RESPONSE SPECTRUM METHOD FOR ESTIMATION OF PEAK FLOOR ACCELERATION DEMAND

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ABSTRACT:

A response spectrum method to estimate peak floor acceleration demands of multi-story buildings subjected to earthquakes and responding elastically is presented in this paper. Available response spectrum methods and modal combination rules such as SRSS or CQC have been used for estimation of displacement demands but cannot be directly used for estimation of peak acceleration. A method has been developed based on absolute modal accelerations and ground acceleration. The proposed method are extensions to CQC method which not only considers the correlation between modal accelerations, but also takes into account the correlation between ground acceleration and modal accelerations. The formulation for estimation of peak floor acceleration has been presented. Closed form solutions for correlations between the modal accelerations as well as modal accelerations and ground accelerations have been derived for filtered white noise inputs. Empirical equations for correlations between modal accelerations as well as modal acceleration and ground acceleration are presented. It is shown that the proposed method can capture the peak floor acceleration obtained from time history analysis.

KEYRORDS: peak floor acceleration, response spectrum method, correlation, continuum model

1. INTROUCTION

Response spectrum methods such as SRSS and CQC (Der Kiureghian 1980), (Der Kiureghian and Nakamura 1993) have been developed for estimation of displacements. In SRSS method, it is assumed that modal displacements are uncorrelated and therefore the total response is simply square root of sum of the square of maximum modal displacements. In CQC, the correlation between modal responses is considered and a better estimation of response can be obtained compare to SRSS method when higher modes have significant contribution to total response of the structure and correlations are not negligible.

Since in estimation of acceleration, we are interested in the absolute floor acceleration and not the relative term (which is the response parameter of interest in estimation of displacement and drift), the usual SRSS and CQC methods cannot be used directly as they were used for estimation of displacement. The major difference between methods to evaluate floor acceleration and floor displacement is that in estimation of floor acceleration, the ground acceleration term has to be included in equations and therefore, the peak ground acceleration as well as correlations that include ground acceleration has to be evaluated.

In this paper, an extension to the CQC method has been presented that writes the peak floor acceleration at a given floor based on the base excitation as well as total modal accelerations and correlations between the base acceleration and modal acceleration as well as correlations between modal accelerations. Empirical equations to estimate the correlations have been presented and compared with those obtained from FWN analysis as well as those obtained from analysis of actual ground motions. A parametric study of estimation of peak floor accelerations is presented and results from the proposed method as well as time history analysis are shown.

2. FORMULATION

The absolute acceleration of k^{th} floor can be written in terms of modal absolute accelerations as follows:

(1)

Where the parameter c_k is the contribution of ground acceleration to the total acceleration at floor k. Based on equation 1, the variance of the total floor acceleration at k^{th} floor can be written as:

(2)

Maximum response of the ith mode as well as maximum floor acceleration and peak ground acceleration can be written as functions of peak factors and standard deviations of maximums:

(3)

(4)

(5)

Substituting equations 3, 4 and 5 into 2, the peak floor acceleration at floor k can be written as:

2ckj=1NΓiϕikσmaxugtσmaxDiTtρugtDiTt12 (6)

The correlation between total accelerations of two modes of vibration can be written as:

(7)

And the correlation between the ground acceleration and total modal acceleration of i^{th} mode will be:

(8)

Where:

(9)

And GF(ω) is the one-sided power spectral density (PSD) function of the base excitation.

3. MODIFIED KANAI-TAJIMI BASE EXCITATION

The simplest representation of a ground motion is a white noise which has constant power in all frequencies. A ground motion can be assumed as a combination of series of harmonic motions with different frequencies and powers. It was known that in real ground motions; most of the power is concentrated in low frequency region and as the frequency increases, the power spectral density function of the ground motion approaches zero. The Kanai-Tajimi power spectral density better matches with the power spectral density of the real ground motions and is usually used in the field of earthquake engineering. The K-T PSD has finite power at zero frequency. The power grows as the frequency increases with a maximum at central frequency (ω_g) and then decreases afterward. The shape of the power spectral density is controlled by bandwidth (ξ_g) while ω_g and G_0 only scales the PSD in horizontal and vertical directions, respectively. The K-T PSD has a deficiency in representing the ground motions in very low frequency region where in recorded ground motions, the power approaches zero while in K-T ground motions, there is a finite power even at frequencies approaching zero (for long periods of vibration). Clough and Penzien (1975) proposed a modification to the original K-T PSD such that the power at zero frequency becomes zero:

(10)

where ωf and ξ_f are the parameters of the filter to attenuate the very low frequency region. Figure 1 shows the average power spectral density function of ground motions and the matching power spectral density of the modified K-T motion. The best match was obtained using the central frequency equal to 1.5 Hz and band width equal to 0.7. The parameters of the modified K-T PSD are ω_f = 0.1 Hz and ξ_f = 1.0, respectively.

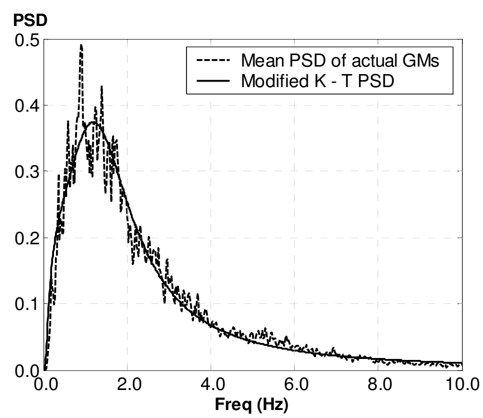

Figure 1 – Average power spectral density of ground motions and fitted modified K-T PSD.

4. CORRELATION OF MODIFIED KANAI-TAJIMI BASE EXCITATION WITH MODAL ABSOLUTE ACCELERATION

The correlation between the ground acceleration and total acceleration of an SDOF system with natural frequency of ω and damping ratio of 1, 3 and 5 percent subjected to a narrow band (ξ_g = 0.2) or wide band (ξ_g = 0.7) filtered white noise is shown in figure 2. The total acceleration response history of an undamped single degree of freedom system with frequency of ω_n when subjected to a sinusoidal acceleration with frequency of ω is:

(11)

For systems with very small natural frequency compared to the frequency of the base excitation, the response is dominated with the first term of equation 11 which means that the system oscillates in its natural frequency. Since the response is a narrow band response, its correlation with the base acceleration approaches zero. On the other side of the spectrum, the correlation of rigid components with the base acceleration approaches one. With help of equation 11, we can see that the response is dominated with the frequency of the ground acceleration and therefore the correlation approaches one. Comparing the correlation of the ground acceleration and SDOF total acceleration shows that the correlation of narrow band inputs approaches one faster than that of the wide band inputs. It is also seen that systems with smaller damping ratio have smaller correlation with the ground acceleration based on the same logic explained for the correlation of ground acceleration and modal relative acceleration.

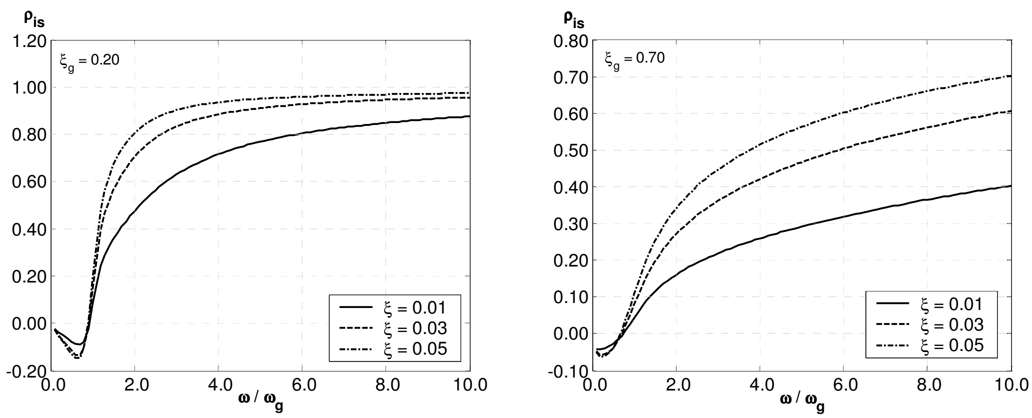

Figure 2 - correlation function between ground acceleration and SDOF system absolute acceleration for wide and narrow band ground motions

5. CORRELATION OF MODAL ABSOLUTE ACCELERATION OF SYSTEMS SUBJECTED TO MODIFIED KANAI-TAJIMI BASE EXCITATION

Figure 3 presents the correlation between two single degree of freedom systems with natural frequency of ω_i and ω_j subjected to filtered white noise inputs with central frequency of ω_g and bandwidth of 0.2 and 0.7. The correlations have been shown for SDOF systems with damping ratio of 0.002 and 0.05. Each of the curves in figure 3 corresponds to a particular value of ω_j. One observation is that the correlation of systems with higher damping is higher compared to low damped systems. This behavior can be explained with the proportionality of damping ratio and bandwidth of the response as the response of systems with higher damping ratio have wider bandwidth and therefore the correlations drop slower compared to less damped systems.

It is also seen that the correlation of two SDOF systems may not necessarily approaches zero when the two frequencies are well separated. For example, correlation of two SDOF systems with damping ratio of 5 percent when subjected to a narrow band input ($\xi_g = 0.2$) remains about one when the frequency of the two systems are very different but larger than twice the central frequency of the ground motion. In fact, as shown in figure 2, the total acceleration of an SDOF system subjected to narrow band input is completely in phase with the input acceleration and therefore, it can be assumed that the correlation of a system with frequency of ω_j with another system with frequency of ω_i when ω_i is much larger than the central frequency of the ground (ω_g) is same as the correlation between the correlation of a system with frequency of ω_j with the ground acceleration. This means that the curves in figure 3 in limit approach to the values shown in figure 2. It is also seen that the correlations of the wide band excitation are smaller than those of the narrow band excitation which matches with the results shown in figure 2.

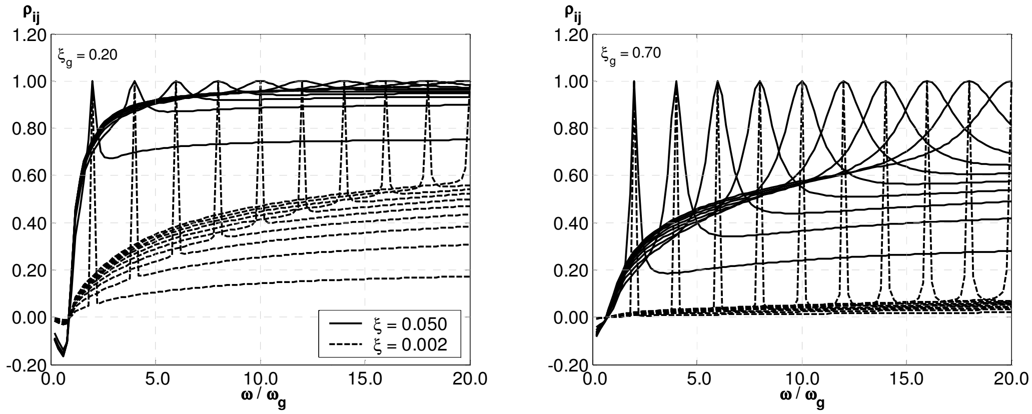

Figure 3 - correlation functions between SDOF absolute accelerations for wide and narrow band ground motions with two values of damping for the structure

6. AVERAGE AND EMPIRICAL CORRELATIONS BASED ON RESPONSE HISTORY ANALYSIS

The average correlations between the ground acceleration and total acceleration of an SDOF system with damping ratios of 1, 3 and 5 percent are shown in figure 4. It can be seen that the variation of correlation with the frequency of the SDOF system is more or less similar to the correlations obtained from wide band FWN in figure 2. The correlation of very flexible components is about -0.2 and as the frequency of the SDOF system increases, the correlation approaches 1.0 and SDOF systems with higher damping ratio have higher correlation with the ground acceleration.

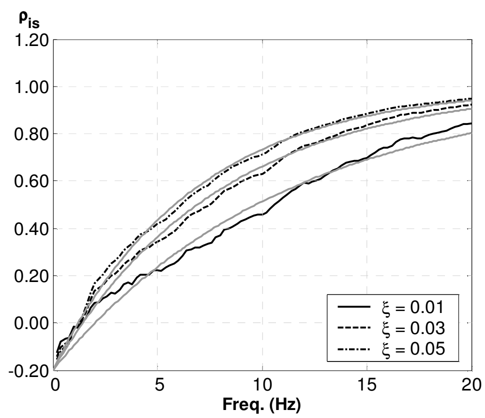

Figure 4 – average and fitted correlation between ground acceleration and SDOF absolute acceleration

The empirical correlation factors that match the correlations based on the response history analysis are shown in figure 4 with continuous lines. For an SDOF system with damping ratio of ξ, the correlation between the ground acceleration and total acceleration of the SDOF system can be estimated with the following expression:

(12)

Correlations of modal absolute accelerations based on the results of the response history analysis of SDOF systems with natural frequency between 0 and 20 Hz and damping ratio of 3 and 5 percent are shown in figure 5. Each curve in figure 5 corresponds to a particular value of ω_j while x axis shows the frequency of the i^{th} mode (ω_i). Similar to figure 3, the correlation between the total acceleration of different modes will not disappear as the frequency of the two modes are well separated and instead the correlation of i^{th} mode with frequency of ω_i and j^{th} mode with frequency of ω_j as ω_i increases approaches to the correlation between the absolute acceleration of an SDOF system with frequency of ω_j and ground acceleration which is close to zero for small values of ω_j and increases as ω_j increases and can be very close to 1 for large values of ω_j.

If the only the left branch of the bell shape curves shown in figure 5 where ω_i is less than ω_j is considered, it is seen that the correlation is controlled with a lower bound curve. In other word, the correlations tends to decrease sharply as the difference between the frequencies of the two systems increases but it cannot be less that a lower bound and this lower bound is defined by the frequency of the first system (ω_i) and damping ratio ξ. Assuming that the modes are well separated, the correlation between the modes is always controlled by the lower bound limit. The lower bounds shown in figure 5 with thick lines can be formulated as:

(13)

where ω is the minimum of ω_i and ω_j.

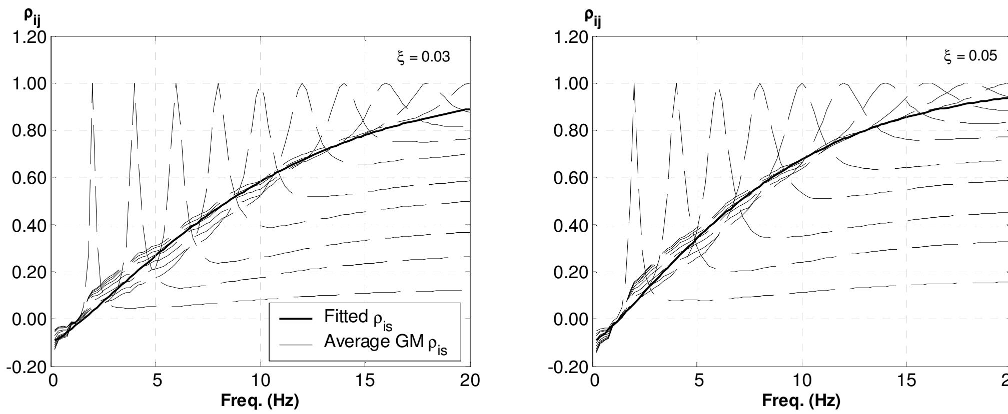

Figure 5 – average and fitted correlation between SDOF absolute accelerations

7. ANALYSIS OF MDOF SYSTEMS

7.1. Analysis Model

The accuracy of the method presented in the previous sections is verified by comparing the peak floor acceleration demands of a set of buildings with different dynamic properties obtained from a response history analysis with those computed from the response spectrum method. Structures are subjected ground motions recorded on firm soils and exact peak floor acceleration demands are computed through response history analysis. Correlations between ground accelerations and modal relative accelerations as well as correlations between relative acceleration of different modes obtained from response history analyses, filtered white noise and empirical equations are used in the response spectrum method and the accuracy of the method with different correlation functions are evaluated. Dynamic characteristics of the structures are approximated with those of the continuum model developed by Miranda and Taghavi (2005). Verification of the method is performed with constant lateral stiffness models with fundamental period of vibration equal to 0.5, 1.5, 2.5 and 3.5 s, a damping ratio of 5 percent and lateral stiffness ratios of 0, 4 and 20.

7.2. Verification of the Response Spectrum Method

Verification of the response spectrum method presented here is performed by comparing the exact peak floor acceleration obtained from response history analysis and approximate peak floor accelerations obtained from response spectrum method. Peak floor accelerations are normalized to peak ground acceleration and the median of peak floor accelerations computed with response history analysis of the continuum model subjected to the ground motions are compared with the peak floor acceleration demands computed using the response spectrum method and correlation coefficients based on filtered white noise and recorded ground motions as well as empirical equations. The first five modes of vibration are used in both response history analyses and response spectrum method.

Figure 6 shows the profile of the median of normalized peak floor acceleration demands based on response history analyses as well as the response spectrum method for structures with fundamental period of vibration equal to 0.5 s. The comparison of methods are shown for shear wall buildings ($\alpha_0 = 0$) and moment resisting frames ($\alpha_0 = 20$). It is seen that the correlations based on response history analysis, FWN and empirical equations provide similar results when used in the response spectrum method. This is in contrast with the relative acceleration version of the response spectrum method where the correlations based on FWN input overestimated the peak floor acceleration demands significantly. Therefore, the correlations based on equations 1 and 2 can be used directly for estimation of the peak floor acceleration although use of empirical equations are easier for practical purposes. The difference between the exact peak floor accelerations and approximates is very small. It is seen that at roof level of a shear beam with fundamental period of 0.5 s, the error is about 10 percent while at most of the height, the response spectrum method can estimate the response with much less error.

As the fundamental period of the structure increases, the difference between the exact and approximate peak floor accelerations becomes smaller. It is seen in figure 7 that for structures with period of vibration equal to 3.5 s, the errors between the results of the exact response history analysis and the response spectrum method are very small. It can also be seen that the response spectrum method usually underestimates the response compared to the response history analysis. Because of the small difference between the exact response and approximate response with the response spectrum method without using the peak factor ratios, it can be concluded that peak factor ratios have small effects on the accuracy of the method and therefore can be neglected.

Coefficients of variation of peak floor acceleration when obtained from response history analysis or response spectrum method for structures with fundamental period of 0.5 and 3.5 s and lateral stiffness ratio of 0 and 20 are presented in figure 8. It can be seen that the coefficient of variation resulted from the response spectrum method is about the same as the coefficient of variation resulted from the response history analysis.

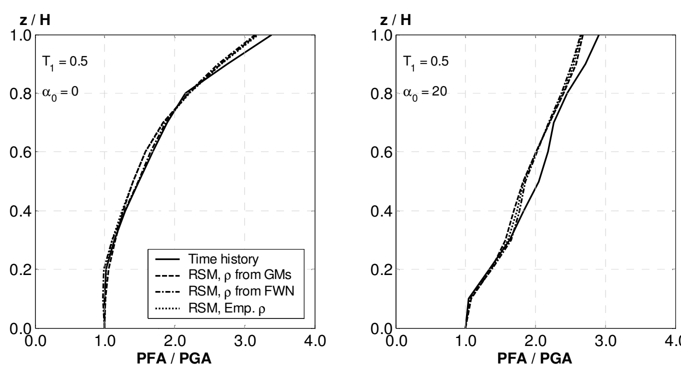

Figure 6 – comparison of peak floor acceleration profiles, time history analysis, FWN and GM, absolute acceleration method for $T_1 = 0.5$ s

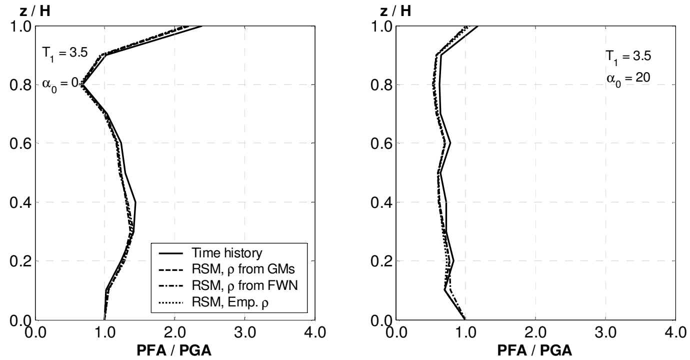

Figure 7 – comparison of peak floor acceleration profiles, time history analysis, FWN and GM, absolute acceleration method for $T_1 = 3.5$ s

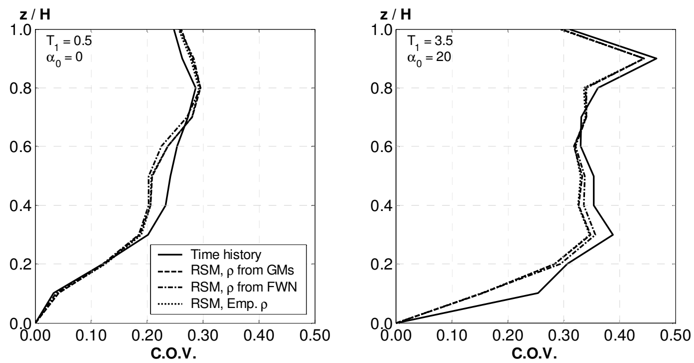

Figure 8 – coefficients of variation between time history results, GM and fitted results, absolute acceleration method

8. SUMMARY AND CONCLUSIONS

A response spectrum method to estimate peak floor acceleration demands of multi-story buildings subjected to earthquakes and responding elastically has been presented in this paper. Available response spectrum methods and modal combination rules such as SRSS or CQC have been used for estimation of displacement demands but cannot be directly used for estimation of peak acceleration. A method has been developed based on absolute modal accelerations and ground acceleration. These proposed methods are extensions to CQC method which not only considers the correlation between modal accelerations, but also takes into account the correlation between ground acceleration and modal accelerations. The formulation for estimation of peak floor acceleration has been presented. Closed form solutions for correlations between the modal accelerations as well as modal accelerations and ground accelerations have been derived for filtered white noise inputs. The modified K-T ground motions were used in this study as the filtered white noise.

The correlations based on FWN input were presented for total acceleration formulation of the response spectrum method. It was seen that the correlation of ground acceleration and total acceleration of a very low frequency SDOF system is about zero and approaches one as the frequency of the system increases with a faster rate in systems with higher damping ratio. It was also seen that the correlation between two modes do not vanish for well separated modes. Since the total acceleration of a relatively rigid system is in phase with the ground accelerations, the correlation of a system with a particular frequency with a very high frequency mode converges to the correlation of that particular mode with the ground acceleration which can be close to zero for low frequency modes and close to one for high frequency modes. Correlations based on the result of the response history analysis are very similar to the correlations based on FWN inputs. Empirical equations for correlation between the total modal accelerations as well as the correlation between the ground and total modal acceleration were presented. The efficiency of the response spectrum method with correlations based on FWN, result of response history analysis and empirical equations were tested against the exact peak floor accelerations obtained from response history analysis. The response spectrum method outputs were about the same and slightly less than the exact results. The closeness of the peak floor acceleration demands estimated with the response spectrum method and those obtained from response history analysis show that the peak factors have small effect on the method and can be neglected.

In summary, the response spectrum method presented in this paper can be effectively used in estimation of peak absolute floor acceleration demands instead of the response history analysis, particularly in cases where the ground motion is specified by a response or design spectrum. The response spectrum method provides relatively accurate estimates of peak floor acceleration demands.

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