms of P- $\Delta$  effects. The performance of the structures was evaluated by means of both pushover and non-linear dynamic analyses. Pushover analyses were carried out considering lateral forces proportional to floor masses multiplied by the corresponding first modal deformation. Non-linear dynamic analyses were performed using a group of seven accelerograms consistent with type 1 Eurocode 8 design spectrum. In particular, the average displacement spectrum of considered ground motions is consistent with design displacement spectrum up to a corner period of 4 sec (Fig. 2). The group is characterized by four recorded ground motions and by three artificial accelerograms. One of the recorded accelerograms was manipulated with a time scale factor for imposing higher demand at larger periods. All accelerograms were scaled to the design PGA. The structural model for non-linear dynamic analyses was characterized also by introduction of damping proportional to tangent stiffness.

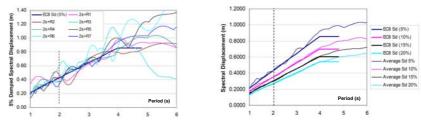


Figure 2 Displacement spectra of the considered ground motions scaled to PGA=0.5 g

#### **5. RESULTS OF ANALYSES**

The average values from non-linear dynamic analyses of maximum roof displacements ( $\Delta_{topNDA}$ ) resulted slightly lower than design values ( $\Delta_{dtop}$ ), but anyway it was found a quite good correspondence between them, especially for frame 3-12 (Tab. 5.1). Pushover curves of structures under study are illustrated in Figure 3, together with bilinear idealization according to EC8. Also design base shear  $(V_{base})$  and value associated to first plastic hinge  $(V_{bl})$ , as well as design displacement and average value from non-linear dynamic analyses, are indicated in the figure. The overstrength of examined frames, in terms of ratio of base shear at elastic limit  $(V_{bu})$ of bilinear idealization to design base shear, resulted quite small for frames 3-6 and 3-9 (Tab. 5.1). For these frames design base shear calculated with contribution of P- $\Delta$  effects resulted almost equal to the elastic limit of bilinear idealization. For the frame 3-12 the maximum base shear from pushover analyses resulted slightly lower than design base shear. This was due probably to the different load vectors applied in design procedure and in pushover analyses, being the latter based on elastic mode shape. Anyway it may be stated that there was a good correspondence between design base shear and calculated lateral strength of buildings. This was due mainly to the use of amplified values of strength of materials for determination of beam flexural capacity and to the choice of neglecting gravity moments in seismic design. The first plastic hinge occurred for all structures at a base shear level significantly smaller than design one. At first hinge a slight decrease of slope of pushover curve was observed. However the global behaviour of the structure was not significantly influenced by earlier plastic hinges. The strong reduction of the slope of pushover curve, associated with global yielding of the structure, occurred after complete formation of plastic hinges in beams. The earlier formation of first plastic hinge and the

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 $\Delta_{dton}$  [mm]

 $\Delta_{topNDA}$  [mm]



V<sub>base</sub> [kN]

Model according to diffe

400 500 600 Displacement  $\delta$  [mm]

V<sub>bu</sub> [kN]

non simultaneous formation of all plastic hinges may be correlated both to the presence, during the analysis, of distributed gravity loads on beams, and to the different load vectors used in design and in pushover analyses.

3-6	292	376	1.28	333	684	606	1.12
3-9	404	555	1.37	315	669	619	1.08
3-12	545	608	1.11	357	725	761	0.95
V <sub>IN</sub> V <sub>IN</sub> V <sub>IN</sub>	600 	<sup>8</sup> (¶mäsy 1 ∂ 2021 1 3251 1		V <sub>b1</sub> =315 300	9 Storeys, 3 Bays RC: 	Frame 1 +	
	0 100 200	300 400 500 6 Displacement $\delta$ [mm]	600 700 800	0 100 20	0 300 400 500 60 Displacementδ[m		00
		Storeys - 3 Bays RC Frame		Total base Shear V [kN] 1000 900 800			
V <sub>b1</sub>	600	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		700 600 500 400 300 4	Timethores, a	i max	
	100	-++ -++ Push	over capacity curve	200	Ti T - Model wi	ith eleloads	

Table 5.1 Values of roof displacement and base shear strength from non-linear analyses and from design

 $V_{bl}$  [kN]

 $\Delta_{dtop}$  / $\Delta_{topNDA}$ 

Figure 3 Pushover curves of structures under study

 bilinearization

 700
 800
 900
 1000

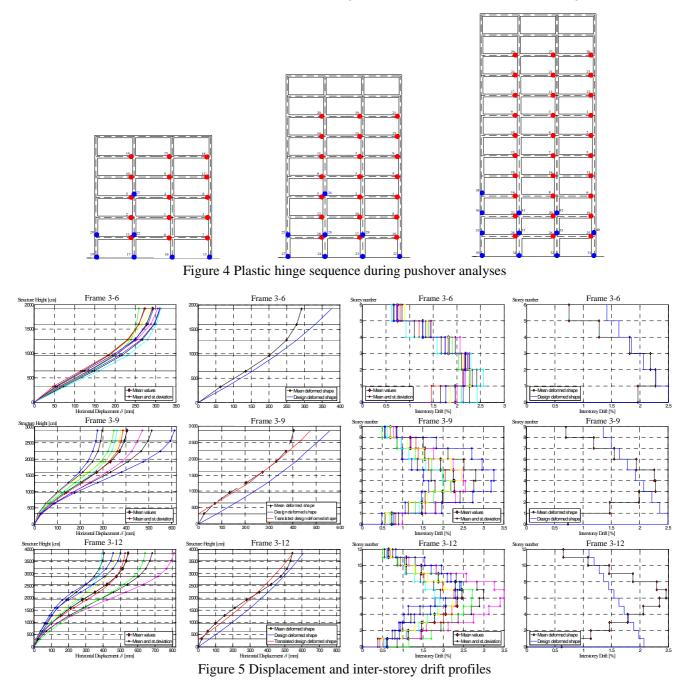
4 400 500 600 Displacement δ [mm]

To investigate the influence of the gravity loads, pushover and non-linear dynamic analyses of the frame 3-9 were repeated considering gravity loads concentrated at beam-column joints. The results were then compared with those obtained considering distributed gravity loads (Fig. 3). It is possible to note that the pushover curves determined with nodal or distributed gravity loads are very similar, especially in terms of base shear strength and of inelastic structural response. The only difference is that with nodal gravity loads formation of first plastic hinge was delayed and plastic hinge in beams tended to occur simultaneously, as considered in design procedure. Moreover, also negligible differences were noted between average values of roof displacement obtained in the two cases from non-linear dynamic analyses. Another investigation was performed by repeating the design of frame 3-9 with inclusion of distributed gravity loads in the seismic design. This new design was carried out by changing only the way to consider gravity loads, and by maintaining all other assumptions and criteria, as the application of simplified method to obtain member forces, the use of amplified material strengths for calculation of flexural capacity of beams and the method for determining capacity design force levels. As a result of this new design, larger quantities of reinforcement were obtained in beams, but this increase regarded mainly lower storeys due to the prevalence of gravity load condition in the design of beams at upper storeys. However a significant increase of reinforcement had to be adopted for exterior columns, since beam moments due to gravity loads were not equilibrated by the presence of adjacent beams. In comparison with previous results, the pushover curve obtained for the frame 3-9 designed considering distributed gravity loads together with seismic action was characterized by a significant increase of the lateral strength and hence of the overstrength related to design base shear. Figure 4 illustrates the configuration of plastic hinges obtained during pushover analysis for the structures under study. The number of each plastic hinge indicates the sequence of formation. Plastic hinges occurred in all beams except at the last two storeys at the top. This was due to the prevalence of gravity load condition in the design of beams at upper storeys. Since distributed gravity loads were considered in the pushover analysis and same reinforcement quantities at the top and the bottom of beams

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were adopted, only plastic hinges at one end of beams were found. From sequence of hinging it is evident that plastic hinges occurred first in all beams, than at the base of columns. Some plastic hinges were found in columns also at other location than at base, but these hinges occurred well into the inelastic range.



The envelopes of storey displacement and of inter-storey drift determined with each earthquake record are illustrated in Figures 5 for the three structures under study. In the same figures also comparisons between average envelopes of non-linear dynamic analyses and design profiles are shown. At the last two storeys non-linear dynamic analyses provided in general smaller values of drifts than design profile. Also this result was due to the prevalence of gravity load condition in the design of beams at upper storeys. The best correspondence between displacement and drift envelopes obtained from design and analysis was found for the 3-6 frames. For the other frames, a tendency to significantly smaller values of drift than those of design profile was noted with non-linear dynamic analyses at the first storeys. This difference, which affected the profiles from non-linear dynamic analyses, may be correlated to the non formation, in these cases, of plastic hinges at the base of all

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columns. Values of drift at the first storey of frame 3-9 and 3-12, in fact, resulted with almost all records well below 1%. This may be correlated to the very low required reinforcement at the base columns of higher buildings, where high values of axial compressive force due to gravity loads are present. Therefore adopted values of reinforcement ratio in these columns, conditioned also by practical considerations, were small but significantly larger than those required. The differences found at the base between profiles from non-linear dynamic analyses and from design may be correlated also to the assumptions of variable dimensions of column cross sections along the height. As a consequence of these results maximum values of drift did not occur at the base but at intermediate storeys. These values resulted below design limit for all frames except for the 3-12, where the concentration of demand at intermediate storeys was significant. A possible way to avoid mentioned differences is to further reduce reinforcement of the first storey columns only at the base. Almost all earthquakes caused smaller top displacements than design value but in one or two cases it was exceeded.

# 6. CONCLUSIONS

A group of plane RC frames, characterized by a variable number of storeys, was designed according to direct displacement-based design approach and its recent developments. The designed structures were then studied by performing several pushover and non-linear dynamic analyses. The application of the design procedure resulted simple and useful. The use of increased material strength and the non consideration of distributed gravity loads in the seismic design of beams allowed to avoid overstrength related to design base shear and to obtain a force-displacement response similar to that assumed in design procedure. Furthermore, little differences were found between top displacement estimates from analyses and design. These results indicate that followed design procedure matched well the requirement of performance-based design to control the inelastic response in the design phase. Distributed gravity loads on beams did not affect significantly the results of non-linear dynamic analyses, but their consideration in seismic design influenced in a relevant way the results of design so to alter the response of the structure and to reduce the ability of controlling it. Obtained drifts from analysis at the first storey resulted smaller than the values predicted in design since plastic hinges did not occurr at the base of all columns, probably as a consequence of dimensioning choices.

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