ENERGY-BASED APPROACH OF STATIC PUSHOVER ANALYSIS

C. Kotanidis[1] and I.N. Doudoumis[2]

[1] *Dipl. Civil Engineer, Dept. of Civil Engineering , Aristotle University of Thessaloniki, Thessaloniki, Greece*
[2] *Professor, Dept. of Civil Engineering , Aristotle University of Thessaloniki, Thessaloniki, Greece*
Email: ckotanid@civil.auth.gr, doud@civil.auth.gr

ABSTRACT :

This paper presents an energy approach of Static Pushover Analysis method. It proposes the replacement of roof displacement that is used in classic "base shear – roof displacement" pushover curve with an equivalent energy-based displacement u_{en}, calculated as a function of the work of external lateral forces acting on the structure. Consistent analytical formulas are provided for the calculation of work of lateral forces acting on the floors of an N-storey frame with rigid floor diaphragms and of the equivalent energy-based displacement. The proposed methodology is applied on characteristic multi-storey concrete frames (moment frame, wall-frame system, coupled shear walls) which are analyzed using Pushover Analysis. Results indicated that classic "base shear – roof displacement" pushover curve could lead to incorrect estimation of the amount of seismic energy the structure is able to dissipate, whereas the proposed energy-based displacement leads to an energy consistent equivalent Single Degree of Freedom (ESDOF) system.

KEYWORDS: Static Pushover Analysis, Nonlinear Static Analysis, Energy, Pushover Curve

1. INTRODUCTION

Modern structures are analyzed and designed mainly by the use of linear static or linear dynamic analysis methods. Those methods are relatively simple in their application and they are explicitly described by the provisions of modern seismic codes. In the case of the evaluation of an existing structure's seismic resistance, or when a more accurate analysis of the building's inelastic seismic response is expected, then two non-linear analysis methods are generally accepted:

- Nonlinear Dynamic Analysis (Time-History) which has universal application to structures.
- Nonlinear Static Analysis (Static Pushover or just Pushover for simplicity) which can be applied to a variety of buildings that satisfy certain criteria. The main advantage of this method is a significant reduction of computational effort while maintaining the credibility of the results at an acceptable level.

Pushover analysis methodologies as presented in Seismic Codes ATC 40 (1996), Eurocode 8 (1998), FEMA 356 (2000) are under continuous development. Gupta and Kunnath (2000) presented an adaptive Pushover method in which external force profile is adjusted in each analysis step taking into account the structure's current dynamic characteristics and appropriate spectra are used for the determination of the seismic loading. Chopra and Goel (2002) developed a Modal Pushover Analysis (MPA) which accounts for the contribution of higher modes by conducting separate Pushover analyses with external force profile proportional to the structure's significant modes and combining results with the SRSS rule.

All Pushover analysis procedures have one common characteristic: They all lead to the formation of the capacity curve of the structure, also known as Pushover curve. A Pushover curve is a plot of base shear versus roof displacement as it is obtained by step by step Pushover analysis and is used to characterize the ability of the structure to dissipate energy, when subjected to an earthquake. Roof displacement is used by code provisions as an overall capacity index of the structure and is compared to code displacement demands resulting from appropriate earthquake spectra.

The use of roof displacement as the overall capacity index of the structure in Pushover analysis, though convenient and practical for the engineer, lacks a consistent theoretical basis. In addition, the area under the Pushover curve represents work of external forces that lacks natural meaning. This has also led to paradoxical analysis results in the case of a load shape vector corresponding to a higher natural mode, which is a part of Modal Pushover proposed by Chopra and Goel (2002). As it was indicated by Goel & Chopra (2002 and 2005), Hernandez-Montes et al (2004), Tjhin, Aschheim & Hernandez-Montes (2005) in certain buildings or in the case of local collapse, it is possible for roof displacement to increase at a decreasing rate or even have a reversal of the Pushover curve in the form of negative displacements, while load increases. That was the reason for the introduction of a “Modified Modal Pushover Analysis” by Chopra, Goel & Chintanapakdee (2004).

To avoid such an unnatural behavior of Pushover analysis, Hernandez-Montes et al (2004) proposed “An energy-based formulation for first- and multiple-mode nonlinear static (Pushover) analyses”. They introduced an energy based displacement for use in the Pushover curve that corresponds to the work of external lateral loads acting on the structure during Pushover analysis, thus leading to a Pushover curve with a clear natural meaning. The area under the “base shear versus energy based displacement” curve represents the total seismic energy absorbed by the building which is equal to the work of the seismic loads acting on the structure.

This paper makes a general energy-based approach of Pushover analysis, providing formulas that make use of typical Pushover analysis results in order to calculate the work of lateral loads acting on the structure and corresponding energy based displacement. The arbitrariness of the use of roof or other floor displacement in Pushover curve is demonstrated by application of the proposed methodology to characteristic examples of multi-storey reinforced concrete frames and comparing the energy-based “base shear versus energy-based displacement” curve to classic “base shear versus single floor displacement” Pushover curves.

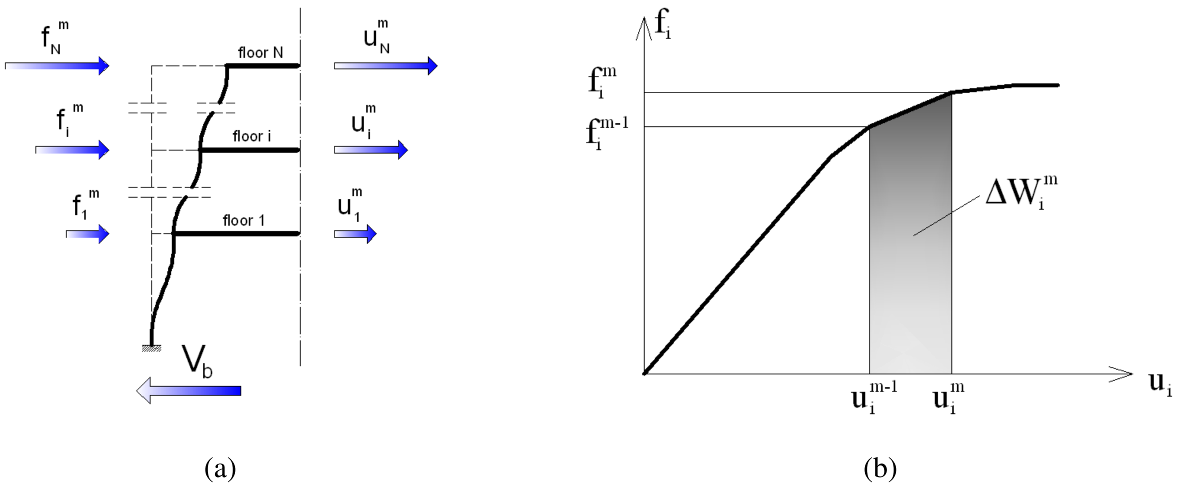


Figure 1: (a) N-floored moment frame with diaphragms at floor levels subjected to lateral forces **F** resulting in horizontal floor displacements **u** at analysis step m. (b) Pushover curve of a single floor i.

2. ENERGY-BASED STATIC NONLINEAR ANALYSIS

The N-storey moment frame with floor diaphragms of Figure 1a is subjected to lateral load profile $\mathbf{F} = [f_1 \;\; f_2 \;\; \ldots \;\; f_N]$, where f_i is the lateral load acting on floor i and N is the total number of floors. The frame is analyzed using Nonlinear Static Analysis (the term “Pushover” will be used for simplicity). Vector **F** gradually increases in size at each analysis step m and either has an invariant shape in the case of classic Pushover or takes into account the model’s current stiffness in the case of adaptive Pushover. The analysis is terminated when any of the structure’s failure criteria is satisfied. Loads **F** cause horizontal roof displacements $\mathbf{u} = [u_1 \quad u_2 \quad \ldots \quad u_N]$, where u_i is the horizontal displacement of floor i and N is the total number of floors.

At each step m of the analysis the total applied force at floor level f_i^m and horizontal displacement u_i^m are available from the analysis results. So Pushover curves for each floor i can be drawn by using pairs of (u_i^m , f_i^m). The area under each curve represents the work of external lateral force f_i. As it is indicated in Figure 1b we can calculate this work for each analysis step, if we consider linear response of the structure between two successive analysis steps. The above assumption is typical for any Pushover analysis. The work ΔW_i^m introduced to the structure by force f_i at step m is equal to the gray area of the graph:

$$\Delta W_i^m = \cdot\left(\frac{f_i^m + f_i^{m-1}}{2}\right)\cdot\left(u_i^m - u_i^{m-1}\right) \tag{2.1}$$

Total work ΔW^m introduced to the structure by the loads acting on all floors at step m is defined as the sum of the corresponding works ΔW_i^m:

$$\Delta W^m = \sum_{i=1}^{n} \Delta W_i^m = \sum_{i=1}^{n}\left(\frac{f_i^m + f_i^{m-1}}{2}\right)\cdot\left(u_i^m - u_i^{m-1}\right) \tag{2.2}$$

Equation (2.2) can also be expressed in matrix form:

$$\Delta W^m = \frac{1}{2}\cdot\left(\mathbf{F^m} + \mathbf{F^{m\text{-}1}}\right)\cdot\left(\mathbf{u^m} - \mathbf{u^{m\text{-}1}}\right) \tag{2.3}$$

where $\mathbf{F^m} = \left[f_1^m \quad f_2^m \quad \cdots \quad f_N^m\right]$, $\mathbf{F^{m\text{-}1}} = \left[f_1^{m-1} \quad f_2^{m-1} \quad \cdots \quad f_N^{m-1}\right]$,
$\mathbf{u^m} = \left[u_1^m \quad u_2^m \quad \cdots \quad u_N^m\right]$, $\mathbf{u^{m\text{-}1}} = \left[u_1^{m-1} \quad u_2^{m-1} \quad \cdots \quad u_N^{m-1}\right]$.

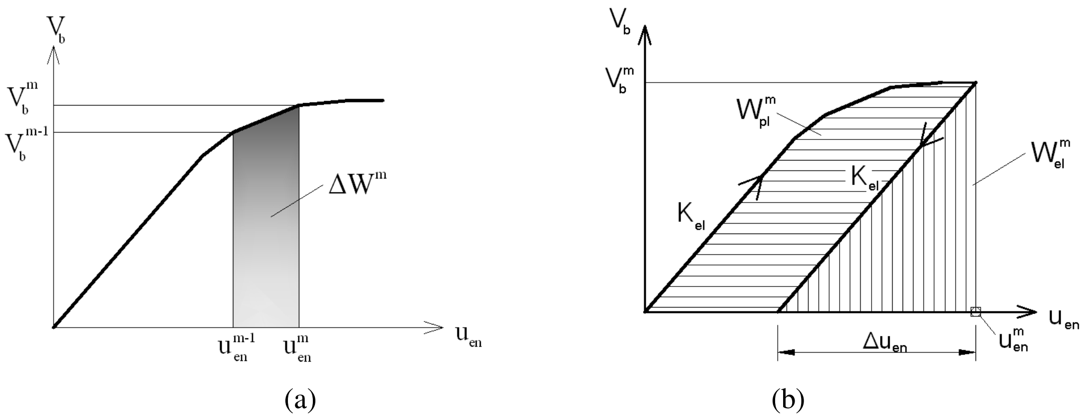


Figure 2: (a) Energy-based Pushover curve of the structure. (b) Calculation of total, elastic and plastic work of external lateral forces as a function of energy displacement u_{en}, base shear V_b and elastic lateral stiffness K_{el}.

Since the energy input during step m was defined by calculating the work of seismic forces **F** during step m, energy-based displacement u_{en} (index “en” indicates energy) of the building can be found with the help of Figure 2(a). Energy-based horizontal displacement u_{en} is defined in such a way that the area under the “base shear versus u_{en}” curve is equal to the total work of external lateral loads **F**.

$$\Delta W^{m} = \Delta u_{en}^{m} \cdot \frac{\left(V_b^{m} + V_b^{m-1}\right)}{2} \Rightarrow \Delta u_{en}^{m} = \frac{2 \cdot \Delta W^{m}}{V_b^{m} + V_b^{m-1}} \tag{2.4}$$

where Δu_{en}^{m} is the energy-based displacement increment of the structure corresponding to the work of lateral loads **F** during analysis step m.

The total energy-based displacement u_{en}^{m} at the end of step m corresponding to base shear V_b^{m} can be calculated by adding the displacement increment Δu_{en}^{m} of step m to the total displacement u_{en}^{m-1} that indicates the response of the structure from the beginning of the analysis (step 0) until step m-1:

$$u_{en}^{m} = u_{en}^{m-1} + \Delta u_{en}^{m} = u_{en}^{m-1} + \frac{2 \cdot \Delta W^{m}}{V_b^{m} + V_b^{m-1}} = u_{en}^{m-1} + \frac{1}{V_b^{m} + V_b^{m-1}} \cdot \left(\mathbf{F^{m}} + \mathbf{F^{m-1}}\right) \cdot \left(\mathbf{u^{m}} - \mathbf{u^{m-1}}\right) \tag{2.5}$$

Equation (2.5) can be applied to any Pushover analysis, regardless of the lateral force profile shape (constant or adaptive). In the case of a constant shape force profile it can be proved that u_{en} is given by equation (2.6).

$$u_{en}^{m} = \frac{\sum_{i=1}^{n} f_i^{m} \cdot u_i^{m}}{V_b^{m}} = \frac{\sum_{i=1}^{n} f_i^{m} \cdot u_i^{m}}{\sum_{i=1}^{n} f_i^{m}} = \frac{1}{V_b^{m}} \cdot \left(\mathbf{F^{m}} \cdot \mathbf{u^{m}}\right) \tag{2.6}$$

3. CALCULATION OF ELASTIC AND PLASTIC WORK IN PUSHOVER ANALYSIS

During a typical Nonlinear Static (Pushover) Analysis, the mathematical model of the structure is subjected to lateral seismic forces gradually increasing in each analysis step. The lateral forces' vector shape typically remains constant throughout the analysis, though gradually increasing in size. The structure "consumes" this input energy by elastic and inelastic deformation of its members.

In a "base shear – energy-based displacement" (V_b-u_{en}) curve, unlike all other typical "base shear – roof displacement" (V_b – u_{roof}) curves – the area between the curve and the displacement axis represents the total input energy which is the work of external forces. During elastic response, all input energy is consumed by elastic deformation of structural members. After the structure is unloaded, it returns to its unstressed condition without any remaining deformation. As soon as external loads exceed the structure's yield point, input energy is being consumed through plastic deformation of yielded structural members and through elastic deformation of the rest of the structural members which remain elastic. As the structure returns to its unloaded condition, plastic deformations remain resident, as it is shown on Figure 3. Elastic work is given by equation (3.1):

$$W_{el}^{m} = \frac{1}{2} \cdot V_b^{m} \cdot \Delta u_{en}^{m} = \frac{1}{2} \cdot \frac{\left(V_b^{m}\right)^2}{K_{el}} \tag{3.1}$$

During any step m of the analysis, plastic work W_{pl}^{m} can be calculated by subtracting elastic work W_{el}^{m} from total work W^{m} computed by the sum of work increments calculated by equations (2.2) or (2.3):

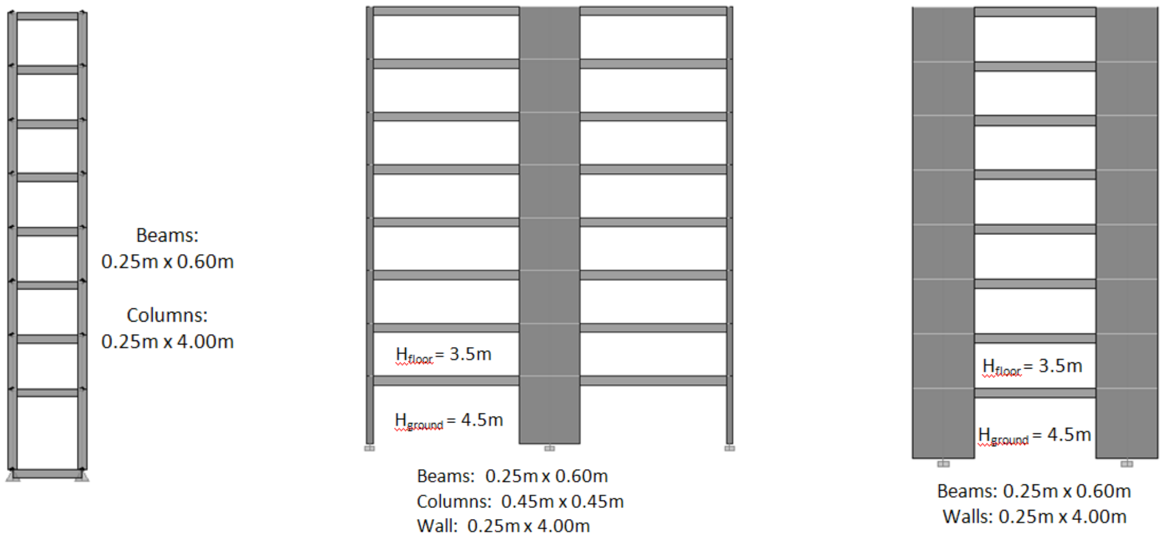
$$W_{pl}^{m} = W^{m} - W_{el}^{m} = \sum_{j=1}^{m} \Delta W^{j} - W_{el}^{m} \tag{3.2}$$

where j is the analysis step number and ΔW^{j} is the work increment corresponding to analysis step j.

It must be noted that all formulae provided for the calculation of the work of external loads or the energy-based displacement u_{en} are utilizing Pushover analysis results that are available at any step of the analysis and are provided by any modern structural analysis software utilized.

4. APPLICATION OF PROPOSED METHODOLOGY TO REINFORCED CONCRETE 2-D FRAMES

The proposed energy approach of Pushover analysis is applied on characteristic reinforced concrete plane structural systems. An 8-floor two-column reinforced concrete frame – Model 1 (Figure 3a), a dual wall-frame system – Model 2 (Figure 3b) and a system of coupled shear walls – Model 3 (Figure 3c) were analyzed using Pushover analysis. Apart from dead load, a 10 kN/m live load acting on floor beams was taken under consideration. The frames were designed using ACI 318 (2005) and IBC 2003 (2004). Section nonlinear properties were defined using bilinear moment-rotation relationships with no post-yield stiffness. Lateral loads shape was chosen to follow FEMA 356 provisions: i) A "1st mode" profile proportional to each frame's fundamental mode and floor masses and ii) a "Uniform" profile analogue to floor masses. Lateral loads are acting along with dead and 30% of live load. Analysis software SAP2000 (2005) was used for the analyses.



(a) (b) (c)

Figure 3: Characteristic r/c plain frames: (a) 8-floor one bay moment frame, (b) 8-floor dual (wall – frame) system, (c) 8-floor frame with coupled shear walls.

From the analysis input data (floor heights, seismic force profile) and the obtained analysis results (base shear, floor horizontal displacements) energy based horizontal displacement u_{en} is calculated and plotted versus building's base shear for all models. On Figure 4a floor horizontal displacements of Model 1 are plotted versus base shear V_b. For comparison, energy-based displacement u_{en} versus V_b is plotted on the same diagram. It is obvious that the shape of the base shear versus floor displacement u_i curve, which in its typical form utilizes roof displacement u_{roof}, depends on the floor displacement chosen. This choice affects the properties of the pushover curve of the equivalent single degree of freedom (ESDOF) system, which is defined through the bilinearisation of the structure's pushover curve. For the curves of Figure 4a and for a given base shear the divergence of floor displacement u_i from u_{en} ranges from -68% in the case of 1st floor to +24% in the case of roof displacement.

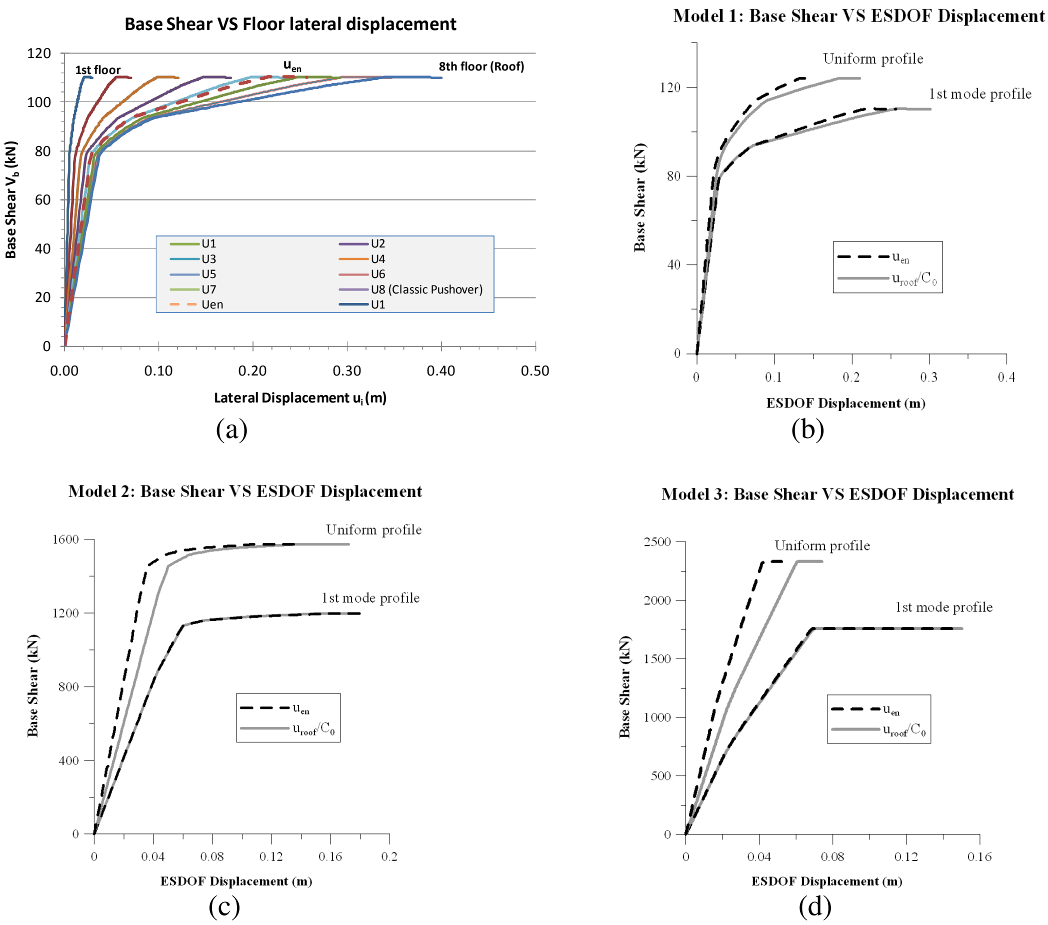


Figure 4: (a) Model 1 base shear V_b versus horizontal floor displacement u_i and energy-based displacement u_{en}. (b), (c), (d) Base shear vs ESDOF horizontal displacement (u_{roof}/Γ_1 and u_{en}) for Models 1, 2 and 3 respective.

Classic Pushover methodologies concern the use of an Equivalent Single Degree of Freedom (ESDOF) system with properties defined from the Pushover curve of the Multi-Degree of Freedom (MDOF) system. For example FEMA 356, (2000) uses coefficient C_0 in order to convert ESDOF displacement demand to Pushover analysis compatible roof target displacement. In the case of external load profile proportional to the fundamental mode, coefficient C_0 is considered equal to the first modal participation factor Γ_1:

$$u_{roof}^{target} = C_0 \cdot u_{ESDOF} = \Gamma_1 \cdot u_{ESDOF} \qquad (4.1)$$

On Figures 4(b), (c) and (d), energy-based horizontal displacement u_{en} is calculated and plotted versus building's base shear for all models and load profiles and is compared to roof horizontal displacement u_{roof} divided by the modal participation factor Γ_1, corresponding to each building's fundamental mode. The difference between u_{en} and u_{roof}/Γ_1 for a specific base shear level is easily understood, if one observes the ending points of respective curves.

Figures 5 (a1), (b1) and (c1) present total work of lateral forces versus base shear. Total work is analyzed to its elastic and plastic counterparts and is calculated as a function of lateral force and horizontal displacement of each floor. It is also equal to the area under the "V_b - u_{en}" curve. Figures 5 (a2), (b2), and (c2) refer to the "work" calculated by the area under the classic "V_b – u_{roof}/C_0" ESDOF Pushover curve. The lack of natural meaning of the work represented by the area under the classic Pushover curve that was mentioned earlier leads to the calculation of plastic and elastic work different from the actual works calculated from the load and displacement data of the analysis.

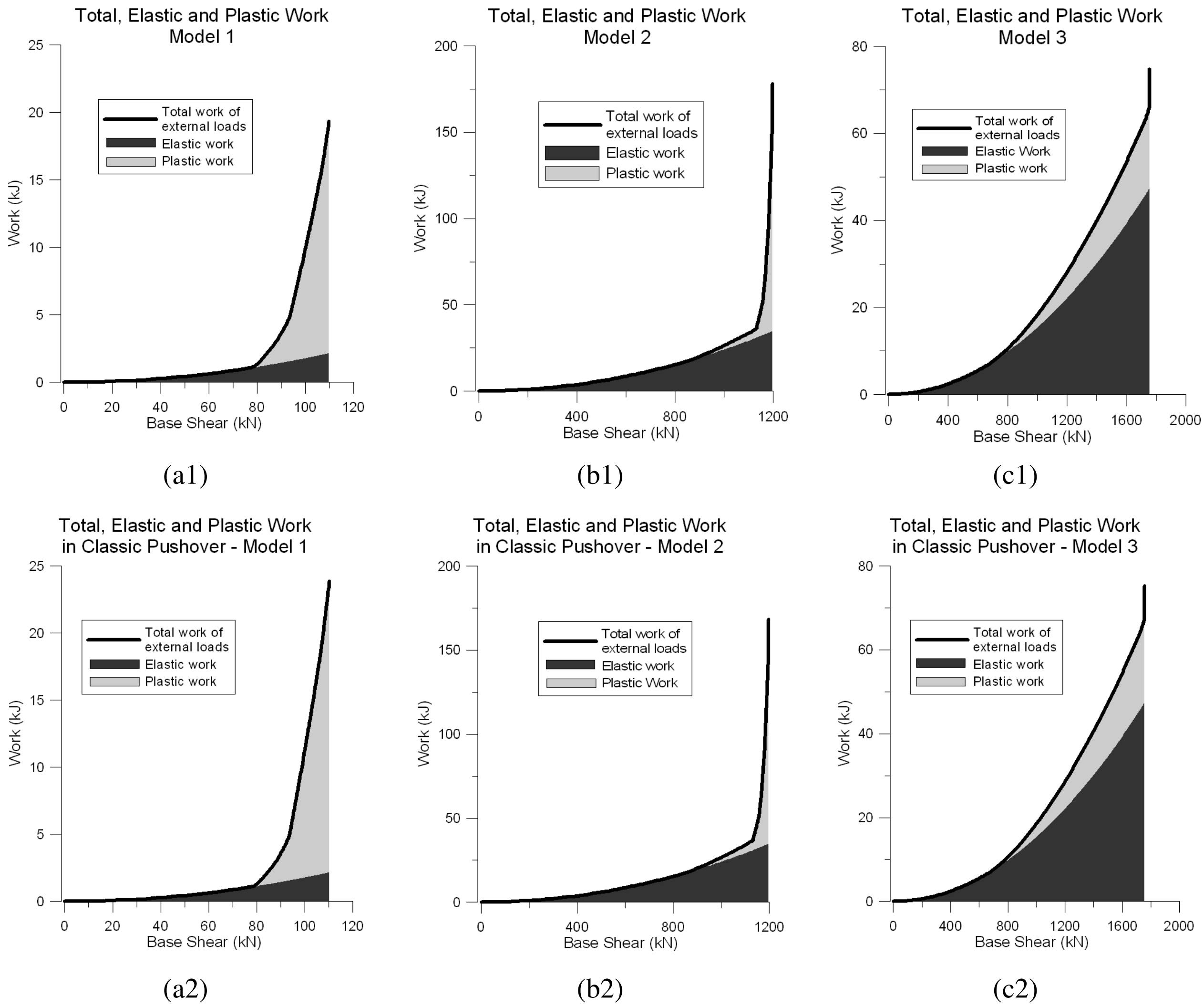


Figure 5: Elastic, plastic and total work compared to "work" of classic "V_b - u_{roof}/C_0" pushover curve.

Table 4.1: Divergence of horizontal displacements and energy

Structural Model	**Average divergence of ESDOF of classic pushover (u_{roof}/C_0) from energy based u_{en}**		**Average divergence of "work" of classic pushover from the work of external lateral loads**	
	"1st mode"	"Uniform"	"1st mode"	"Uniform"
Model 1	16.95%	38.02%	18.64%	41.63%
Model 2	-4.52%	16.06%	-5.45%	14.43%
Model 3	0.69%	39.55%	0.50%	37.50%

Since a building's seismic performance is characterized by its ability to absorb seismic energy while it is being subjected to earthquake loads, using the displacement of a single floor as the structure's response index leads to over- or under-estimation of its actual seismic response, as it is indicated by Table 4.1. For the models of our case study, classic "V_b - u_{roof}/C_0" curve presents a rather small divergence from the energy-based one when "1st mode" profile is considered. However, larger inaccuracies are noted in the case of "Uniform" load profile, where the use of a coefficient (C_o, Γ_1) for the correlation of ESDOF horizontal displacement and roof displacement yields unsatisfactory results (divergence around 40% for work and ESDOF displacement), thus the need for use of energy-based pushover curve becomes eminent.

5. CONCLUSIONS

Many efforts have been made by numerous researchers in order to improve the reliability of Pushover analysis. This paper provided consistent formulas for the calculation of the work of lateral loads acting on a structure during Pushover analysis so that determination of an energy based displacement is possible. This energy-based displacement aims to replace roof displacement on classic "base shear - u_{roof}" curve, leading to a Pushover curve that represents the actual work of external lateral loads and thus has a natural meaning. Conducting Pushover analysis on various types of reinforced concrete frames and comparing classic Pushover curves that plot base shear versus single floor (typically roof) horizontal displacement to energy-based Pushover curve that plots base shear versus energy-based displacement u_{en}, showed the arbitrariness of using displacement of a single floor as an overall seismic response index of the structure. Defining properties of equivalent SDOF system using energy-based Pushover curve should yield more reliable results than using classic roof displacement Pushover curve, an argument that should be the subject of further research.

6. AKNOWLEDGEMENTS

Parts of the research presented on this paper are realized with the support of the European Union and the Greek Ministry of Education for the Pythagoras II project (E.P.E.A.E.K.).

REFERENCES

ACI Committee 318 (2005). Building code requirements for structural concrete (ACI 318-05) and commentary (ACI 318R-05), American Concrete Institute, Farmington Hills, Mich.

ATC-40 (1996). Applied Technology Council, Comartin, C.D., Niewiarowski, R.W., Rojahn, C. & California. Seismic Safety Commission 1996, Seismic evaluation and retrofit of concrete buildings, Applied Technology Council, Redwood City, Calif.

Chopra, A.K. and Goel, R. K. (2002). A modal Pushover analysis procedure for estimating seismic demands for buildings, *Earthquake Engineering and Structural Dynamics 2002* **31**, 561–582.

Chopra, Anil K., Goel, R. K. and Chintanapakdee, C. (2004). Evaluation of a modified MPA procedure assuming higher modes as elastic to estimate seismic demands, *Earthquake Spectra* **20:30**, 757–778.

Eurocode 8, (1998). "Design of Structures for Earthquake Resistance - Part 1: general rules, seismic actions and rules for buildings", EN, vol. 1, pp. 2004.

FEMA 356 (2000). Prestandard and Commentary for the Seismic Rehabilitation of Buildings, prepared by American Society of Civil Engineers for the Federal Emergency Management Agency, Washington, DC.

Goel, R. K. and Chopra, A. K. (2002). Reversal in higher mode Pushover curves and its implications on implementation of the MPA procedure," Report distributed to participants in project ATC-55, Applied Technology Council, Redwood City, California, December 2, 19 pp.

Goel, R. K. and Chopra, A. K. (2005). Role of Higher-"Mode" Pushover Analyses in Seismic Analysis of Buildings, *Earthquake Spectra* **21:4**, 1027–1041.

Gupta, B. and Kunnath, S. (2000). Adaptive Spectra-Based Pushover Procedure for Seismic Evaluation of Structures, *Earthquake Spectra* **16:2**, 367-390.

Hernandez-Montes, E., Kwon, O. S. and Aschheim, M. A. (2004). An energy-based formulation for first-and multiple-mode nonlinear static (Pushover) analyses, *Journal of Earthquake Engineering* **8:1**, 69-88.

IBC 2003 (2004). International Code Council 2004, International Building Code, 2003, International Code Council, Country Club Hills, IL.

SAP2000, Computers and Structures Inc (2005). Extended 3D Analysis of Building Systems, Berkeley, USA.

Tjhin, T., Aschheim, M. and Hernandez-Montes, E. (2005). Estimates of peak roof displacement using "equivalent" single degree of freedom systems, *Journal of Structural Engineering* **131:3**, 517-522.