OPTIMAL SENSOR PLACEMENT FOR STRUCTURAL DAMAGE IDENTIFICAITON WITH MODAL MEASUREMENTS

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ABSTRACT :
A methodology of optimum sensor placement for structural health monitoring, which simultaneously meets the requirements of the damage identifyability and the modal differentiability, is presented in the paper. The modal differentiability and damage identifiability are two crucial issues for structural health monitoring. However, the optimum sensor locations based on the modal differentiability contradicts the results based on the damage identifiability in the previous research. Firstly, the output of the damaged structure is expressed in time domain as a function of mode shapes, eigenvector sensitivity and modal coordinates, which relates the modal differentiability with the damage identifiability. Secondly, the formulation for the FIM is presented, and the optimal criterion is proposed to maximize the 2-norm of the FIM and to minimize the condition number of the output matrix. Thirdly, an iterative algorithm is developed to get the optimal sensor placement for the number of available sensors. Finally, numerical examples are carried out to demonstrate the feasibility and effectiveness of the proposed approach. The results show that the damage can be accurately identified by the information of the sensors placed by the proposed approach. The novel method effectively overcomes the contradiction between the modal differentiability and the damage identifiability.

KEYWORDS:
Optimal sensor placement, Structural damage identification, Modal measurement, Modal differentiability, Damage identificability

1. INTRODUCATION

The structural damage identification based on modal measurements has received much attention over the past several decades because of its important in structural health monitoring (Doebling et.al., 1996). The existing methodologies rely on the fact that the occurrence of the damage in a structure leads to the changes in the modal characteristics, i.e. modal frequency, mode shape and modal damping ratio, of the structural system. The structural damage identification is typically a kind of inverse problem in nature. According to the differences of the theoretical formulation, the techniques of the structural damage identification are mainly categorized into three types: signature analysis or pattern recognition approaches, finite-element model updating or system identification approaches and neural networks approaches.

No matter what kind of approach is adopted, the accuracy of the damage identification would highly depend on the modal measurements before and after the damage occurrence. Therefore, the high-spatial resolution and noise-free measurements are expected to provide the sufficient information that can be extracted from the measured data. Unfortunately, due to the cost-effective considerations, the instrument limitations and the ambient disturbance, only sparse and noisy measurements can be obtained from the limited sensor locations in a real structure. Consequently, the optimal selection of the number and location of sensors has become a fundamental problem for structural damage identification (Udwadia, 1994).

Structural damage identification with modal measurements involves in not only the structural parameter identification but also the modal identification. Therefore, the optimal sensor placements (OSP) for structural damage identification with modal measurement should simultaneously take consideration of the modal differentiability and the parameter identifiability. However, to author's knowledge, the two objectives are studied individually at present. (Cobb and Liest, 1996) presented a method of the optimal sensor placement for the purpose of the structural damage identification. (Shi et al., 2000) proposed a methodology of optimal sensor placement for structural damage identification by maximizing the FIM, which is related with the eigenvector sensitivity analysis. It is noted that the aforementioned research has an implicit assumption that the modal data, which is used to form the FIM by using of the eigenvector sensitivity, are clearly identified or differentiated. In fact, the maximization of the FIM based on the eigenvector sensitivity cannot maximize the FIM based on the modal matrix, which is proposed by references (Kammar, 1991). This contradiction leads to that the sensor placement optimized by eigenvector sensitivity is impossible to meet the requirements of the modal differentiability and identifiability. In a result, the poorly identified modal data will make the damage identification failure.

A novel methodology of optimal sensor placement for structural damage identification, which can simultaneously meet the requirements of modal differentiability and damage identifiability, is presented in the paper. In this study, first, the output of the damaged structure is expressed in time domain as a function of mode shapes, eigenvector sensitivity and modal coordinates, which relates the modal differentiability with the damage identifiability. Then, the formulation for the FIM is presented, and the optimal criterion is proposed to maximize the 2-norm of the FIM and to minimize the condition number of the output matrix. The maximization of the FIM results in an efficient unbiased estimator of the modal coordinates and the damage coefficients. And the minimization of the condition number makes the estimator more robust because that the parameters of the undamaged structure are not exactly known while the parameters of the damaged structure are unknown at all. Third, an iterative algorithm is developed to get the optimal sensor placement for the number of available sensors. Final, numerical examples are carried out to demonstrate the feasibility and effectiveness of the proposed approach.

2. THEORITICAL FORMULATION

The governing differential equation of motion for a N-dof structural system is given as

$$M\ddot{X} + C\dot{X} + KX = F\left(t\right) \tag{2.1}$$

where M, C and K are the mass matrix, damping matrix and stiffness matrix, respectively; $\ddot{X}$, $\dot{X}$ and X are the acceleration vector, velocity vector and displacement vector, respectively.

In common, the damage in a structure leads to the change only in the stiffness but not the mass. Because that the civil structure is often slightly damped, the stiffness changes would not significantly affect the damping properties of the structure. The stiffness, modal eigenvalues and mode shapes of damaged structure can be expressed as

$$K^d = K + \Delta K \quad \Lambda^d = \Lambda + \Delta\Lambda \quad \Phi^d = \Phi + \Delta\Phi \tag{2.2}$$

where the superscript u denotes the damaged case; ΔK is the small change in stiffness due to damage in a structure; $\Delta\Phi$ and $\Delta\Delta$ are the small changes in the mode shapes and eigenvalues, respectively. It is noted that the assumption of the small perturbation is reasonable for the incipient damage. In structural health monitoring, it is normally desired to obtain information when there are relatively small changes in stiffness, prior to a large change that is catastrophic. Therefore, the assumption of the small perturbation is sufficient for optimal sensor placement. The changes in stiffness can then be expressed as a fractional change of the elemental stiffness elements.

$$\Delta K = \sum_{i=1}^{NE} \delta_i K_i \quad i = 1,2,\cdots,NE \tag{2.3}$$

where K_i and δ_i are the i-th elemental stiffness matrix and the corresponding damage coefficient, respectively;

NE is the total number of the structure. This model is suitable in the case that the change in stiffness due to damage is proportional to the elemental stiffness. Even though in the case that the change is not proportional, the model works well since this assumption does not change the essence of the requirement of the damage localization.

For a small perturbation, the equation of the naturally un-damped vibration of the damaged structure becomes

$$(K+\Delta K)(\Phi+\Delta\Phi)=M(\Lambda+\Delta\Lambda)(\Phi+\Delta\Phi) \tag{2.4}$$

By using of the modal technique, the changes in i-th mode shape due to structural damage can be expressed as

$$\Delta\phi_i = S_i\cdot\delta \tag{2.5}$$

where S_i is the sensitivity matrix of the i-th mode shape with respect to the damage coefficient vector δ.

$$S_i=\left(\sum_{\substack{r=1\\r\neq i}}^{N}\frac{\phi_r^T K_1\phi_i}{\lambda_i-\lambda_r}\phi_r \quad \sum_{\substack{r=1\\r\neq i}}^{N}\frac{\phi_r^T K_2\phi_i}{\lambda_i-\lambda_r}\phi_r \quad \cdots \quad \sum_{\substack{r=1\\r\neq i}}^{N}\frac{\phi_r^T K_{NE}\phi_i}{\lambda_i-\lambda_r}\phi_r\right) \tag{2.6}$$

$$\delta=(\delta_1 \quad \delta_2 \quad \cdots \quad \delta_{NE})^T \tag{2.7}$$

The changes in all mode shapes can be rewritten in a matrix form as

$$\begin{aligned}\Delta\Phi&=\begin{bmatrix}\Delta\phi_1 & \Delta\phi_2 & \cdots & \Delta\phi_N\end{bmatrix}\\&=\begin{bmatrix}S_1 & S_2 & \cdots & S_N\end{bmatrix}\begin{bmatrix}\delta & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \delta & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \delta\end{bmatrix}=S\Omega\end{aligned} \tag{2.8}$$

where $\mathbf{0}$ is a $NE\times 1$ vector with zero in each element.

3. FISHER INFORMATION MATRIX FOR DAMAGED STRUCTURE

According to Kammer, the measurement equation of structural system can be expressed as

$$Y(t)=\Phi q(t)+v(t) \tag{3.1}$$

where q is the vector of modal coordinates at time t; $v(t)$ is the error vector due to measured noise and modeling error, which is assumed as a stationary Gaussian white noise process with zero mean and a variance of Ψ_0^2. For simplification, the error vector $v(t)$ is assumed that the measured noise and modeling error are uncorrelated and possesses identical statistical properties in each sensor.

For structural damage identification or health monitoring, the current state is unknown so that the measurement equation cannot be directly obtained by Eqn. 3.1. By using of Eqn. 2.2, the measurement equation of unknown state is given as follows.

$$\begin{aligned}Y^d(t)&=\Phi^d q^d(t)+v^d(t)\\&=(\Phi+\Delta\Phi)q^d(t)+v^d(t)\end{aligned} \tag{3.2}$$

By substituting Eqn. 2.8 into Eqn. 3.2, the measurement equation can be expressed as

$$Y^d(t) = (\Phi + S\Omega) q^d(t) + v^d(t) = [\Phi \quad S] \begin{Bmatrix} q^d(t) \\ \Omega q^d(t) \end{Bmatrix} + v^d(t) \tag{3.3}$$

The above equation is rewritten in a compact form as

$$Y^d(t) = \Gamma\Theta + v^d(t) \tag{3.4a}$$

$$\Gamma = [\Phi \quad S] \tag{3.4b}$$

$$\Theta = \begin{Bmatrix} q^d(t) \\ \Omega q^d(t) \end{Bmatrix} \tag{3.4c}$$

By compared with Eqn. 3.1 there are two types of variables in Eqn. 3.4, in which the former is the state vector q (i.e. the modal coordinates) and the latter is the damage coefficient matrix Ω. Eqn. 3.4 not only relates the system states with the structural damage but also connect the undamaged target modes with the damaged output measurements. Although Eqn. 3.4 assumes that the sensors measure displacement, the velocity and acceleration measurement can be obtained by the similar way.

For an efficient unbiased estimator, the covariance matrix of the estimate error for damaged or unknown system is given by

$$\begin{aligned} E\left[\left(\Theta - \hat{\Theta}\right)\left(\Theta - \hat{\Theta}\right)^T\right] &= \left[\left(\frac{\partial y^d}{\partial \Theta}\right)^T \left[\Psi_0^d\right]^{-1} \left(\frac{\partial y^d}{\partial \Theta}\right)\right] \\ &= \left[\frac{[\Phi \quad S]^T [\Phi \quad S]}{\left(\Psi_0^d\right)^2}\right]^{-1} = Q^{-1} \end{aligned} \tag{3.5}$$

where Q is the Fisher information matrix; E denotes the expected value; $\hat{\Theta}$ is the efficient unbiased estimator of Θ; Ψ_0^d is the variance of error vector for damaged or unknown structure. The Fisher information matrix can then be expressed as

$$Q = \frac{[\Phi \quad S]^T [\Phi \quad S]}{\left(\Psi_0^d\right)^2} = \frac{A_0^d}{\left(\Psi_0^d\right)^2} \tag{3.6}$$

Because of the uncorrelated and identical statistical properties of the error vector, A_0^d will be referred to as the Fisher information matrix,

$$A_0^d = [\Phi \quad S]^T [\Phi \quad S] = \Gamma^T \Gamma \tag{3.7}$$

where $\Gamma = [\Phi \quad S]$ is the output matrix of the damaged structure. It is noted that the proposed FIM is different from the FIM defined by previous literature. In this study, the FIM includes two types of information: the modal partitions Φ make the target modes of undamaged structure linearly independent while the sensitivity partitions S

reflect the sensitivity of target modes with respect to structural damage. To obtain the best estimator, the maximization of FIM (Eqn. 3.6) is carried out in a certain norm, which will result in the minimum covariance of state vector q and damage coefficient matrix Ω.

By introducing the individual goal of optimal sensor placement, the proposed FIM can deduce two special cases:

The first case (Modal Differentiability): The partitions S are omitted from the output matrix Γ. The FIM is then derived as follows

$$A_0 = \Phi^T \Phi \tag{3.8}$$

Eqn. 3.8 is as same as Effective Independent (EfI) method proposed by (Kammer, 1991), which only renders the modal partitions Φ linearly independent or observable.

The second case (Damage Identifiability): The partitions Φ are omitted from the output matrix Γ. The FIM is then derived as follows

$$A^d = S^T S \tag{3.9}$$

Eqn. 3.9 is as same as the one proposed by (Shi et. al., 2000), which only renders the sensitivity partitions S identifiable based on the assumption that the modes are observable or differentiable. This method is abbreviated as DS in this study.

From Eqn. 3.8 and 3.9, the formulations of FIMs are obviously different since modal partitions are not equal to sensitivity partitions (i.e. $\Phi \neq S$). Thus, maximization of A^d is impossible to lead to maximization of A_0. This fact will result in that the measurements in sensor locations optimized by damage identifiablity have poorly modal differentiability. In this study, the contradiction between modal differentiability and damage identifiability in accordance with the individual optimal goal will be numerically illustrated in the Section 5.

4. ALGORITHM OF OPTIMAL SENSOR PLACEMENT

If m sensors are available, the optimal sensor locations are can then be obtained by selecting m locations, out of possible N, so that a certain norm of the FIM A_0^d is maximized. According to Eqn. 3.4 and 3.7, the optimal criterion is given as bellow

$$J_1 = \max \left\| A_0^d \left(\Theta\right) \right\| \tag{4.1}$$

where operator $\|\cdot\|$ is a suitable scalar norm.

In nature, optimal sensor placement and damage identification are typically inverse problem. Therefore, the quality of the estimate is seriously dependent with the condition number of output matrix Γ or FIM. The condition number gives a measure of the estimate's robustness to sensor noise and modeling errors due to discretization error and inaccurately nominal parameters. It is noted that the maximization of FIM cannot guarantee the maximization of the condition number of the output matrix or FIM. The smaller values of the condition number are preferred to best estimator. For optimal sensor placement, the condition number should be considered in the optimal criterion, i.e.

$$J_2 = \min \left\{ cond \left[\Gamma \left(\Theta \right) \right] \right\} \tag{4.2}$$

Since the condition number is positive, the optimal criterion can be rewritten as

$$J_2 = \max\left\{cond^{-1}\left[\Gamma(\Theta)\right]\right\} \tag{4.3}$$

where $cond$ is the condition number of output matrix Γ.

By simultaneously considering norm and condition number, the optimal criterion can be defined as

$$J = \max\left\{\left\|A_0^d(\Theta)\right\| + cond^{-1}\left[\Gamma(\Theta)\right]\right\} \tag{4.4}$$

To maximize the optimal criterion (Eqn. 4.4) in a certain norm and condition number, the goodness and robustness of the estimator can be obtained simultaneously, which is important for damage identification in a real structure.

According to the definition of 2-norm and the condition number, the optimal criterion can be rewritten as

$$J = \max\left\{\sigma_1^2\left[\Gamma(\Theta)\right] + \frac{\sigma_r\left[\Gamma(\Theta)\right]}{\sigma_1\left[\Gamma(\Theta)\right]}\right\} \tag{4.5}$$

In Eqn. 4.5, the first term is not equal to the second term in the order of magnitude. To avoid that the smaller term is masked by the bigger term, the scalar transform should be carried out before the optimal process. The 2-norm of FIM and the condition number of output matrix Γ is normalized by the maximum and minimum values of the corresponding terms into the interval $[0,1]$, so that two terms in the optimal criterion can then be calculated in a same order of magnitude.

Once the number of sensors m is given, according to the optimal criterion (Eqn. 4.5), the algorithm of optimal sensor placement is proposed in the following steps:

1. Determine the number of the target modes, and compute the target modes and the corresponding sensitivity by using FEM.
2. Set $k = 1:N$, delete the k-th row of output matrix Γ that corresponds to the k-th sensor location, and calculate the normalized optimal criterion J^k.
3. Sort $J^k\left(k = 1, 2, \cdots, N\right)$ in a descending order.
4. Select the largest J^k, which means that the sensor in this location does not contribute substantially and is not robust.
5. Delete the sensor corresponding to the largest J^k.
6. Set $N = N - 1$ and form output matrix without the row deleted in step 5.
7. Repeat step 2 to 6 until the number of the remaining sensors is m, and m sensor locations corresponding to the optimal criterion give the optimal sensor placement.

5. NUMERICAL SIMULATION

To demonstrate the feasibility and effectiveness of the proposed method, the numerical examples of a truss structure were carried out in the study. The structure, which was also used to validate the algorithm by Shi et. al., is a two-dimensional truss shown in Fig. 1. The model consists of 31 spar elements, 14nodes and 28 degrees-of-freedom. The material parameters of the structure are as follows: elastic modulus $E = 70GPa$, and mass density $\rho = 2770\, kg/m^3$. The section area of the spar is $A = 0.001m^2$.

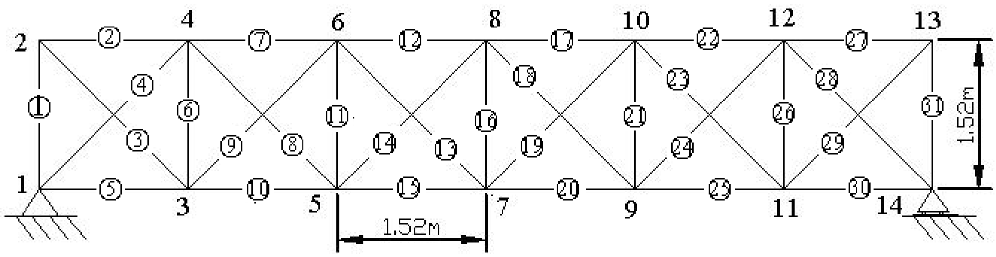

Figure 1 Planar Truss Model

In the numerical simulations, the target modes are the first three modes and the number of the sensors is 15. Firstly, the OSP was studied by the proposed method (abbreviated as DRS), EfI method and DS method. Then, the damage identification was performed with modal measurements on the optimal locations to compare the OSP effectiveness.

With the first three modes, the sensor locations were optimized by EfI and DS method, respectively. The optimal results are both listed in Table 1. From Table 1, the sensor locations are not same, which indicates that the different OSP may lead to the contradiction between the modal differentiability and damage identifiability. To remedy the contradiction, the OSP was studied by the DRS method (listed in Table 1). From Table 1, the sensor locations optimized by the proposed method are different from the results of FfI method as well as the results of Shi's method.

Table 1 Optimal sensor locations based on three different methods

	Optimal sensor locations (DOF)														
EfI	4	6	8	10	12	14	16	17	18	19	20	21	22	23	25
DS	1	3	6	7	10	11	12	13	14	15	17	18	19	20	22
DRS	1	4	6	7	8	9	10	11	13	15	16	18	20	22	23

To validate the performance of the optimal sensor locations for damage identification, the damage identification were carried out by sensitivity analysis method. The damage coefficients are solved by the following equation.

$$\hat{\delta} = [\mathbf{S}^T\mathbf{S}]^{-1}\mathbf{S}^T\Delta\Phi_i \tag{5.1}$$

The damage was assumed with a reduction in the stiffness of bars in the structure. In this study, the damage elements were bar 15 and bar 17, and the damage severity were 30% and 20%, respectively. The identification results were shown in Fig. 2. The results of the proposed method (DRS) clearly identify the damaged elements in bar 15 and 17. However, the identification results based on EfI method display that the damages occur in not only element 15 and element 17but also elements 4, 23, 27 and 29. Moreover, the identification results based on Shi's method show that the damages occur in damage elements 15 and 17 as well as undamaged elements 4, 6, 11 and 18. The numerical simulations demonstrate that the proposed method can select the sensor locations where not only the modal differentiability but also the damage identifiability are both optimal.

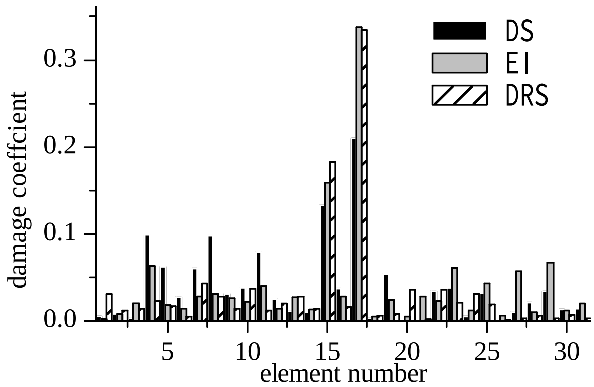

Figure 2 Damage identification with modal measurements on different sensor locations

6. CONCLUSIONS

A novel methodology of optimal sensor placement for structural damage identification, which can simultaneously meet the requirements of modal differentiability and damage identifiability, is presented in the paper. Firstly, the output of the damaged structure is expressed in time domain as a function of mode shapes, eigenvector sensitivity and modal coordinates, which relates the modal differentiability with the damage identifiability. Secondly, the formulation for the FIM is presented, and the optimal criterion is proposed to maximize the 2-norm of the FIM and to minimize the condition number of the output matrix. Thirdly, an iterative algorithm is developed to get the optimal sensor placement for the number of available sensors. Finally, numerical examples are carried out to demonstrate the feasibility and effectiveness of the proposed approach.

According the numerical results, it clearly shows that the optimal criterions, based on modal differentiability and damage identifiability respectively, can leads to the contradiction of the optimal sensor placement. The proposed FIM includes not only the modal partitions which make the target modes of undamaged structure linearly independent but also the sensitivity partitions which reflect the sensitivity of target modes with respect to structural damage. The novel FIM effectively remedies the contradiction between the modal differentiability and damage identifiability. Moreover, the maximization of the FIM results in an efficient unbiased estimator of the modal coordinates and the damage coefficients. And the minimization of the condition number makes the estimator more robust because that the parameters of the undamaged structure are not exactly known while the parameters of the damaged structure are unknown at all. The numerical simulations demonstrate that the proposed method is a promising tool of the optimal sensor placement for structural damage identification with modal measurements.

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