INELASTIC SEISMIC DISPLACEMENT RESPONSE PREDICTION OF MDOF SYSTEMS BY EQUIVALENT LINEARIZATION

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ABSTRACT :

An equivalent linearization procedure is developed as a simple approximate method for predicting the inelastic seismic displacement response of MDOF structural systems. The procedure employs response spectrum analysis and capacity analysis, and is mainly based on reducing the stiffness of structural members that are expected to respond in the inelastic range. The method is verified on a 12 story RC concrete frame, under 96 ground motion components.

KEYWORDS: Equivalent linearization, capacity analysis, displacement, drift, chord rotation,

1. INTRODUCTION

The analysis procedures employed for determining the earthquake performance of buildings can be grouped as linear static, linear dynamic (response history and response spectrum), nonlinear static and nonlinear dynamic. Among these, nonlinear dynamic response history analysis is accepted as the most rigorous simulation of seismic response. However, nonlinear dynamic analysis tools are not standard as linear elastic analysis tools. Nonlinear dynamic procedures may suffer from stability or convergence problems, and require considerable amount of run time and post processing efforts. Linear elastic procedures on the other hand are simple, standard, stable, and well accepted in engineering practice. However they have limited capacity in simulating inelastic seismic behavior.

In this transition period from linear to nonlinear analysis and from force-based to deformation-based assessment and design, an equivalent linearization procedure may serve as an efficient tool for seismic assessment. Properly developed equivalent linearization procedures for MDOF systems permit change of stiffness in the inelastic response range under increasing lateral forces, and fully consider the contribution of all significant vibration modes.

An equivalent linearization procedure which utilizes the familiar response spectrum analysis and capacity analysis is developed in this study for the inelastic seismic displacement response prediction of MDOF systems. The procedure is applied on a twelve story reinforced concrete plane frame for which higher mode effects are important. The predictions of the equivalent linearization procedure are compared with the nonlinear response history analysis results by utilizing 96 ground motions. Displacement and deformation response predictions from two other approximate procedures, which are the conventional linear response spectrum analysis (Chopra, 2001) and the pushover analysis according to FEMA-356 (ASCE, 2000), are also presented. Basic deformation response parameters; namely roof displacement demands, interstory drift ratios and chord rotations at member ends are used in the comparative evaluations.

2. EQUIVALENT LINEARIZATION PROCEDURE

The procedure mainly consists of reducing the stiffness of structural members that are expected to respond in the inelastic range. Combined results of demand and capacity analyses are employed for the construction of an equivalent linear system with reduced stiffness. Equivalent damping is not explicitly used in this study. Instead, maximum modal displacement demands are determined either from the equal displacement rule, or from the independent nonlinear response history analysis of SDOF systems representing inelastic modes. Hence, response spectrum analysis is utilized as the analysis tool in the implementation of the proposed equivalent linearization procedure. Further details are explained in the following paragraphs.

2.1. Elastic Demand Analysis

Demand analysis consists of gravity analysis and response spectrum analysis. In response spectrum analysis, earthquake ground excitation is expressed by its 5% damped linear elastic pseudo acceleration response spectrum. Axial forces in the columns and other members obtained from gravity analysis are employed in the capacity analysis. Moment demands obtained from both gravity analysis and response spectrum analysis are employed for determining the final yielding distribution, and accordingly the stiffness reduction.

2.2. Capacity Analysis

Capacity analysis is conducted in order to determine the member capacities first, and then to identify those member ends which have a yielding potential. In addition, base shear capacity of the building at the fundamental mode is obtained. Capacity analysis is composed of five basic steps. In step 1, positive and negative flexural capacities of beam end sections are calculated by using nominal material strengths. Step 2 is the calculation of column axial forces and moment capacities. Axial forces are required for calculating the moment capacities of columns and walls. The total axial force in a column is equal to the sum of the axial forces due to gravity loading and due to earthquake loading. Axial forces due to gravity loading can be obtained by conducting linear elastic analysis. However it is misleading to calculate the axial forces due to earthquake loading by conducting linear elastic analysis, because they are bounded by the maximum shear forces that can be transmitted from the spanning beams. Hence it is adequate to calculate the axial forces due to gravity loading by conducting linear elastic analysis, and axial forces due to earthquake loading by conducting limit analysis. Limit analysis leads to axial force values at the limit state of the structure when it reaches its lateral load capacity. This is acceptable, since most of the structures reach their base shear force capacities under a strong earthquake excitation for which they are seismically assessed or designed. The moment capacity is then calculated under the total axial force N by using the interaction diagram of the associated column.

Column-to-Beam Capacity Ratios (CBCR) are calculated at all joints in step 3 by dividing the total moment capacity of the column-ends, with the total moment capacity of the beam-ends connecting to the joint, in the direction consistent with the earthquake direction. Step 4 is the identification of potential yielding member-ends. If CBCR at a joint is smaller than 0.8, column-ends are considered as the potential yielding member-ends. If CBCR is greater than 1.2, beam-ends are considered as the potential yielding member-ends. Finally, if CBCR is between 0.8 and 1.2, all member ends connecting to the joint are considered to have yielding potential. Bottom ends of ground story columns are accepted to have yielding potential inherently. Base shear capacity of the building is estimated in step 5. Axial forces of the ground story columns due to earthquake loading and the moment capacities of these columns were calculated in Step 2. Assuming that the bases of entire ground story columns yield, base shear capacity of a structure can be estimated by considering the global moment equilibrium about a column base as shown in Figure 1. The base shear capacity estimated accordingly corresponds to the first mode for 2D models. There is no need for considering gravity loads in this analysis.

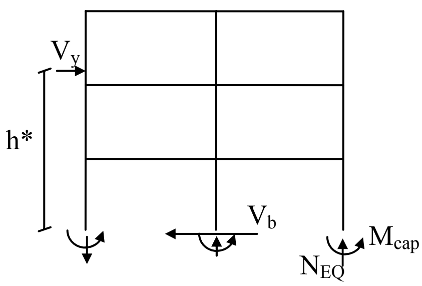

Figure 1. Calculation of base shear capacity for 2D models

2.3. Determination of Final Yielding Distribution

Potential yielding member ends were identified as a result of capacity analysis. Final yielding distribution is determined by involving the demand analysis results. Member-ends which do not have yielding potential are inherently non-yielding member ends. A potentially yielding member end is considered as a yielding member-end if the Demand-to-Capacity Ratio (DCR, Equation 1) at that end is greater than unity. Otherwise, it is considered as a non-yielding member-end.

$$\mathrm{DCR} = \frac{M_E}{M_{rc}} \tag{1}$$

In Equation 1, M_E is the earthquake moment obtained from response spectrum analysis and M_{rc} is the residual capacity moment at the considered member end. Residual capacity moment at a member end is calculated by excluding the gravity moment from the capacity moment, by considering the directions consistently. If the earthquake moment and the gravity moment are in opposite directions, gravity moment is added to the capacity moment in order to calculate the residual capacity moment. Gravity moment is subtracted from the capacity moment in the reverse case. It is assumed that the directions of the earthquake moments at member ends are controlled by the fundamental vibration mode in the considered earthquake direction, as stated previously.

2.4. Stiffness Reduction

Stiffness of the structure is reduced by reducing the flexural stiffnesses of structural members so that it corresponds to their secant stiffness to the peak elastic deformation demand. Member stiffness reduction is achieved by reducing the moment of inertia.

Considering that beams and columns of the structural frames are prismatic members, it can be accepted that moment of inertia 'I' along the member length is constant. Reduced moment of inertia of a member is calculated by considering the relation between the sum of moments and the sum of chord rotations at the member ends. Chord rotation at a member end is defined as the angle between the chord which connects the two ends of a member, and the tangent to the deflected shape at the considered member end, which is illustrated in Figure 2.

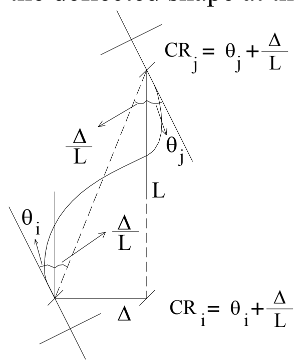

Figure 2. Definition of chord rotation at the i and j ends of a frame member

The relationship between chord rotations and moments at the ends of a linear elastic prismatic member is expressed as

$$\begin{bmatrix} M_i \\ M_j \end{bmatrix} = \frac{2EI}{L}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} \theta_i \\ \theta_j \end{bmatrix} \tag{2}$$

In Equation 2, M_i and M_j are the moments and θ_i and θ_j are the chord rotations at the i and j ends of a member, respectively. E is the modulus of elasticity, L is the clear span length and I is the moment of inertia. Summing the first and second rows in Equation 2 gives

$$M_i + M_j = \frac{6EI}{L}\left(\theta_i + \theta_j\right) \tag{3}$$

Equation 3 can then be used in order to relate the sum of elastic moments and sum of chord rotations at the member-ends obtained from response spectrum analysis.

$$\sum M_E = \frac{6EI}{L} \sum \theta_E \tag{4}$$

"Σ" in Equation 4 designates the sum of moments or chord rotations at both ends of a member. M_E and θ_E denote the earthquake moment and earthquake chord rotation obtained from response spectrum analysis, respectively. The relationship between the sum of residual capacity moments and the sum of chord rotations calculated from response spectrum analysis can be expressed similarly, by employing the equal displacement rule (Figure 3).

$$\sum M_{rc} = \frac{6EI'}{L} \sum \theta_E \tag{5}$$

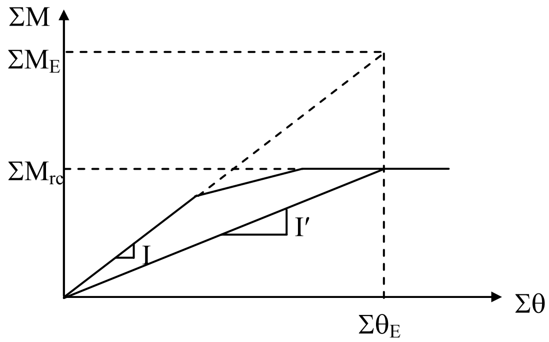

Figure 3. The variation of total end moments with total chord rotations at member-ends in the linear elastic and inelastic ranges

I' is the effective (reduced) moment of inertia and ΣM_{rc} is the sum of residual capacity moments at both ends of the member in Equation 5. Dividing Equation 4 by Equation 5 and rearranging, one obtains

$$I' = \frac{I}{R_M} \tag{6}$$

where R_M is the reduction factor defined by

$$R_M = \frac{\sum M_E}{\sum M_{rc}} \tag{7}$$

R_M expressions for different yield states are presented in Table 1.

Table 1. RM corresponding to different yielding situations

Yielding Situation	**Reduction Factor**
Both ends non-yielding	$R_M = 1$
One end yielding	$R_M = \frac{\sum M_E}{M_{rc,i} + \alpha M_{rc,j}} \quad \alpha = \begin{cases} CBCR & \text{if } CBCR < 1 \\ 1/CBCR & \text{if } CBCR > 1 \end{cases}$ *
Both ends yielding	$R_M = \frac{\sum M_E}{M_{rc,i} + M_{rc,j}}$

CBCR is the column-to-beam capacity ratio around the joint to which the non-yielding end connects

2.5. Spectral Displacement Demands for the Equivalent Linear System

The global displacement demand of the reduced equivalent linear system can not be determined by conducting response spectrum analysis under a ground motion unless an increased ("equivalent") damping is imposed for reflecting the hysteretic energy dissipation of the actual inelastic system. Global displacement demands are directly obtained from the spectral displacement demands in each mode of the equivalent linear system in this paper. These spectral displacement demands are called the target spectral displacements Sdn*.

Target spectral displacement in the first mode is calculated via nonlinear response history analysis of the equivalent SDOF system representing this mode (Sdni*). When the hysteresis behavior of the SDOF system is elastoplastic, three parameters are required in each mode n for conducting nonlinear response history analysis. These are the period T_n , the base shear capacity ratio and the viscous damping ratio (5%). T_n is obtained from the eigenvalue analysis of the original, unreduced-stiffness structure whereas the base shear capacity in the first mode is estimated as a result of capacity analysis (see Figure 1).

Target spectral displacements for the higher modes are assumed to be equal to the elastic spectral modal displacements of the original unreduced-stiffness structure (Sdne*). Spectral displacements corresponding to the higher modes usually remain in the elastic range under most ground excitations. The procedure introduced herein is referred as EQL-NL.

2.6. Response Spectrum Analysis of the Equivalent Linear System

Response spectrum analysis of the equivalent linear system is conducted after determining the target spectral modal displacements, Sdn* through one of the two alternative procedures above. In the implementation of response spectrum analysis, modal force vector at any mode ($\mathbf{f}_n'$) should be determined such that the n'th mode SDOF system is subjected to a maximum displacement that is equal to the target spectral displacement calculated for that mode. For this purpose, the pseudo acceleration (PSan*) corresponding to the n'th mode target spectral displacement (Sdn*) is calculated from Equation 8.

$$\mathrm{PSa}_n^* = \mathrm{Sd}_n^* \left(\frac{2\pi}{T_n'} \right)^2 \tag{8}$$

T_n' in Equation 8 is the n'th mode vibration period of the reduced system. Then the n'th mode lateral force vector ($\mathbf{f}_n'$) is calculated from,

$$\mathbf{f}_n' = \Gamma_n' \mathbf{m} \boldsymbol{\varphi}_n' \mathrm{PSa}_n^* \tag{9}$$

In Equation 9, $\boldsymbol{\varphi}_n'$ and Γ_n' are the mode shape vector and modal participation factor at the n'th mode of the equivalent linear system respectively, and m is the mass matrix. Maximum responses in each mode are finally calculated by applying the modal force vector in Equation (10) to the equivalent linear structure. Then these modal responses are combined by using statistical combination rules, SRSS or CQC. Implementation of the equivalent linearization method is demonstrated schematically in Figure 4.

2.7. Determination of Response Parameters

As a result of gravity and response spectrum analysis conducted for the equivalent linear system, displacement response parameters including story displacements, interstory drift ratios and chord rotations are calculated.

3. GROUND MOTIONS EMPLOYED IN CASE STUDIES

96 near-fault ground motion components are employed in the case studies. They possess PGV values greater than 35 cm/s, and their closest distance to fault plane is less than 20 km. All of the 96 ground motion records were downloaded from the PEER strong motion database. The acceleration response spectra of the ground motions are shown in Figure 5, along with the code design spectrum. It can be observed that the mean spectrum matches with the code spectrum quite well along the entire period range.

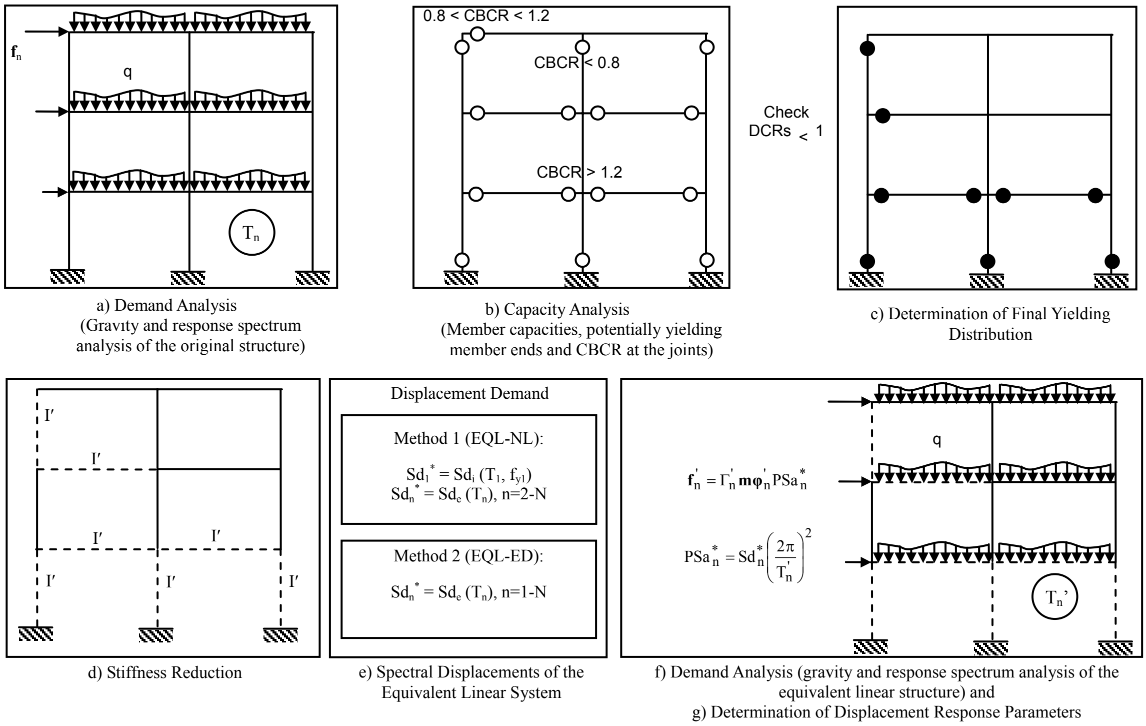

Figure 4. Schematical demonstration of the equivalent linearization procedure

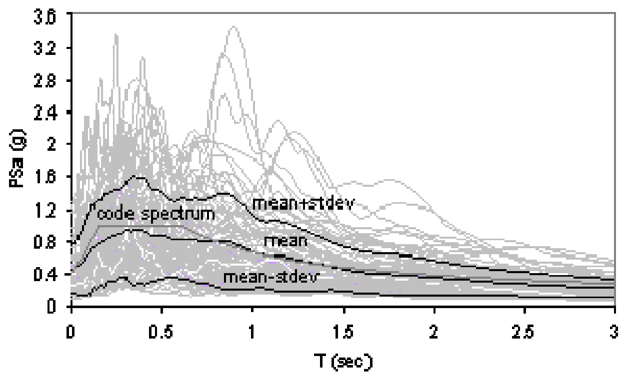

Figure 5. Acceleration response spectra for the 96 ground motion

4. CASE STUDY: 12 STORY RC SYMMETRICAL FRAME

The case study is a twelve story reinforced concrete frame building with the floor plan shown in Figure 6. The building is designed according to the regulations of Turkish Earthquake Code (2007) in accordance with the capacity design principles. An enhanced ductility level is assumed for the building. The design spectrum is shown in Figure 5. Concrete and steel characteristic strengths are 25 MPa and 420 MPa, respectively. Slab thickness for all floors is 140 mm and live load is 3.5 kN/m2. Dimensions of the beams at the first four, the second four and the last four stories are 300x550, 300x500 and 300x450 mm2 respectively, whereas dimensions of the columns at the first four, the second four and the last four stories are 600x600, 550x550 and 500x500 mm2 respectively. There is no basement; height of the ground story is 4 m while the height of all other stories is 3.2 m. A plane model consisting of Frames A and B is constructed for the analysis.

The plane frame is modeled by using the nonlinear analysis software Drain-2DX (1993). All required nonlinear and linear analyses are conducted with this software. Inelasticity is restricted to the lumped flexural plastic hinges at the member ends. Cracked section stiffness is employed for the initial linear segment of the moment-curvature relations. Gross moments of inertia are multiplied with 0.6 and 0.5 for the columns and beams, respectively, in order to represent cracking. Free vibration periods and effective modal mass ratios for the first three modes of the twelve story frame are 1.95, 0.70, 0.40 seconds, and 0.777, 0.125, 0.045, respectively. Although the effect of higher modes may not be apparent regarding the effective modal mass ratios, their contribution to seismic response is considerable for most ground motions due to higher response accelerations at the vibration periods of these modes.

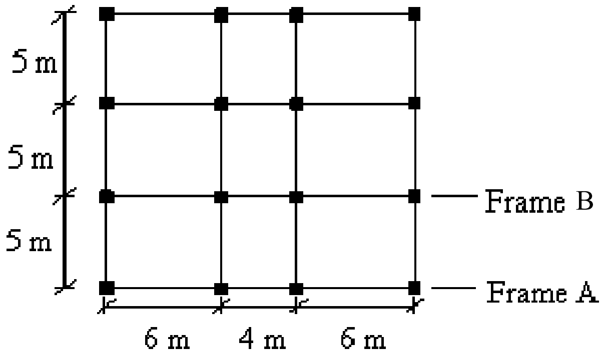

Figure 6. Story plan of the 12 story building

Maximum values of story displacements, interstory drift ratios and chord rotations are obtained for each ground motion record. The results from nonlinear response history analysis (NRHA), pushover analysis according to FEMA-356 (PO-1), the equivalent linearization procedure (EQL-NL) and linear elastic response spectrum analysis (RSA) are presented comparatively.

All approximate methods considered herein have their own ways of calculating a roof displacement demand. However, comparison of local response parameters is conducted at the same roof displacement with NRHA. The purpose for such an adjustment is to observe the accuracy of the approximate methods in estimating the distribution of local response parameters within the structure.

Median values of interstory drift ratios and beam chord rotations obtained from the employed analysis methods are plotted in Figure 7. For each method and each ground motion, the maximum value of a response parameter is obtained, and then the statistical parameters are calculated for each method by using all ground motions in the set. It should be noted that, for a ground motion, the maximum values of different response parameters are obtained at different times in response history analyses. Beam chord rotation at a story level indicates the average of the chord rotations at the beam ends at that story level.

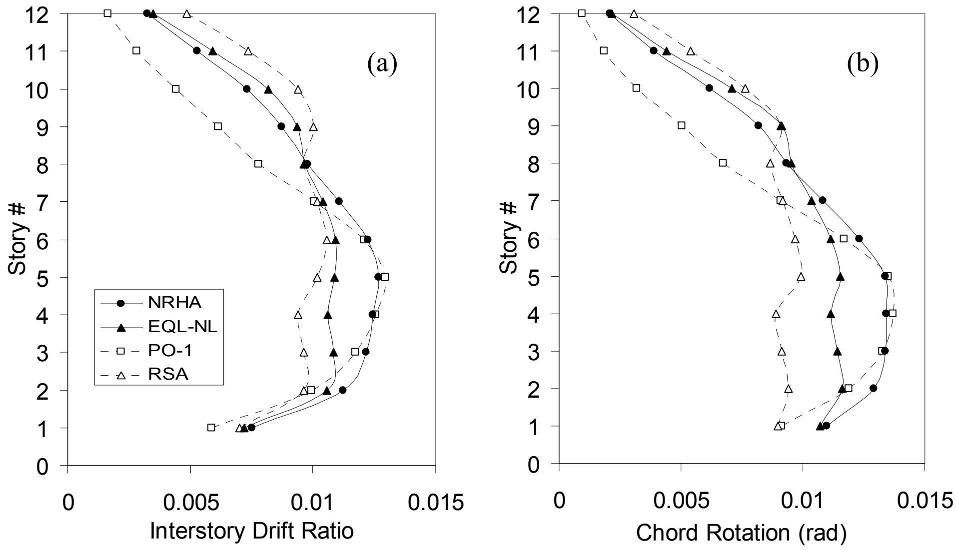

Figure 7. Median values of a) interstory drift ratios, b) beam chord rotations.

Interstory drift ratios obtained from the approximate methods normalized by those obtained from NRHA are plotted in Figure 8 for all ground motions. In this figure, each thin gray line corresponds to a different ground motion and the thick gray lines indicate the median and median ± standard deviation values. Beam chord rotations produce similar results; hence they are not presented herein.

It can be observed from Figures 7 and 8 that equivalent linearization procedure significantly improves the response predictions of RSA. Improvement is substantial for the interstory drift ratios and beam chord rotations, those especially at the upper stories. RSA overestimates beam chord rotations and interstory drift ratios at the upper stories. Equivalent linearization procedure is successful in bringing these values closer to NRHA results. On the other hand, PO-1 underestimates interstory drift ratios and beam chord rotations at the upper stories since higher mode contributions are not considered in PO-1.

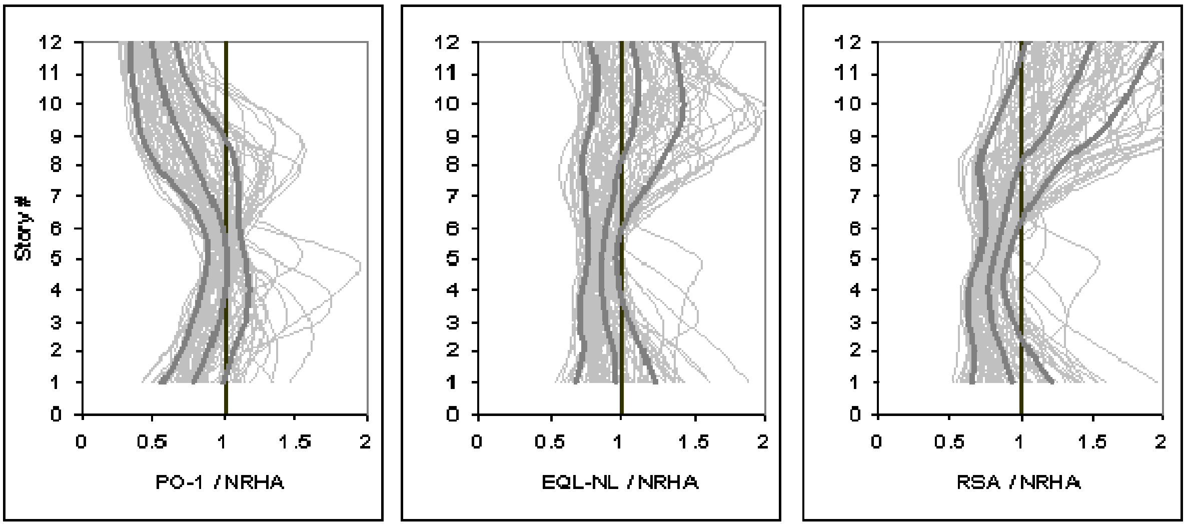

Figure 8. Interstory drift ratios of the approximate methods normalized with those of NRHA for all ground motions

5. CONCLUSIONS

Equivalent linearization procedure is successful in predicting the interstory drift ratios and beam chord rotations. It improves the results of standard response spectrum analysis significantly. Improvements in beam chord rotations are more pronounced than the improvements in interstory drift ratios, especially in members which undergo larger inelastic deformations. Considering the conceptual simplicity and conventional analysis tools used in its implementation, equivalent linearization procedure is an effective tool in predicting inelastic seismic displacement response parameters of regular and irregular structures with sufficient accuracy.

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