TORSIONAL RESPONSE IN BUILDINGS EXPOSED TO GROUND MOTIONS INDUCED BY VRANCEA EARTHQUAKES

R. Enache[1], S. Demetriu [2] and E. Albota[3]

[1]*Lecturer, Dept. of Theoretical Mechanics, Statics and Dynamics of Structures, Technical University of Civil Engineering Bucharest, Romania*
[2]*Professor, Dept. of Theoretical Mechanics, Statics and Dynamics of Structures, Technical University of Civil Engineering Bucharest, Romania*
[3] *Lecturer, Dept. of Theoretical Mechanics, Statics and Dynamics of Structures, Technical University of Civil Engineering Bucharest, Romania*
Email: enacheruxandra@msn.com, demetriu@utcb.ro, *e_albota@yahoo.com*

ABSTRACT

Natural torsion appears in systems with no coincidence between the mass and the stiffness centre. The dynamic response in this case couples torsion and translation on one or two orthogonal directions, depending on the existence of a symmetry axis. These systems are named torsional coupled systems. But torsion (known as accidental torsion) may occur even in symmetrical systems due to several reasons, as rotational components of the ground motion or uncertainties in stiffness or mass distribution. In this paper is analysed the seismic response of non-symmetrical systems (relative to both axes) acted simultaneously by the registered translational and the computed rotational components of the ground motion during several important Vrancea earthquakes (1977, 1986 and 1990). There are outlined the influence of the eccentricities and of the rotational component on the torsional response. Torsional response spectra are drawn for different site conditions.

KEYWORDS: natural torsion, accidental torsion, rotational components.

1. INTRODUCTION

Natural torsion occurs in systems where the mass center and the stiffness center lie in different points. If there is no symmetry axis, torsion and translations on two orthogonal axes are coupled in the dynamic response.It is possible to appear torsion even in symmetrical buildings, known as accidental torsion, which may be induced by the rotational components of the ground motion during an earthquake.

The rotational components can be obtained from the translational components records. The rotation about a vertical axis is proportional with the spatial derivatives of the ground motion velocities with respect to orthogonal horizontal directions. Angular displacements, velocities and accelerations will depend on the site conditions, characterized by the shear wave velocity. The results' values are in acceptable errors limits if the ground motion records are made in dense networks. Although, the spatial derivatives cannot be always used because the recording devices are often not so close one to another and the differences between the records of the same seismic event involve also the local geological conditions of the site.

2. THE ANALYSED DYNAMIC SYSTEMS

There are considered two dynamic systems (Figure 1a and b). Both systems have one level, with the mass distributed to the rigid floor supported by massless columns or shear walls. The coordinate axes have the origin in the mass centre (CM).

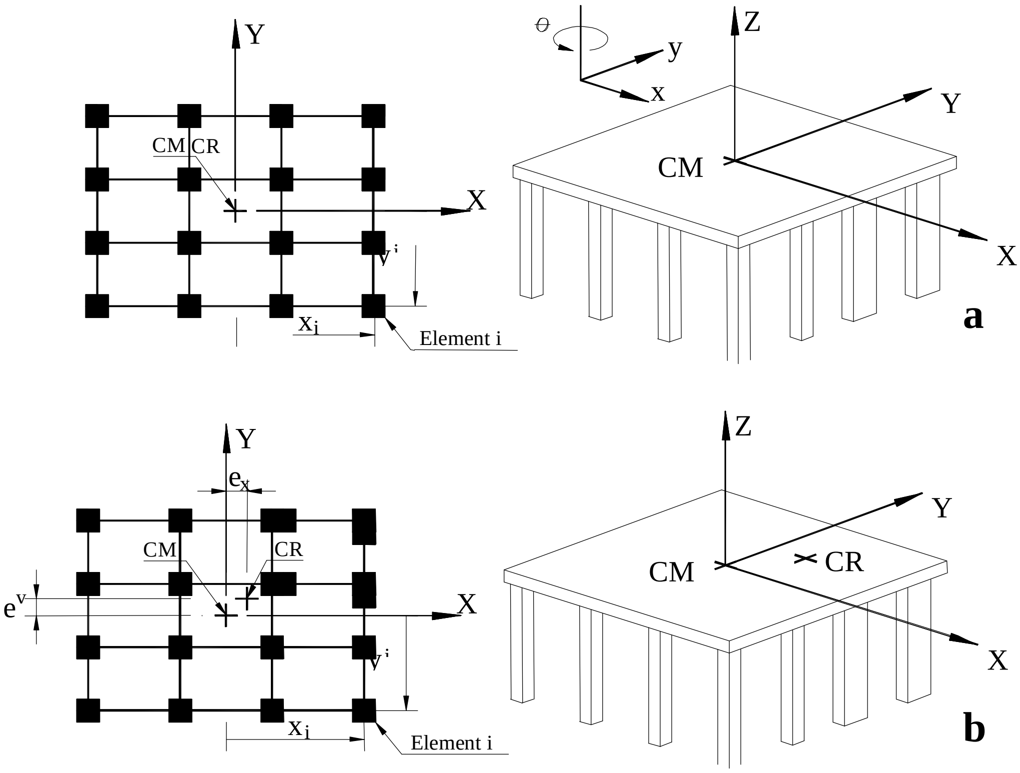

Figure1. The analysed systems

The system from Figure 1a has coincidence between mass and stiffness centre while in the system from Figure 1b these centres do not lie in the same point. Between the stiffness centre (CR) and the mass centre there are the static eccentricities e_x and e_y :

$$e_x = \frac{1}{R_y}\sum_i x_i R_{y,i} \qquad \text{and} \qquad e_y = \frac{1}{R_x}\sum_i y_i R_{x,i} \tag{2.1}$$

where $R_{x,i}$ and $R_{y,i}$ are the translational stiffnesses of the i^{th} element and x_i and y_i define the position of this element about the mass centre.

Both systems have three degrees of freedom. The dynamic coordinates associated to these degrees of freedom correspond to two horizontal translations and a rotation about a vertical axis. The system from Figure 1a is named the reference uncoupled system because there is no coupling between the degrees of freedom. The system is characterized by the circular frequencies:

$$\omega_x = \sqrt{\frac{R_x}{m}} \quad , \quad \omega_y = \sqrt{\frac{R_y}{m}} \quad \text{and} \quad \omega_\theta = \sqrt{\frac{R_\theta}{mr^2}} \tag{2.2}$$

where r is the gyration radius about a vertical axis passing through CM, R_x and R_y are the translation stiffnesses on x and y and R_θ is the torsional stiffness of the structure about CM:

$$R_x = \sum_i R_{x,i} \qquad R_y = \sum_i R_{y,i} \quad \text{and} \quad R_\theta = \sum_i R_{x,i} y_i^{\,2} + \sum_i R_{y,i} x_i^{\,2} \tag{2.3}$$

The three degrees of freedom of system from Figure 1b are coupled because of the static eccentricities. The natural frequencies, the eigenvectors and the dynamic behaviour of the coupled system depend on four dimensionless parameters: ω_θ / ω_x, ω_y / ω_x, e_x/r and e_y/r.

3. THE COMPONENTS OF GROUND MOTION

The translational accelerations are recorded in a site on *x* and *y* horizontal directions. The ground motion is considered to be the result of the superposition of two independent, non-dispersive waves propagating along the

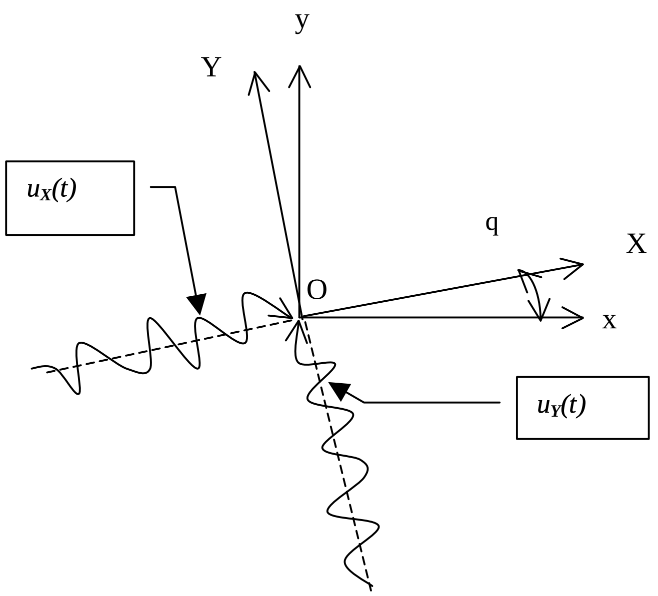

Figure 2

principal directions *X* and *Y*. The principal direction *X* makes an angle q with the recording direction *x* (Figure 2).

The displacements of the ground on *X* and *Y* directions are:

$$u_X(t) = g(Y - v_S \cdot t) \tag{3.1}$$

$$u_Y(t) = f(X - v_S \cdot t) \tag{3.2}$$

where v_S is the shear wave velocity. If the waveform doesn't change from a site to another, the spatial derivatives can be replaced by the time derivatives divided by the shear wave velocity. Rotation about a vertical axis *Z* is

$$f(t) = \frac{1}{2}\left(\frac{\partial u_Y}{\partial X} - \frac{\partial u_X}{\partial Y}\right) \tag{3.3}$$

From (3.1) and (3.2) it results:

$$f(t) = \frac{1}{2v_S}(\dot{u}_X(t) - \dot{u}_Y(t)) \tag{3.4}$$

Translational components on *X* and *Y* are given by

$$\begin{aligned} u_X(t) &= u_x(t)\cos q + u_y(t)\sin q \\ u_Y(t) &= -u_x(t)\sin q + u_y(t)\cos q \end{aligned} \tag{3.5}$$

From relationships (3.4) and (3.5) there can be obtained the following characteristics of rotational component:

The angular displacement

$$\begin{aligned}\varphi(t) &= \frac{1}{2v_S}\left(\dot{u}_x(t)\cos\theta + \dot{u}_y(t)\sin\theta + \dot{u}_x(t)\sin\theta - \dot{u}_y(t)\cos\theta\right) = \\ &= \frac{1}{\sqrt{2}\cdot v_S}\left[\dot{u}_x(t)\cos\left(\theta - \frac{\pi}{4}\right) + \dot{u}_y(t)\sin\left(\theta - \frac{\pi}{4}\right)\right]\end{aligned} \tag{3.6}$$

The angular velocity

$$\dot{\varphi}(t) = \frac{1}{\sqrt{2}\cdot v_S}\left[\ddot{u}_x(t)\cos\left(\theta - \frac{\pi}{4}\right) + \ddot{u}_y(t)\sin\left(\theta - \frac{\pi}{4}\right)\right] \tag{3.7}$$

The angular acceleration

$$\ddot{\varphi}(t) = \frac{1}{\sqrt{2}\cdot v_S}\left[\dddot{u}_x(t)\cos\left(\theta - \frac{\pi}{4}\right) + \dddot{u}_y(t)\sin\left(\theta - \frac{\pi}{4}\right)\right] \tag{3.8}$$

There have been considered the translational components on two horizontal directions, recorded in different stations in Bucharest, Focsani and Cernavoda during March 4, 1977, August 30, 1986, May 30 and 31, 1990

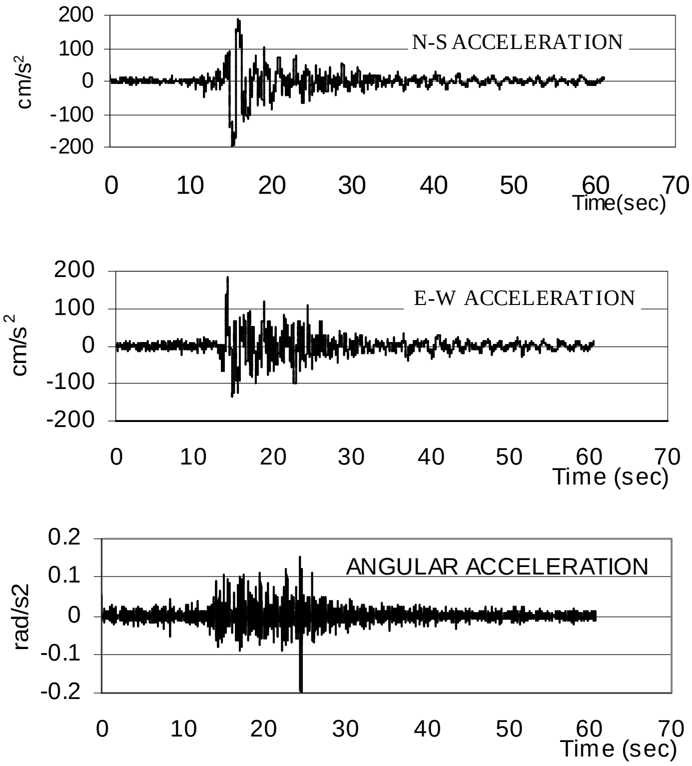

Figure 3. Time histories of the ground accelerations for Vrancea 1977, Bucharest, INCERC

Vrancea earthquakes. For each record it has been determined the angle q between the principal direction X and the recording direction x, corresponding to a zero correlation factor of the translational components of velocities projected on two orthogonal directions. The X direction is the direction for maximum dispersion and Y is the direction of minimum dispersion.

Table 1

Earthquake	Hypocenter depth (km)	Mag. M_{G-R}	Station	Component	Peak acceleration cm/s^2	Peak velocity cm/s	Peak displacement cm	Record Duration (s)
Vrancea 4 March 1977	109	7.2	Bucharest INCERC	NS	206.8	67.95	16.19	60.65
				EW	188.5	30.3	9.22	
Vrancea 30 August 1986	133	7.0	Bucharest Metalurgiei	N37W	40.7	4.82	1.01	67.37
				N127W	71.7	14.75	3.12	
			Bucharest Militari	N178E	72.1	8.42	2.26	56.65
				N92W	100.6	13.97	3.43	
			Bucharest IMGB	N30W	69.3	12.78	2.83	42.91
				N120W	59.4	7.05	1.59	
			Bucharest Panduri	N139W	96.2	15.03	2.83	51.79
				N131E	89.4	8.15	1.44	
			Bucharest Titulescu	N45E	83.8	7.52	1.37	48.86
				N45W	87.5	15.38	3.23	
			Bucharest INCERC	NS	97.0	15.51	3.75	47.97
				EW	109.1	11.31	2.56	
			Focsani	NS	166.8	27.20	5.90	43.48
				E	237.6	20.73	2.07	
			Cernavoda	NS	54.3	3.72	0.45	70.17
				EW	42.9	3.89	0.80	
Vrancea 30 May 1990	91	6.7	Bucharest INCERC	NS	66.2	6.35	1.06	52.48
				EW	98.9	16.97	2.91	
			Focsani	NS	118.0	17.09	4.41	67.16
				E	71.0	11.33	2.72	
			Cernavoda	NS	100.9	8.88	1.72	56.17
				EW	92.6	8.64	1.13	

The angular velocities and displacements time-histories are determined using (3.6) and (3.7), as a linear combination of the translational acceleration and velocities time-histories. In order to determine the angular accelerations, is either necessary to derive the translational acceleration - using (3.8) or to derive the angular velocities computed before.

The time histories of the recorded translational accelerations and of the computed angular acceleration for Bucharest-INCERC station in 1977 are represented in Figure 3.

4. THE DYNAMIC RESPONSE OF THE CONSIDERED SYSTEMS

For the torsional uncoupled system there are computed the response spectra for translational components on two directions, expressed in spectral accelerations, velocities and displacements and the torsional spectrum of the angular accelerations, velocities and displacements. These spectra are represented in Figure 4 for the components of the ground motion recorded in Bucharest - INCERC station during the 1977 Vrancea earthquake, for different damping ratios.

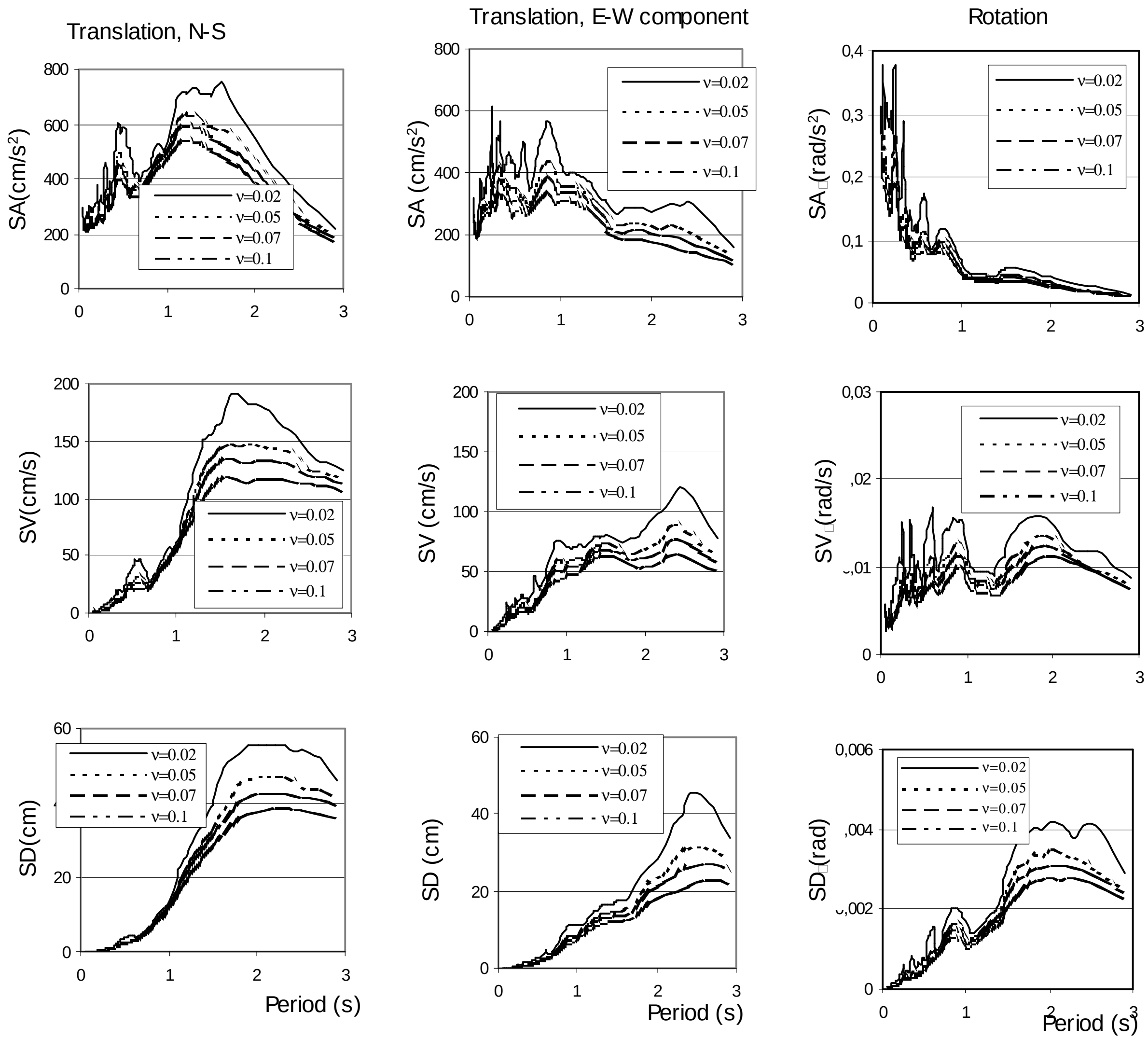

Figure 4. Translational and torsional response spectra for Vrancea 1977, Bucharest, INCERC

The normalized absolute acceleration spectra (b_f spectra) are obtained by dividing the angular acceleration spectra to the peak angular acceleration at the ground surface:

$$b_f = \frac{SA_f}{\ddot{\varphi}_{g,\max}}. \tag{4.1}$$

The use of b_f spectra in order to characterize the structural effects generated by the rotational components eliminates the dependence of the spectral values on the shear wave velocity.

The dynamic amplification spectra for torsion are presented in Figure 5 for rotational components obtained from records in Bucharest-INCERC station for 1977, 1986 and 1990 earthquakes. There are significant amplifications for systems having a period less than 0.4 s for 1990 earthquake.

An average value of b_f spectrum and a mean plus one standard deviation for the six records in Bucharest stations are presented in Figure 6. Amplifications of the response can be observed for systems with torsional periods between 0.2 and 0.5 sec.

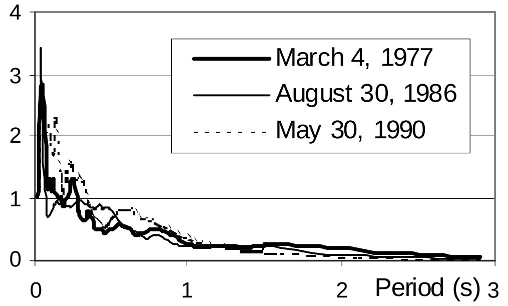

Figure 5. Dynamic amplification spectra for torsion for rotational components obtained from records in Bucharest-INCERC

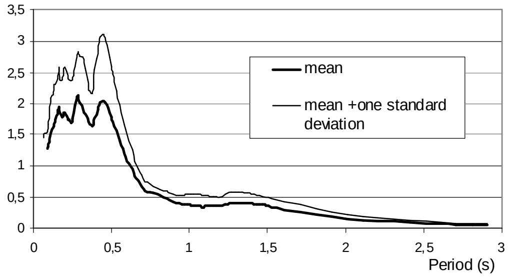

Figure 6. Average value of b_f spectrum for Bucharest , 1986.

As a case study, it is considered a coupled system with same translational stiffness on x and y ($w_x = w_y$), the same eccentricities ($e_x = e_y$) and damping ratio of 5%. The translational periods of the uncoupled system ($T_x = T_y$) have values between 0.2 and 2.2 sec. The floor is square of 10x10m. The system is excited simultaneously by the translational and the rotational components. The response of the coupled system is expressed in normalized shear forces on x and y and normalized torsional moment.

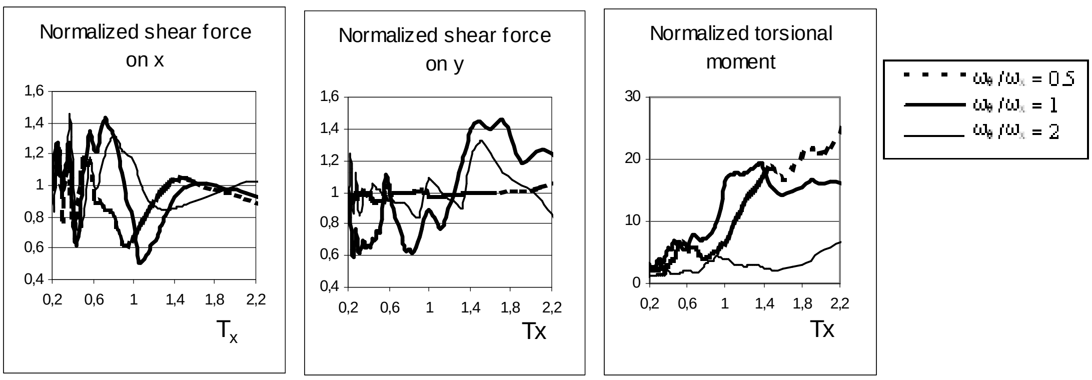

Figure 7. Eccentricity influence on the shear force and on the torsional moment

The maximum responses for systems with eccentricities of 0.3r on both directions normalized to the corresponding response of perfect symmetrical systems are represented in Figure 7 in order to see the effect of the eccentricities.

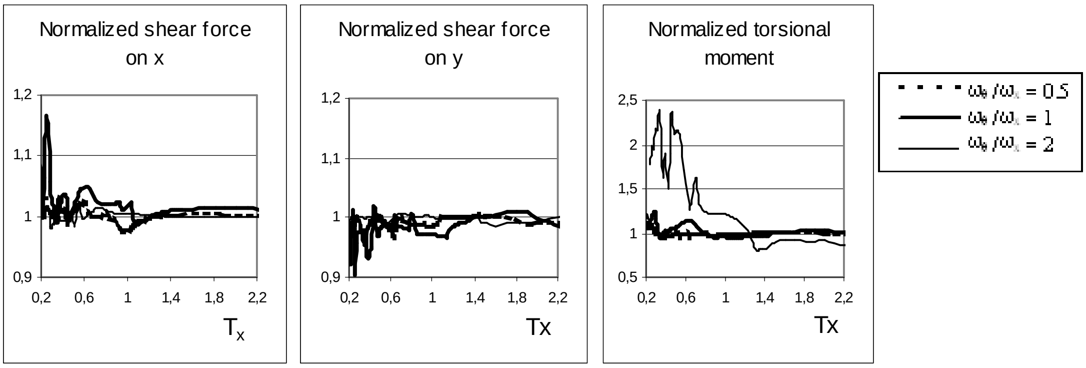

Figure 8. Rotational component influence on the shear force and on the torsional moment

The effect of the rotational component of the ground motion on the response is outlined in Figure 8 by normalizing the maximum response of a system with eccentricities of 0.3r to the corresponding response of the same system acted only by the translational components.

5. CONCLUSIONS

1. The ground motion consists of translational and rotational components. The rotational components can be obtained using linear combinations of the recorded translational components, if some assumptions are considered.
2. Translational and rotational response spectra are drawn for uncoupled systems. The torsional response spectra have dynamic amplifications for shorter periods than the translational spectra, for the analysed earthquake ground motions.
3. Eccentricities may increase or decrease (till 40 % for the studied case) the shear forces on both directions, depending on the value of the translational periods. A significant increase is produced to the torsional moment, especially for torsional flexible systems ($\omega_\theta / \omega_x = 0.5$) or with coincidence between the translational and the torsional frequencies ($\omega_\theta / \omega_x = 1$).
4. The rotational component has almost no influence on the value of the shear forces but increases the torsional moment in torsional rigid systems ($\omega_\theta / \omega_x = 2$).

REFERENCES

Castellani A., Boffi G. (1986). Rotational Components of the Surface Ground Motion During an Earthquake. *Earthquake Engineering and Structural Dynamics*.**14**, 751-767.

Enache R. (2003). Influența fenomenului de torsiune generala asupra răspunsului seismic al structurilor în cadre. Ph.D.Thesis, Bucureşti, UTCB.

Enache R., Demetriu S.(2004). Torsional response spectra of Vrancea earthquake ground motions. *Performance based Engineering for 21st Century*, Iasi, 353-359.

Lee W., Trifunac M. (1985). Torsional Accelerograms. *Soil Dynamics and Earthquake Engineering*. **4**, 132-139

De la Llera J.C., Chopra A. K. (1994). Accidental and natural torsion in earthquake response and design of buildings. *Report no. UCB / EERC* - 94/07.

Nathan N.D., MacKenzie J.R., Kevitt W.E. (1975) Structural Response to Translational and Rotational Ground Motions., *Earthquake Engineering and Structural Dynamics*, 105-115

Newmark N.M., Rosenblueth E.(1971). Fundamentals of earthquake Engineering., Prentice-Hall.

Newmark N.M. ,(1969). Torsion in Symmetrical Buildings, *Proceedings of the Fourth World Conference on Earthquake Engineering.* 19-32.

Tso W.K. ,Hsu T.-I. (1978). Torsional Spectrum for Earthquake Motions. *Earthquake Engineering and Structural Dynamics*.**6**, 375-382.