IDENTIFICATION OF STIFFNESS LOSS AS A MEASURE OF DAMAGE IN BUILDINGS

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ABSTRACT :

A method to locate and evaluate damaged structural and no structural elements in buildings is presented. In order to detect damage the proposed method uses as data the modal shapes and vibration frequencies of the structures. The severity of the damage is measured in terms of the changes of stiffness of the structural elements. In order to do it, the method uses the analytical model of the structure to represent its initial state without damage. The modal shapes and vibration frequencies of the damaged state of the structure are used to fit their lateral stiffness matrix. Whit this matrix and by using an iterative process, the damaged elements of the structure are detected. Application of the method is illustrated evaluating different damage states of building models. Additionally, the effect of incomplete modal information is evaluated and discussed.

KEYWORDS: Damage detection, Stiffness loss, Transformation matrix, Structural damage, damaged elements.

1. INTRODUCTION

It is of great interest for engineers to know the damage that a structure subjected to natural phenomena undergoes. For long time, the damage state of the structures has been considered subjectively through the experience. Today, several procedures based on different kinds of tests and analyses are being developed and provide quantitative estimations.
Because of their physical properties, the dynamic characteristics of the structures may vary. For this reason, these have been studied in order to quantify the observed damage. In this way, the variations in modal shapes and vibration frequencies are related to the loss of stiffness. Thus, by using the lateral stiffness of the structure, which can be obtained from its dynamic response and initial stiffness, it is possible to locate and assess its damage by comparing both states.
In this paper the problem of damage detection in building frames from its dynamic characteristics (modal shapes and vibration frequencies), is studied. In order to detect damage in structural and non-structural elements, the Transformation Matrix method, TMm (Escobar et al., 2001) is proposed. Since the changes in the stiffness of the elements that compose a structure influence directly their lateral stiffness, a geometric transformation matrix is used to locate and assess damage in these elements.
The method requires modal shapes and vibration frequencies of the current state of the structure to adjust the condensed stiffness matrix by using the procedure of Baruch and Bar-Itzhack (1978).
In this paper the damage is expressed as the loss of stiffness. With this method it is possible to locate where the structure has been damaged and to compute the percentage of its stiffness degradation.

2. BACKGROUND

All structures (buildings, bridges, offshore structures, communication towers, etc.), accumulate gradual damage through their lifetime. Thus, it is of great importance to locate and assess the damage that can be observed. In this way, it could be possible to improve its safety level by reinforcing or replacing the damaged structural elements.
When using dynamic tests to determine the vibration frequencies of structures, any reduction in their values can be interpreted as a loss of stiffness. Nevertheless, in order to detect, with certain precision level, the damage of a structure, it can be necessary to record changes in their vibration frequencies, as minimum, 5% approximately (Creed, 1995). It is important to mention that, in some structures, changes in their vibration frequencies do not imply the existence of damage. This is the case of concrete and steel bridges with changes that exceed 5% in their vibration frequencies values due only to environmental conditions (Aktan et al., 1994). On the other hand, if some repair to the structure has not been carried out, values of vibration frequencies, greater than the expected, can indicate that the stiffness of its supports increased (Morgan and Oerstele, 1994).
From results of vibration tests of reinforced concrete plane frames (Salawu, 1997), it has been observed that the degree of reduction of the natural vibration frequency, depends on the relative position of the damage with respect to modal shapes. Also, when the damage affects zones of high stresses in frames, reductions until 15% in their vibration frequencies can be observed. In the opposite case, when damaged zones present display low stress levels, damage detection using vibration frequencies is not very reliable (Salawu, 1997).
Analytical models of prestressed concrete structures, have shown that vibration frequencies are little sensitive to changes in stiffness to simulate structural damage (Camomilla et al., 1993). Whereas in real structures, if vibration frequencies are determined from ambient vibration tests where essentially only the dead load participates, losses until 50% of the prestress force are not detected by means of changes on frequencies. This is because the loss of the prestress force only reduces the load to which the excessive tension in the concrete would open cracks in the structure. So, if the tests are carried out for lower values of load they will not generate changes in its frequencies of vibration.

3. FITTING OF THE STIFFNESS MATRIX OF STRUCTURES

Most of the algorithms to detect damage in structures carry on a comparison between the analytical stiffness matrix of the structure in an initial state without damage, and the stiffness matrix of the damaged structure. The goal is to observe variations between both states in order to compute damage in the structure. One way to obtain the stiffness matrix of a structure without damage is starting from its analytical model, by means of its structural blueprints.
In this paper the global stiffness matrix is condensed to the primary degrees of freedom. In the case of buildings, the primary degrees of freedom are the lateral displacements, thus, the condensed stiffness is a lateral stiffness matrix.
To fit the stiffness matrix of a damaged structure modal parameters obtained experimentally can be used (Acevedo, 2005). This author observed that among the current methods to fit stiffness matrices of structures, the one proposed by Baruch and Bar Itzhack (1978) provides the smaller values of relative error. This method reconstructs the stiffness matrix of a damaged structure, accepting that the masses matrix is constant. The reconstructed stiffness matrix of the damaged structure, can be obtained diminishing the norm of the error between this matrix and the one of the structure without damage. This is:

$$
\begin{aligned}
[\mathrm{K_U}] = [\mathrm{K_A}] - [\mathrm{K_A}][\Phi_\mathrm{X}][\Phi_\mathrm{X}]^\mathrm{T}[\mathrm{M_A}] - [\mathrm{M_A}][\Phi_\mathrm{X}][\Phi_\mathrm{X}]^\mathrm{T}[\mathrm{K_A}] + [\mathrm{M_A}] \\
[\Phi_\mathrm{X}][\Phi_\mathrm{X}]^\mathrm{T}[\mathrm{K_A}][\Phi_\mathrm{X}][\Phi_\mathrm{X}]^\mathrm{T}[\mathrm{M_A}] + [\mathrm{M_A}][\Phi_\mathrm{X}]\left[\omega_\mathrm{X}^2\right][\Phi_\mathrm{X}]^\mathrm{T}[\mathrm{M_A}]
\end{aligned} \tag{1}
$$

where $[\mathrm{K_U}]$ is the fitted stiffness matrix of order nxn; $[\mathrm{K_A}]$ is the condensed analytical stiffness matrix of order nxn; $[\Phi_\mathrm{X}]$ is the experimental modal matrix of order mxn with $m \le n$; $[\omega_\mathrm{X}^2]$, of size mxm is a diagonal matrix whose elements are the square value of the angular frequencies; $[\mathrm{M_A}]$ is the nxn mass matrix.

4. THE TRANSFORMATION MATRIX METHOD

In figure 1, the flow chart of the Transformation Matrix method algorithm, TMm, to detect damage is shown (Galiote and Escobar, 2006).

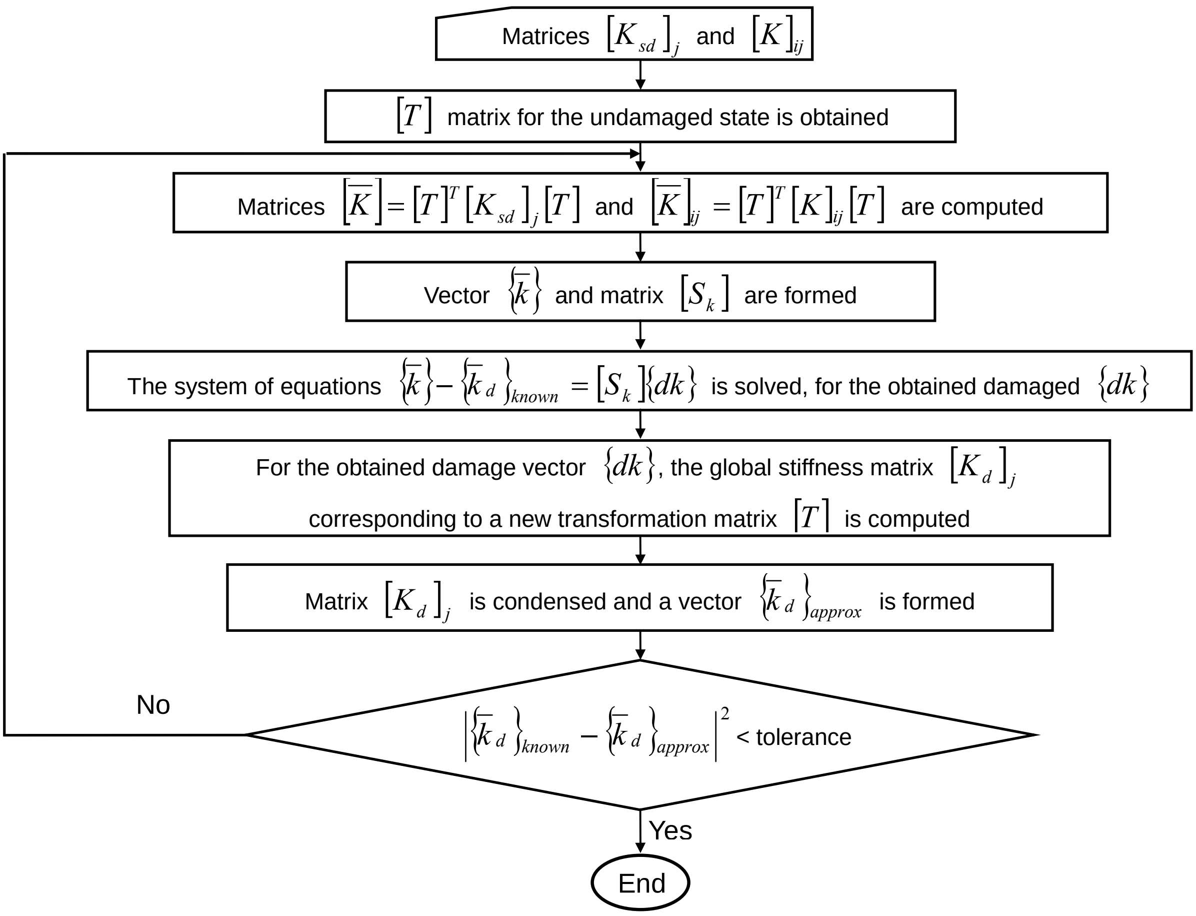


Figure 1. Algorithm of the TMm for detection of damage in structures (Galiote and Escobar, 2006).

5. CALIBRATION OF THE TRANSFORMATION MATRIX METHOD

In order to calibrate TMm, the structure of the Mass Transport System building, STC, was studied (Martinez, 2007). This was a reinforced concrete building damaged during the September 1985 earthquakes in Mexico City. It was finally demolished. The building had frames in the longitudinal direction and shear walls in the transverse one. Thus, an inner frame was analyzed (figure 2).
Dimensions of beams were 40*x*90 cm in all floors; outer columns in all storeys and inner columns in storeys one and two, 50*x*90 cm; storeys three and four, columns of 50*x*80 cm; storeys five and six, 50*x*70 cm; storeys seven to ten, 50*x*60 cm. The mass of storeys one to the nine is 15 t-m/s^2, and of storey ten is 12 t-m/s^2. An elastic modulus of 221360 kg/cm^2 was considered.

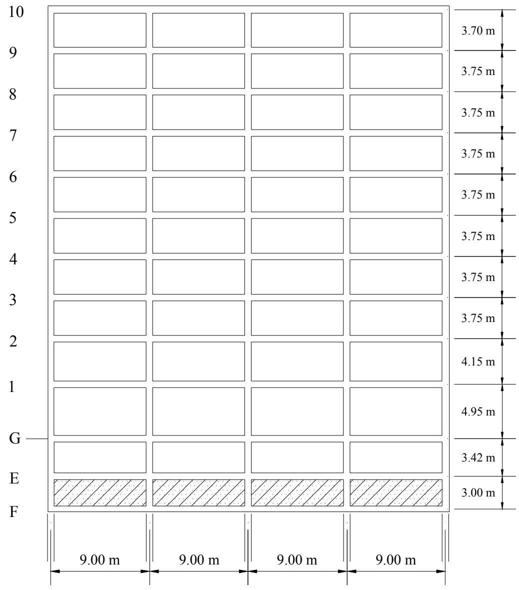


Figure 2. STC frame studied.

In order to evaluate the effect of incomplete modal information on TMm, the damage state of figure 3 was simulated. In this figure, the percentage of the simulated loss of stiffness of the structural elements is indicated.

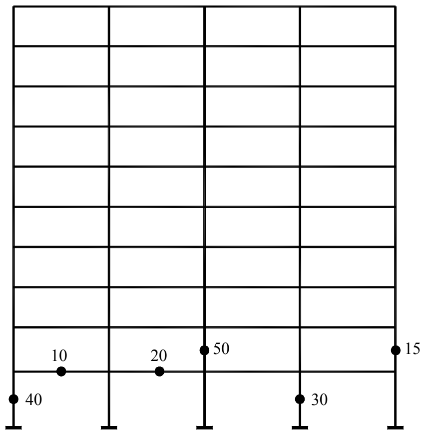


Figure 3. Simulated damage in the STC frame.

Figure 4 shows that the results of the TMm are exact when the modal information is complete. The precision of the method increases as the amount of modal shapes and vibration frequencies, used in the Baruch and Bar-Itzhack equation, is increased.
In figure 5 the same influence of mode shapes and frequencies can be observed. Relative error of the terms of the diagonal of the stiffness matrix of the structure with damage, fitted with the Baruch and Bar-Itzhack equation, with different amount of modal shapes and vibration frequencies, is presented. It can bee seen that as more modal shapes and vibration frequencies were included in the calculation, the error decreased. In this case, element K(2,2) presented the greatest value of the relative error.

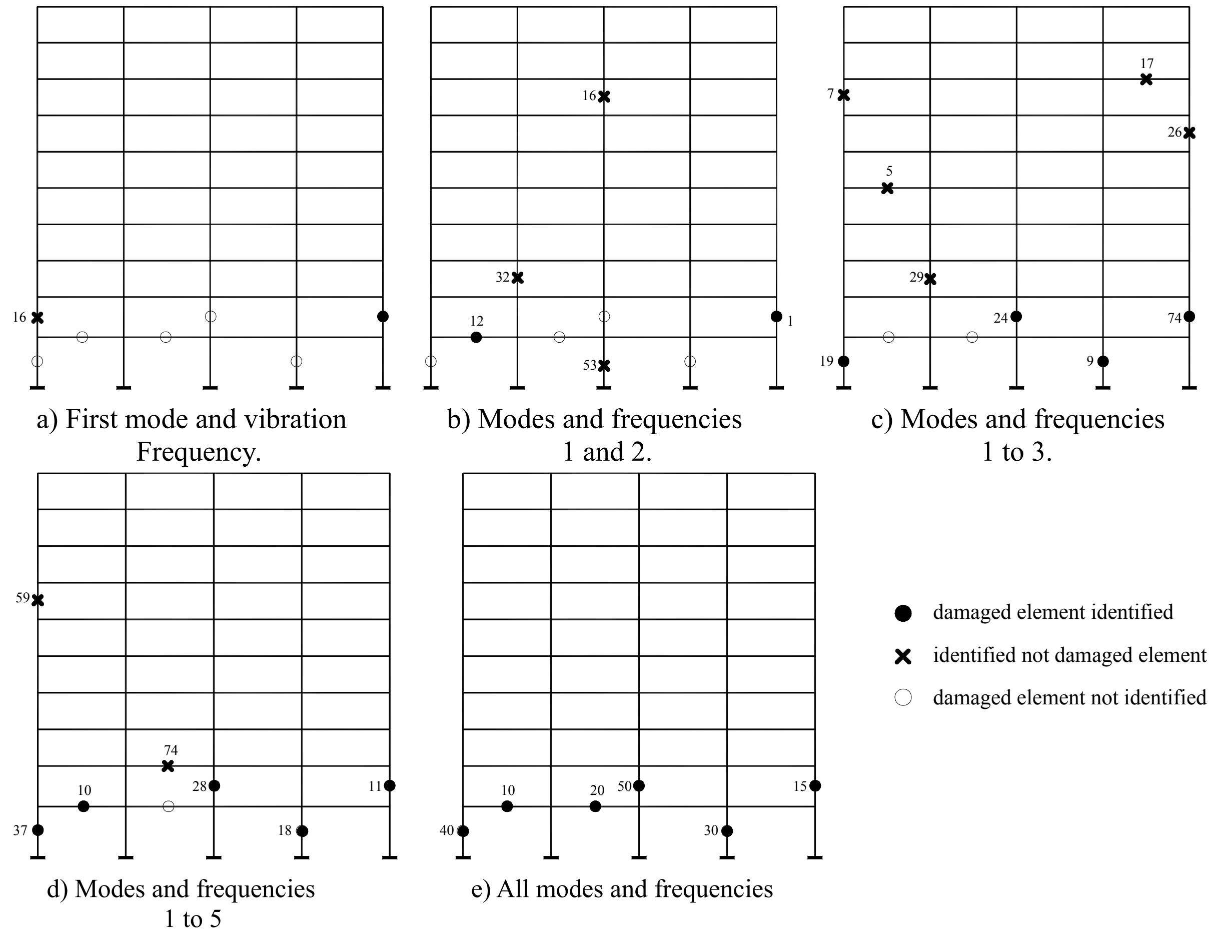


Figure 4. Detected damage in the STC frame with TMm by using different amount of modal shapes and vibration frequencies.

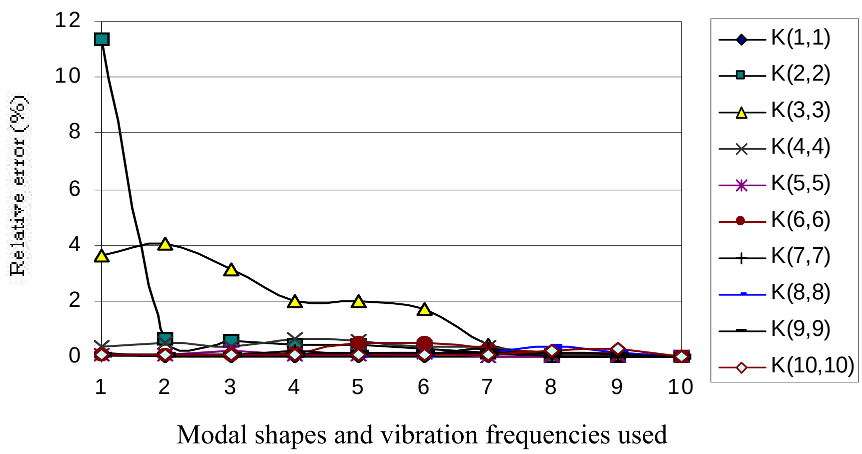


Figure 5. Relative error in the terms of the diagonal of the stiffness matrix of the damaged structure reconstructed with the Baruch and Bar-Itzhack equation.

Thus, the errors in damage detection with TMm are due to the approach used for the fitting of the stiffness matrix of the structure. In order to tackle this issue, two matrices of corrective factors were proposed and evaluated (Mendoza, 2007). The corrective factors matrix M_1 was obtained empirically and is defined as:

$$M_1 = \left(\left(\frac{1}{n^2} \right) \left(n^2 + j^2 - i^2 \right) \right) \; para \; j \geq i$$
$$M_1 = \left(\left(\frac{1}{n^2} \right) \left(n^2 + i^2 - j^2 \right) \right) \; para \; i < j \tag{2}$$

where n is the number of primary degrees of freedom of the structure (in this case, the number of storeys); i and j the row and column of the matrix, respectively.
The second corrective factors matrix M_2 was obtained from the covariance among a group of data. For instance, if A is a matrix of order mxn, its covariances matrix is (Jennings, 1992):

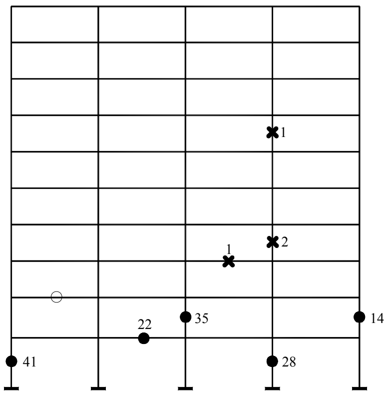
$$C = \frac{1}{m-1} X^T X \tag{3}$$

where m it is the number of rows of matrix X which is computed with the average of the values contained in each column of A, subtracting from each value of the column, the average of the same one. Matrix C is an nxn symmetric matrix. In order to obtain the correlation matrix of A, the scaled matrix X^T is needed. Thus, the matrix of corrective factors is obtained as (Mendoza, 2007):

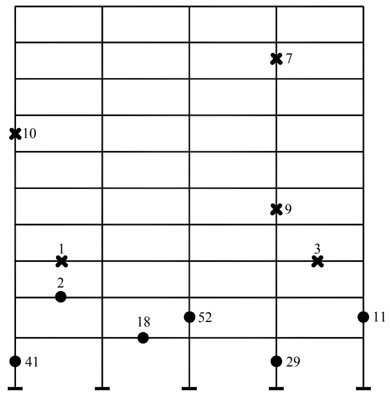
$$M_2 = \left(\frac{1}{m-1} X_m^T X \right)^{\frac{1}{n-1}} \tag{4}$$

where X_m^T is matrix X scaled.
These matrices of adjustment factors were used for the different structural models. M_1 provided better results when applied to regular plane frames and M_2 to irregular (Mendoza, 2007). For the case of frame STC the matrix of adjustment factors M_1 was used.
In order to exemplify the correction of the matrix of reconstructed stiffness, the matrix of adjustment factors was applied to the matrices reconstructed with modes 1, 1 and 2, 1 to 3 and 1 to 5, since generally, vibration tests of real structures only the first four or five modes can be obtained with precision (Galiote, 2006).
In figure 6 the damage calculated with TMm is presented. It can be observed that when fitted the stiffness matrix using different number of modes and vibration frequencies and corrected with the matrix M_1, the location of damaged elements improved and the obtained magnitude presented variations of loss of stiffness between 1 and 10% with respect to the simulated damage.
Figure 6a, presents the damage of frame STC calculated with TMm using the first mode and vibration frequency when the matrix of adjustment of reconstructed stiffness was used. It can be seen that five damaged elements were detected, a beam was not detected and three not damaged elements with percentage of loss of stiffness smaller than 2% were located.
Damage obtained using only TMm with the first two modes and frequencies of vibration and the matrix of reconstructed stiffness is presented in figure 6b. It is possible to observe that all the damaged elements were detected and additionally five elements no damaged with percentage smaller than 10%.
When TMm with the first three modes and frequencies of vibration was used, with the stiffness matrix reconstructed, five out of the six elements damaged were located with percentage of stiffness loss very

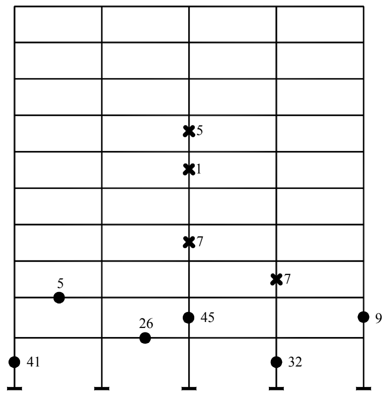
approximated to the simulated damage (figure 3). The element that was not detected was a beam and elements no damaged with percentage of damage lower than 7% were also detected.



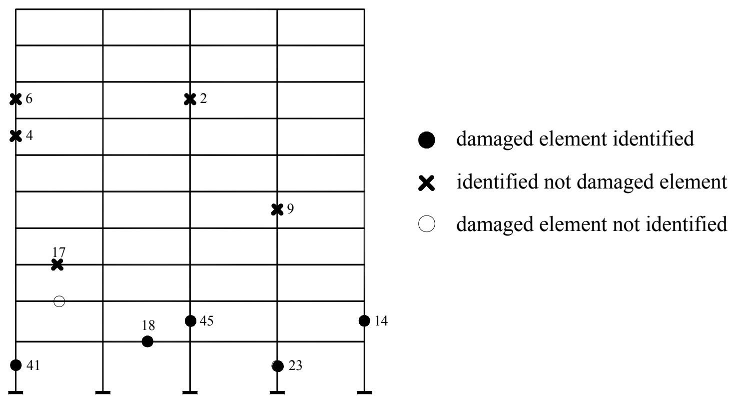
a) First mode and vibration frequency.



b) Modes and frequencies 1 and 2.



c) Modes and frequencies 1 to 3.



d) Modes and frequencies 1 to 5.

Figure 6. Damage detected with TMm in frame STC using different amount of modes and vibration frequencies to reconstruct the stiffness matrix of the structure with damage.

When TMm with the first three modes and frequencies of vibration was used, with the matrix of reconstructed stiffness fit, five out of the six elements damaged were located with percentage of loss of stiffness very approximated to the simulated damage (figure 3). The element that was not detected is a beam and elements non-damaged were detected with percentage of damage lower than 7%.
Figure 6d presents damage calculated with TMm when fitted the stiffness matrix reconstructed with the first five modes and vibration frequencies and the matrix M_I. In this case, the magnitude of the calculated damage presented variations of loss of stiffness between 1 and 7% with respect to the simulated damage. A damaged beam was not detected. In addition, not damaged elements were located.

6. CONCLUSIONS

The location and estimation of damage in structures by means of changes in its dynamic characteristics was studied.

In order to do this, the Transformation Matrix method, TMm, to locate and estimate structural damage, defined as the loss of stiffness of the structural and nonstructural elements in buildings modeled in two and three dimensions, was proposed. The location and estimation of the magnitude of the damage in the elements of the structures were made in independent form for each one of them.
The effect of non-complete modal information on the damage detection of damage by using on the TMm, was studied.
From the obtained results, it can be concluded that the TMm locates the damaged elements of a structure and determines correctly its magnitude of damage, expressed as a percentage of the loss of stiffness.
The cases presented here are part of a project which looks for establishing the relationship between the damage at local level of the structural elements, expressed as loss of stiffness, with the physical state of a structure subjected to earthquakes.

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