FLAT-BOTTOM GRAIN SILOS UNDER EARTHQUAKE GROUND MOTION

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ABSTRACT :

This paper proposes an innovative methodology for the seismic design of flat-bottom silos containing granular, grain-like material. In the general issues concerning the actions provoked by earthquake ground motion on the walls of flat-bottom grain silos, the assessment of the horizontal actions seems to be of particular interest.
Up to date, the horizontal actions due to the seismic event are usually evaluated under the hypotheses (i) of stiff behavior of the silo and its contents and (ii) that the grain mass corresponding to the whole content of the silo except the base cone with an inclination equal to the internal friction angle of the grain is balanced by the horizontal actions provided by the walls (supposing that the seismic force coming from the base cone is balanced by friction and does not push against the walls). The analyses reported here, which are developed by simulating the earthquake ground motion with constant vertical and horizontal accelerations, lead to the subdivision of the ensiled material into three different portions depending on the interaction with the container, by means of plain dynamic equilibrium considerations with reference to the above mentioned accelerations. Two portions push into the silo walls, while the third one does not push into the silo walls. The findings indicate that, in the case of silos characterized by specific (but usual) height/diameter slenderness ratios, the portion of grain mass that interacts with the silo walls turns out to be noticeably lower than the total mass of the grain in the silo.

KEYWORDS: flat-bottom silos, seismic action, grain mass, analytical developments

1. INTRODUCTION

In the general issues concerning the actions provoked by earthquake ground motion on the walls of flat-bottom grain silos, the assessment of the horizontal actions seems to be of particular interest. These actions are usually evaluated under the following hypotheses:
(i). stiff behaviour of the silo and its contents (which means considering the silo and its contents to be subjected to ground accelerations);
(ii). the grain mass corresponding to the whole content of the silo except the base cone with an inclination equal to the internal friction angle of the grain is balanced by the horizontal actions provided by the walls (supposing that the seismic force coming from the base cone is balanced by friction and therefore does not push against the walls).
This design approach is not supported by specific scientific studies; as a matter of fact, even though there are many papers on the behaviour of liquid silos under earthquake ground motion (Hamdan 2000, Nachtigall 2003), there are no examples of scientific investigation into the dynamic behaviour, let alone under earthquake ground motion, of flat-bottom grain silos.
The main goal of the paper is to present analytical developments devoted to the evaluation of the effective behaviour of flat-bottom silos containing grain, as subjected to constant horizontal acceleration and constant vertical acceleration. In more detail, the developments presented here, keeping the validity of the hypothesis (i), aim to assess the effective horizontal actions that rise on the silo walls due to the accelerations, by means of analytical studies and on the basis of dynamic equilibrium considerations. The analyses reported here are developed by simulating the earthquake ground motion with constant vertical and horizontal accelerations (time-history dynamic analyses are not carried out). The results obtained show how these horizontal actions are far lower with respect to those that can be obtained using the hypothesis (ii).
To better understand the physical meaning of the results obtained, a physical representation of the results in terms of portions of grain mass which actually weigh (in terms of horizontal actions) upon the silo walls is also provided, in addition to the analytical expression of the horizontal actions. The results obtained are then used to formulate a procedure for the seismic design of silos.

2. STATIC CONDITIONS

A silo with radius R and filled with grain up to height *H*, as represented in Figure 1, is considered, assuming the hypotheses of (a) horizontal free surface of the grain, (b) absence of vertical additional acceleration and (c) absence of horizontal additional acceleration (with respect to the acceleration of gravity).

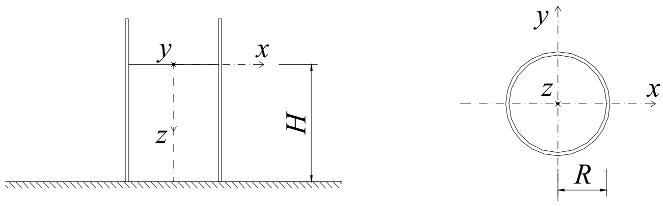

Figure 1. Geometry of a flat-bottom grain silo and reference system adopted

It is well known that (Pozzati and Ceccoli 1972) the grain provides a vertical push onto the silo walls. According to Janssen (1895) and Koenen (1896), for the vertical translational equilibrium of a grain portion at a generic height *z* the vertical pressures at the base of this portion are equally distributed over the whole surface.
It can be hypothesized that the vertical pressures actually tend to diminish from the core of the grain portion until they disappear when the grain meets the silo walls. A limiting schematisation (that will be useful for the assessment, to guarantee safety, of the actions induced on the silo walls by the vertical accelerations, as illustrated in the following sections) is one where the grain is divided into two "equivalent" portions composed of (*i*) grain completely leaning against the layers below (central portion) and (*ii*) grain completely sustained by the walls (and therefore characterized by a null vertical pressure between one grain and another).
Therefore, with reference to a horizontal layer of grain with height *dz* at a generic height *z* measured from the

free surface, it can be divided into two portions:

– an "internal disk" with a diameter of $2r$ (corresponding to the grain leaning against the layers below),
– an "external torus", highlighted in Figure 2 with red hatching, with a thickness s_0 (corresponding to the grain sustained by the walls).

The dimensions of the internal disk and the external torus vary with height z measured from the free surface of the grain, in that the thickness s_0 of the external torus varies with z. An internal disk D_0 with a height dz and a radius $r=R- s_0$ is considered. It is supposed that it is placed at a depth z measured from the free surface of the grain, namely at height $h=H-z$ from the ground. A sector of the circular annulus ("slice" of the external torus) with a central angle $d\theta$ is now considered. The sector of the circular annulus is identified by the central angle *6* measured clockwise from the negative semi-axis of x, as indicated in Figure 2. A system of auxiliary coordinates $\xi-\eta$ on the horizontal plane is also defined, in which ξ represents the radial direction (perpendicular to the lateral surface of the silo) and η represents the direction perpendicular to ξ (tangent to the lateral surface of the silo), as indicated in Figure 2.

In the absence of additional accelerations, each sector of the circular annulus (that is, a portion of the external torus) will later be indicated as element E_0. Each element E_0 has height dz and thickness s_0.

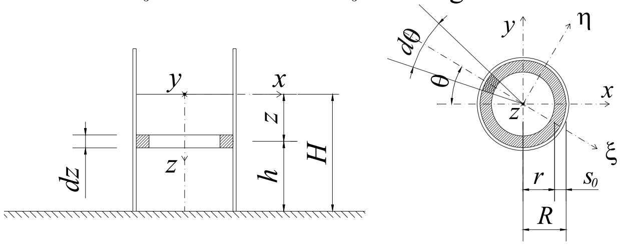

Figure 2. The sector of circular annulus (dark grey hatching) identified by the central angle θ of the external torus of the grain (grey hatching)

The vert. and hor. actions operating on two symmetrical elements E_0 are represented schematically in Fig. 3.

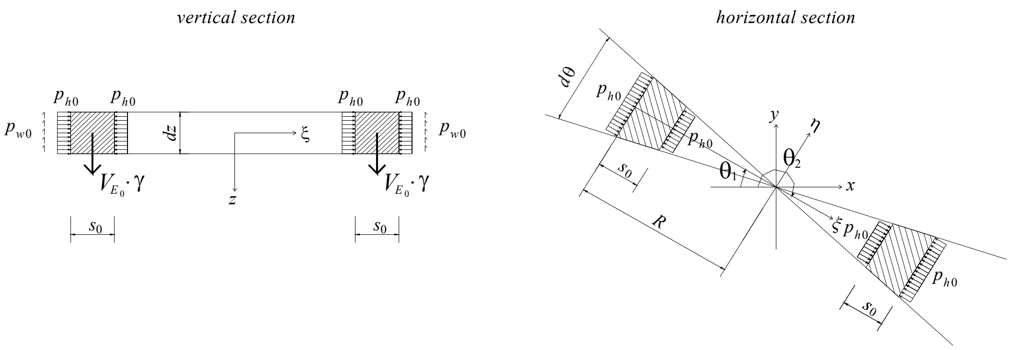

Figure 3. Vertical and horiz. actions operating on two symmetrical elements E_0 under static conditions

Horizontal pressure $p_{h0}(z)$ is equal to (Pozzati and Ceccoli 1972) $p_{h0}(z)=p_{v0}(z)\lambda=\gamma z\lambda$ where λ is the pressure ratio of the grain. Vertical frictional shear stress $p_{w0}(z)$ is given by $p_{w0}(z)= p_{h0}(z)\mu= \gamma z\lambda\mu$ where μ is the friction coefficient between the grain and the silo wall. Analytical developments based upon the dynamic equilibrium conditions of disk D_0 and elements E_0, which are not reported here for sake of conciseness, allow to obtain the mathematical expression of the thickness $s_0(z)$ of the external torus as: $s_0=R-\sqrt{(R^2-2Rp_{h0}\mu/\gamma)}$.

3. ACCELERATED CONDITIONS

The following hypotheses are considered:

1 horizontal free surface of the grain;
2 presence of constant vertical additional acceleration $a_{g\text{-}vert}$ towards z (positive upwards);

3 presence of constant horizontal additional acceleration $a_{g\text{-}orizz}=a_g$ towards x (positive to the right);
4 the inertial forces acting on internal disk D, due to the horizontal additional acceleration, are completely balanced by the resultant of the shear (tangential) stresses developing on the lower surface of the disk (at the disk foundation), which will then be indicated as τ_{base} (absence of horizontal sliding of the grain);
5 horizontal pressure, $p_h(z)$, between element E and disk D in the presence of vertical and horizontal additional accelerations is equal to horizontal pressure, $p_{h0}(z)$, between element E_0 and disk D_0 in the presence of vertical additional acceleration only;
6 negative variation (depression) of the horizontal pressure, $\Delta p_h(z)$, between element E and the silo wall is so that it cannot completely annihilate horizontal pressure, $p_h(z)$, between element E and the silo wall (hypothesis of not annihilating the pressure).

For safety reasons, the analyses are not carried out here with reference to Janssen and Koenen's hypotheses (Pozzati and Ceccoli 1972), but to the subdivision of the grain into disks and elements, which, as will then be seen, implies the rising of additional forces exchanged between the silo wall and the grain, in the presence of horizontal accelerations. In the following analyses, the elements and the disk are indicated with E and D respectively (namely without subscript), in order to differentiate them from the portions identified in the static case (which were indicated with subscript 0). Figures 4 and 5 schematically represent the actions that elements E, element D and the silo walls exchange, due to constant horizontal acceleration a_g and constant vertical acceleration $a_{g\text{-}vert}$, in the following limiting cases, respectively:

- *limiting case A*: every disk is horizontally balanced by means of the transmission of shear actions to disk D below;
- *limiting case B*: every disk is horizontally balanced by means of the transmission of shear actions not only to disk D below, but also to elements E below.

In both figures, the direction of the additional acceleration (towards x) is rotated by an angle θ on the horizontal plane compared to the direction (towards ξ) perpendicular to the external vertical surface of element E.

3.1. Limiting Case A

Figure 4 shows the mutual actions that disk D, elements E and the external walls of the silo exchange. It must be noticed that, in addition to the vertical and horizontal forces present in the case of static load, there are also the vertical and additional horizontal forces reported hereafter:

Δp_h overpressure (or depression) due to the effects of horizontal additional acceleration and to be added to (or subtracted from) p_{h0} between element E and the silo wall;

$a_g V_E \gamma$ inertial force coming from the centre of mass of element E and acting towards x, due to the effect of acceleration a_g (inertial force to the left as acceleration $a_{g\text{-}orizz}$ has been assumed to be positive to the right)

$a_{g\text{-}vert} V_{D0^*} \gamma$ inertial force coming from the centre of mass of disk D_0* and acting towards z due to acceleration $a_{g\text{-}vert}$ (downward inertial force, acceleration $a_{g\text{-}vert}$ having been assumed to be positive upwards).

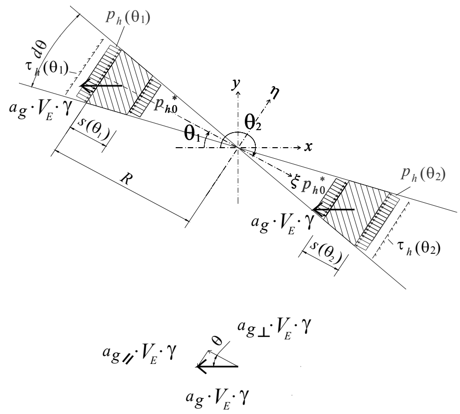

Figure 4. Vertical and horizontal actions operating on disk D and symmetrical elements E placed at different heights in the presence of additional horizontal and vertical acceleration

The free-body diagram shows that:

- as far *the* horizontal translational equilibrium of disk D is concerned, the weight of disk D multiplied by a_g must be equal to the resultant of the horizontal shear stresses that develop on the lower surface of D;
- as far as the horizontal translational equilibrium of element E is concerned, the resultant of the horizontal overpressure Δp_h must be equal to the weight of element E multiplied by a_g;
- as far as the vertical translational equilibrium of disk D is concerned, the resultant of vertical pressures $p_{v0}*$ on the lower surface must be equal to the weight of disk D multiplied by $(1+ a_{g\text{-}vert})$;
- as far the vertical translational equilibrium of element E is concerned, the resultant of vertical frictional shear stresses p_w must be equal to the weight of element E multiplied by $\left(1+a_{g-vert}\right)$.

Element E placed at a generic height z is considered. In the presence of vertical and horizontal additional accelerations, the total pressure, p_h, between element E and the silo wall is given by $p_h = p_{h0}* + \Delta p_h$ and the vertical frictional shear stress, in this case, is given by $p_w = p_h\mu$. As far as the single element E is concerned, the horizontal equilibrium equation along the radial direction (namely, along the normal to the external surface of element E) provides $\Delta p_h \cdot A_E = a_{g\perp} \cdot V_E \cdot \gamma$ where $a_{g\perp}$ represents the component normal to the external vertical surface (towards ξ) of element E (Fig. 6) of horizontal acceleration a_g, which is equal to $a_{g\perp} = a_g\cos\theta$.. The expression of p_h can be obtained as follows:

$$p_h = p_{h0}^* + \Delta p_h = \left[\frac{1}{\nu\left(1-\nu\cdot a_g\cos\theta\cdot\mu\right)}\right]\cdot\gamma\cdot\lambda\cdot z \qquad (3.1.1)$$

The horizontal shear stresses exchanging the external torus of grain and the silo walls are indicated by τ_h. As far as the single element E is concerned, the horizontal equilibrium equation along the tangential direction (namely along the tangent horizontal to the external surface of element E) gives $\tau_h A_E = a_{g\parallel}\gamma V_E$ where $a_{g\parallel}$ represents the component tangent to the external vertical surface (towards η) of element E of horizontal acceleration a_g, that is equal to $a_{g\parallel} = a_g \sin\theta$. It's therefore possible to obtain $s=R-\sqrt{(R^2-\beta Rz)}$.

3.2. Limiting Case B

Figure 5 shows the mutual actions that disk D, elements E and the external walls of the silo exchange. In addition to the mutual actions exchanged by the elements placed in correspondence with the free surface, it is necessary to consider also the horizontal shear stresses τ that the grain portion placed directly above the portion hung by the external walls transmits to the one below, hung by the external walls:

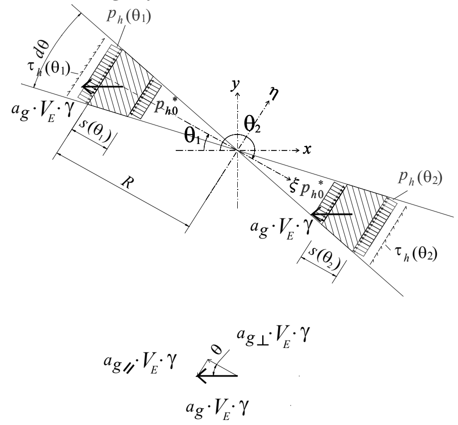

Figure 5. Vertical and horizontal actions operating on disk D and symmetrical elements E placed at different heights in the presence of additional horizontal and vertical acceleration

The free-body diagram shows that:

- as far the horizontal translational equilibrium of disk D is concerned, the weight of disk D multiplied by a_g must be equal to the difference between the resultant of horizontal shear stresses τ that develop on the lower

surface of disk D and the resultant of horizontal shear stresses τ that develop on the upper surface of disk D;
- as far as the horizontal translational equilibrium of element E is concerned, the resultant of horizontal overpressure Δp_h must be equal to the sum of the weight of element E and the weight of the grain portion directly positioned above the portion hung by the external walls multiplied by a_g;
- as far as the vertical translational equilibrium of disk D is concerned, the difference between the resultant of vertical pressures p_{v0}* on the lower surface of disk D and the resultant of vertical pressures p_{v0}* on the upper surface of disk D must be equal to the weight of disk D multiplied by $(1+a_{g\text{-}vert})$;
- as far as the vertical translational equilibrium of element E is concerned, the resultant of vertical frictional shear stresses p_w must be equal to the sum of the weight of element E and the weight of the grain portion directly positioned above the portion hung by the external walls multiplied by $(1+a_{g\text{-}vert})$.

Element E placed at a generic height z is considered. In the presence of vertical and horizontal additional accelerations, total pressure p_h between element E and the silo wall is given by $p_h= p_{h0}*+ \Delta p_h$. The vertical frictional shear stress, in this case, is given by $p_w= p_h\mu$. The expression of p_h is obtained as follows:

$$p_h = p_{h0}^* + \Delta p_h = \left[\frac{1}{\nu\left(1-\nu \cdot a_g \cos\theta \cdot \mu\right)}\right] \cdot \gamma \cdot \lambda \cdot z \qquad (3.2.1)$$

The horizontal shear stresses exchanging between the external torus and the silo walls are indicated by τ_h. As far as the single element E is concerned, the horizontal equilibrium equation along the tangential direction gives:

$$\tau_h \cdot A_E = a_{g//} \cdot \gamma \cdot \left(V_E + \left(R - s - \frac{ds}{2}\right) d\theta \cdot ds \cdot z\right) \qquad (3.2.2)$$

where $a_{g\|}$ represents the component with a direction (direction η) tangential to the external vertical surface of element E (Fig. 11b) of horizontal acceleration a_g, which is equal to $a_{g\|}=a_g sin\theta$. It's therefore possible to obtain $s(z)=R-\sqrt{(R^2-\beta Rz/2)}$.

Both the limiting cases lead to the identification of the same additional horizontal pressures Δp_h thus proving the validity of the result obtained.

3.3. The portions into which the grain can be divided in relations to its behaviour under accelerated conditions

The grain must be divided into different portions in order to make physical sense of the findings obtained on the actions transmitted from the grain to the silo walls under accelerated conditions (vertical and horizontal accelerations), so the designer may immediately understand the developing action of the grain portions on the wall.

Portion A1 of *Limiting Case A* is the amount of grain leaning on the grain below up to the silo foundation. This portion of the grain does not interact with the silo walls. From a geometrical point of view, it coincides with the vertical axis truncated cone (overturned) solid (Fig. 6), with, as a minor base, the one obtained by drawing the curve $\gamma(z,\theta)=R-s(z,\theta)=R-z\beta(\theta)/2$ for $0°\le\theta\le 360°$ on the plane $z=H$ (therefore at the silo foundation) and, as a major base, the silo circumference (placed at height z=0). Portion A2 of *Limiting Case A* is the amount of grain that is completely sustained by the lateral walls of the silo. This portion of the grain interacts with the silo walls. From a geometrical point of view, it coincides with the vertical axis cylindrical annulus (Fig. 6) and thickness $s= s(z,\theta)$, which is variable according to height z and angle θ on the horizontal plane.

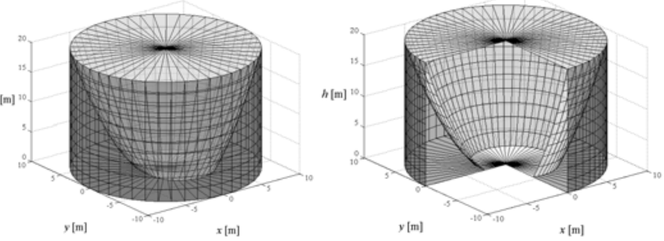

Figure 6. Three-dimensional views of portion A1 (in light grey) and of portion A2 (in dark grey) of the flat-bottom grain silo: overview and sectioned view

Portion B1 of *Limiting Case B* is the amount of grain leaning on the grain below up to the silo foundation. This portion of the grain does not interact with the silo walls. From a geometrical point of view, it coincides with the vertical axis cylinder (Fig. 8) that can be obtained by translating along the *z*-direction the plane geometrical figure which can be obtained by drawing curve $\gamma(z,\theta)=R-s(z,\theta)=R-z\beta(\theta)/2$ for $0°\leq\theta\leq 360°$ on the plane $z=H$ (therefore, at the silo foundation). Portion B2 of *Limiting Case B* is the amount of grain that is completely and directly sustained by the lateral walls of the silo. This portion of the grain interacts with the silo walls. From a geometrical point of view, it coincides with the vertical axis cylindrical annulus (represented in red in Fig. 7) and thickness $s=s(z,\theta)$, which is variable according to height z and angle θ on the horizontal plane.
Portion B3 of Limiting Case B is the amount of grain that is completely and indirectly (through the grain portion B2) sustained by the lateral walls of the silo. This portion of the grain interacts with the silo walls indirectly. From a geometrical point of view, it coincides with the vertical axis cylindrical annulus (Fig. 7) and thickness $t(z,\theta)=s(z=H,\theta)-s(z,\theta)$, which is variable according to height z and angle θ on the horizontal plane. This portion represents the remaining part of the grain cylinder once portions B1 and B2 are taken away.

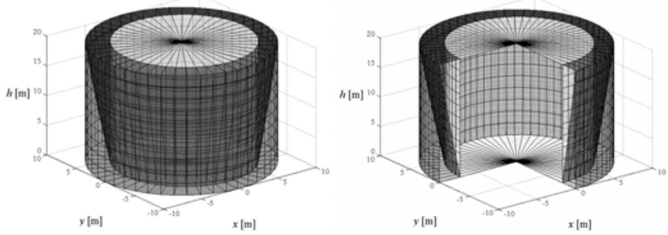

Figure 7. Three-dimensional views of portion B1 (in light grey) and of portion B2 (in medium-light grey) and of portion B3 (in dark grey) of the flat-bottom grain silo: overview and sectioned view

4. METHODOLOGY PROPOSED FOR THE ASSESMENT OF SEISMIC ACTION ON FLAT-BOTTOM GRAIN SILOS

Structures characterized by high values of vertical and horizontal stiffness, such as silos, do not amplify or diminish the acceleration induced by the earthquake ground motion at their base and they are therefore subjected to the stresses caused by the inertial forces that arise due to the accelerations at the base that, during seismic events, vary with continuity in time. It is clear that, in the case of very stiff structures, for which the hypothesis of the accelerations not amplifying is considered valid, the highest stresses induced by seismic action are those that derive from the maximum ground acceleration PGA (Peak Ground Acceleration).
Therefore, as far as the seismic design of silos is concerned, it is reasonable to schematise, as adopted in current design practice, the action induced by earthquake ground motion as a couple of horizontal and vertical additional accelerations, which we can indicate (in conformity with the analytical development reported in the first part of this paper), respectively as $a_{g\text{-}orizz}$ and $a_{g\text{-}vert}$.
It is a generally acknowledged hypothesis in seismic analysis of structures that $PGA_o=PGA_v=PGA$. In addition, when the horizontal acceleration reaches its maximum value, the vertical one is equal to 30% of its maximum value and vice versa. Therefore, the design verifications can be reasonably carried out with reference to the condition of max $a_{g\text{-}orizz}$ and 30% of the max $a_{g\text{-}vert}$ and to the condition of max $a_{g\text{-}vert}$ and 30% of the max $a_{g\text{-}orizz}$.
By schematising seismic action on silos as two additional, constant, horizontal and vertical accelerations equal to $a_{g\text{-}orizz}$ and $a_{g\text{-}vert}$ their effect can be evaluated with reference to the results obtained through the analytical elaborations described in previous sections, which indicate that the following actions develop on the silo walls:
- horizontal radial pressure (with positive direction towards the outside of the silo) $p_h=\beta'\gamma z\lambda$;
- horizontal shear stress $\tau_h=\beta'\gamma z\lambda\tan\theta$;
- total vertical frictional shear stress $p_w=\beta'\gamma z\lambda\mu$, where $\beta'=\gamma(1+a_{g\text{-}vert})^2/(1+a_{g\text{-}vert}-a_{g\text{-}orizz}\mu\cos\theta)$, γ is the specific weight (density) of the grain, z is the height considered, starting from the free surface of the grain (with positive downward direction), λ is the pressure ratio of the material, μ is the friction coefficient between the grain and the lateral silo wall, θ is the angle between the entrance direction of the earthquake ground motion (horizontal acceleration) and the perpendicular to the portion of the lateral silo surface considered;
- all the accelerations are to be expressed in fractions of g (acceleration of gravity).

It must be highlighted that the actions developing on the silo walls are not axial-symmetrical. The base shear is

$$T = a_g \cdot \gamma \cdot \pi R H^2 \left(\frac{\lambda \cdot \mu}{1 - \nu^2 \cdot a_g{}^2 \cdot \mu^2} \right) \tag{4.1}$$

Traditional calculation of flat-bottom silos would lead to the following actions at the silo foundation equal to $T_{trad.} = a_g \cdot \gamma \cdot V_{tot} = a_g \cdot \gamma \cdot \pi R^2 H$. For the immediate assessment of the benefits that the methodology presented gives with respect to traditional calculation, it is appropriate to define the following ratio between the shear obtained from the formulation presented and that obtained from traditional calculation:

$$\rho_T = \frac{T}{T_{trad.}} = \frac{H}{R} \left(\frac{\lambda \cdot \mu}{1 - \nu^2 \cdot a_g{}^2 \cdot \mu^2} \right) \tag{4.2}$$

As an illustrative example, for a silo (*i*) characterized by a height equal to the diameter containing granular material with pressure ratio λ=0,5 and grain-silo wall friction coefficient μ=0,37 and (*ii*) subjected to the horizontal additional acceleration a_g=0,3g and the vertical additional acceleration $a_{g\text{-}vert}$=0,1g, the portion of grain mass interacting with the silo walls proves to be equal to 37% of the total grain mass.
Hypothesis 4 reported in section 3, proves to be confirmed when:

$$PGA \le \frac{\mu_{base}}{(1 + 0.3 \cdot \mu_{base})} \tag{4.3}$$

As far as the limit slenderness ratios for which the maximum thickness of the portion of the grain sustained by the walls is equal to the radius of the silo are concerned, the following condition should be verified:

$$\Delta = \frac{H}{2R} < \frac{1 - \nu \cdot a_g \cdot \mu}{2 \cdot \lambda \cdot \mu} \tag{4.4}$$

5. CONCLUSIONS

In this paper, the actions provoked by earthquake ground motions on the walls of flat-bottom grain silos have been studied analytically. In the first part, the mutual actions exchanged between the grain and the silo walls under static and accelerated conditions (constant vertical acceleration and constant horizontal acceleration) have been assessed. In the second part, the analytical results obtained in the first part have been used to formulate some synthetic suggestions aimed at assessing seismic effects on the walls of flat-bottom grain silos. The results indicate that, in the case of silos characterized by specific (but normal) height/diameter slenderness ratios, the portion of grain mass interacting with the silo walls proves to be noticeably lower than the total grain mass contained in the silo.

ACKNOWLEDGEMENTS

Financial supports of Department of Civil Protection (Reluis 2005 Grant – Task 2: “Assessment and mitigation of the vulnerability of r.c. existing buildings”) is gratefully acknowledged.

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